SNS-B COURSE: BRAID GROUPS AND MAPPPING CLASS GROUPS

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Master Program: Fundamental structures and applications to algebra, geometry and topology. Level:M1

Course Outline:

Mapping Class Groups are fundamental objects in Geometry, Topology, Analysis, and Algebra. The course aims to introduce the readers to the basic properties of Mapping Class Groups, starting with the particular case of Braid Groups, and to prepare them to be able to read advanced courses and graduate-level literature in hyperbolic geometry, geometric topology, Teichmüller theory, moduli spaces, dynamics of homeomorphisms, etc.

There are two main references for this course proposal:

- Christian Kassel, Vladimir Turaev, *Braid Groups*, Graduate Texts in Mathematics **247**, Springer (2008), ISBN 978-0-387-33841-5
- Benson Farb, Dan Margalit, *A Primer on Mapping Class Groups*, Princeton University Press (2012), ISBN 978-0-691-14794-9

In addition, some very specific material would be used from:

- Allen Hatcher, *Algebraic Topology* (2000)
- W. B. Raymond Lickorish, *An Introduction to Knot Theory*, Graduate Texts in Mathematics **175**, Springer (1997), ISBN 0-387-98254-X

There would be 24 courses of 2 hours each. All but the first and last course would be 1 hour of lecture + 1 hour of problems. Outline:

1. Algebraic and geometric braid groups [KT, §1.1, 1.2]
2. Pure braid groups [KT, §1.3]

3-4. Covering spaces, fundamental group, fibre buundles, $K(\pi, 1)$ [KT, Appendix A; L, §7, 11; H, §1]

5. Configurations spaces, braid automorphisms of free groups [KT, §1.4, 1.5]
6. Braid groups as mapping class groups [KT, §1.6, 1.7]

7-8. Homotopy, isotopy, transversality, homology groups (we shall need only H_1 and H_2) [H, §2]

9. Knots, links, Seifert surfaces [KT, §2.1; L, §2]
10. Closed braids, Alexander's theorem [KT, §2.2, 2.3]
11. Links as closures of braids, Markov's theorem [KT, §2.4, 2.5.1]
12. Burau representation, 1 [KT, §3.1, 3.2.1-3.2.2]
13. Burau representation, 2 [KT, §3.2.3-3.2.5, 3.3]
14. Alexander-Conway polynomial [KT, §3.4]

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15. Lawrence-Krammer-Bigelow representation (an outline) [KT, §3.5]
16. Symmetric groups [[KT, §4.1]
17. Young diagrams [KT, §5.1, 5.2]

18-19. Poincare models of 2-dimensional hyperbolic geometry

20. Simple closed curves on compact connected surfaces [FM, §1.2-1.4]
21. The Mapping Class Group of the torus [FM, §2.1, 2.2]
22. Dehn twists and half-Dehn twists [FM, §3, 9]
23. The complex of curves [FM, §4]