Application of the Wang-Landau algorithm to the 3-Matrix-Model

Thomas Kaltenbrunner

Dublin Institute for Advanced Studies
School of Theoretical Physics

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Outline

Properties of 3-Matrix-Model

$$S[X] = NTr\{-\frac{1}{4}[X_a, X_b]^2 + \frac{i\alpha}{3}\epsilon_{abc}X_a[X_b, X_c]\}$$

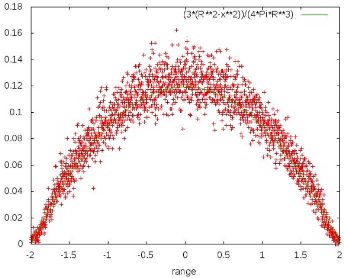
- X_a's NxN matrices, where $a, b = 1, 2, 3$
- equations of motion:
  $\frac{\delta}{\delta X_a}$: $Tr[X_b, -[X_a, X_b] + i\alpha\epsilon_{abc}X_c] \stackrel{!}{=} 0 \rightarrow X_a = \alpha L_a$
- L_a's are SU(2) generators of N-dimensional irreducible representation $[L_a, L_b] = i\epsilon_{abc}L_c$
  form one set of solutions (another e.g.: commuting matrices)
- classical solution: $S[L] = -N\alpha^4 \sum_L d(L)\frac{C_2^{su(2)}(L)}{6}$
- $\tilde{\alpha}^4 = \beta$ interpreted as inverse temperature, where $\tilde{\alpha} = \sqrt{N}\alpha$

Matrix Phase I

- high-temperature phase
- system governed by Yang-Mills term
- matrices have random entries, distributed around zero
- ground state has positive energy with solutions around commuting saddle point
- no background geometry

Matrix Phase II

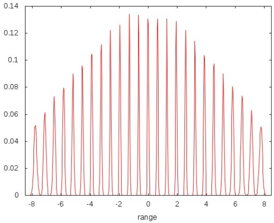
- EV-distribution continuous within solid ball
- distribution fits parabola $f(x) = \frac{3(R^2 - x^2)}{4\pi R^3}$



Eigenvalue Distribution for X_3 of size 25x25

Fuzzy Phase (I)

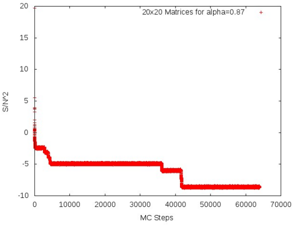
- Low-temperature phase
- Chern-Simons Term becomes important
- Geometry of a fuzzy sphere emerges
- ground state with negative energy
- forms fuzzy S^2 with radius $R = \frac{1}{2}\alpha\sqrt{N^2-1}$
- EV's distribute around fuzzy sphere due to SU(2)
- classical sphere emerges when $N \to \infty$

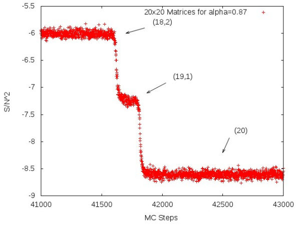


Eigenvalue distribution for X_3 of size 25x25

Fuzzy Phase (II)

- multi fuzzy sphere solutions possible $X_a = \alpha L_a^{(k)} \otimes 1_k$ where $N = \sum_{i=2}^{N} n_i k_i$ with n fuzzy spheres of size i
- ground state given by the N-dimensional irrep of SU(2)





Adding Fluctuations

- expand around minimum: $X_a \to X_a = \alpha\phi L_a + A_a$
- solve $Z = \int dX_a e^{-S[X]}$ up to first order
- integrate out, divide by N^2 and let $N \to \infty$ to find F $=$ -$\frac{1}{\beta}$lnZ
  $\frac{F}{N^2} = \frac{3}{4}\ln[\tilde{\alpha}^4] + \frac{\tilde{\alpha}^4}{2}\left(\frac{\phi^4}{4} - \frac{\phi^3}{3}\right) + \ln[\phi]$ where $\tilde{\alpha} = \sqrt{N}\alpha$
- find $\tilde{\alpha}_{crit} \simeq 2.087$
- average of action:
  $\frac{<S>}{N^2} = \tilde{\alpha}^4 \frac{d}{d\tilde{\alpha}^4}\left(\frac{F}{N^2}\right) = \frac{3}{4} - \frac{\tilde{\alpha}\phi^3}{24}$
- Entropy for $\tilde{\alpha} = 0$, $S = \frac{3}{4}$
  for $\tilde{\alpha} = \tilde{\alpha}_c$, $S = \frac{5}{12}$
- jump in Entropy at $\tilde{\alpha}_{crit}$ with $\Delta S = \frac{1}{9}$ per DoF

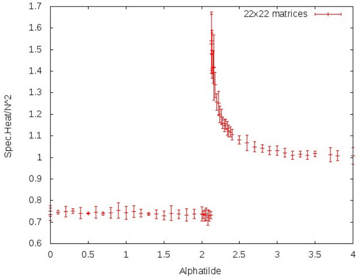
Specific Heat

when approaching from low temperature phase:

- jump in specific heat; diverges at point of phase transition

when approaching from high temperature phase:

- no divergent specific heat



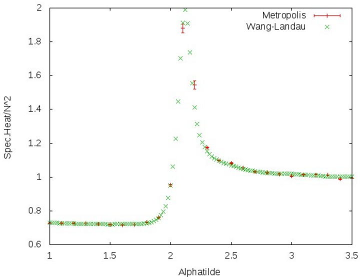
Metropolis-Algorithm

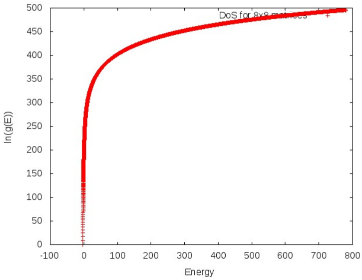
- generates Markov chain of microstates
- accept new state with probability $P = min(1, e^{-(E_{old} - E_{new})})$
- system will eventually decay to ground state
- the set of microstates obtained in the simulation constitutes an estimation of the canonical ensemble of the system
- can calculate macroscopical properties by averaging over this set of microstates i with inner energy E_i

Wang-Landau-Algorithm: Main Features

- estimates Density of States of system $Z = \sum_{\text{states i}} e^{-\beta E_i} = \sum_E g(E) e^{-\beta E}$
- can compute partition function and thus the free energy of the system
- made for systems with 1st order phase transition
- introduces bias for visited energies $\rightarrow$ forces system in regions of energy space that have not been probed yet; enhances "tunneling probability"
- developed in solid state physics; first applied to spin systems which have discrete parameter E
- in our system continuous parameter E $\rightarrow$ have to make bins small enough to capture all important properties of the system

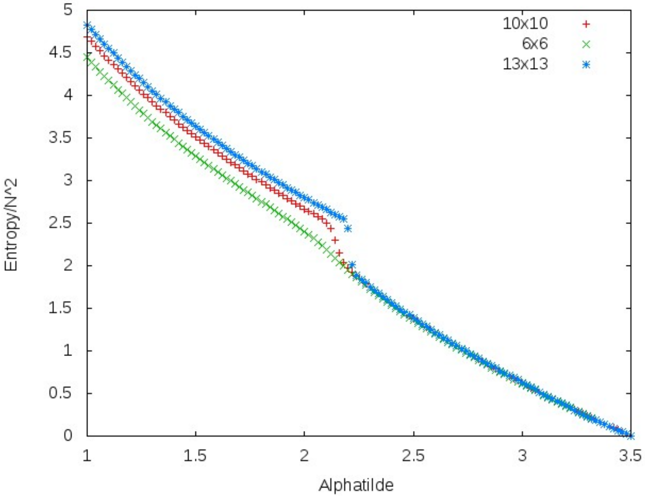
Comparison with Metropolis algorithm



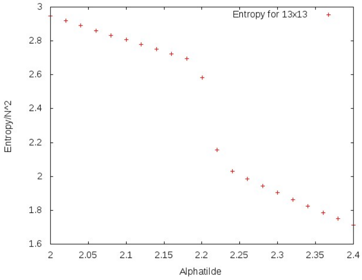


- graphs show specific heat and density of states for 8x8 matrices
- agreement around $\tilde{\alpha}_{crit}$ very good

Results for Entropy



Estimation of ΔS



- taking values from $\tilde{\alpha} = 2.20$ to $\tilde{\alpha} = 2.22$, $\Delta S \simeq 0.14$ while $1/9 \simeq 0.11$
- not possible to distinguish between 1^{st} and 2^{nd} order phase transition
- would need to test for bigger matrices

Conclusion and Outlook

- Wang-Landau allows to measure entropy directly from simulation data
- specific heat fits good with metropolis results
- confirms jump in entropy

- improve algorithm so states near the ground state in fuzzy phase are visited more often
- test for bigger matrix sizes
- behaviour of 1^{st} or 2^{nd} order phase transition?
- determine ΔS more accurately

References

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