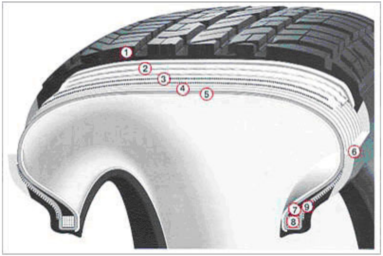
Dynamics of Pneumatic Tyres and Linear Operators

R.S. Yakushev

Department of Theoretical Mechanics, Kazan Federal University, Republic of Tatarstan, Russian Federation

03.07.2010

A tyre is a complex construction consisting of several areas. Their materials have different mechanical properties.



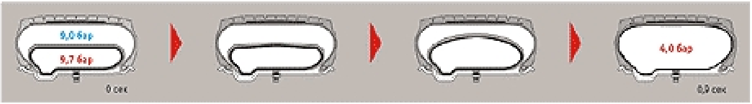
A tyre is a complex construction consisting of several zones whose materials have different mechanical properties.

Constructive elements of the tyre:

- 1.*Protector*;
- 2.*Bandage*;
- 3.*Steelcordbelt* − *layers*;
- 4.*Textilecordgaskets*;
- 5.*Innerlayer*;
- 6.*Sidebands*;
- 7.*Foiltape*;
- 8.*Steelring*;
- 9.*Sideprotectivetape*.

Basic tyre elements – carcass and breaker – are made of the cord-based composite.

The complexities of the tyre mathematical modeling depend on many factors:
physical and chemical properties of the materials;
constructive characteristics of tyres, e.g. the ratio between height and width of the tyre profile;
the tyre cross-section form;
the mixed-layer carcass cord and breaker threads intersection angle;
additional safety cameras, i.e. autonomous closed cavities; etc.



In the previous works the authors considered the tyre as elastic ring in the simplified model or as multi-layer shell made of different materials in more complex model.

The complexity of the problem defines the solution's methods, that are mainly numerical methods, such as FEM, method of local variations and finite differences method.

In our work we use the spatial setting of the problem. (We proceed from setting the problem of the elasticity theory.)

The stress-strain state of the pneumatic tyre govern by a non-linear equations.
Following [1] we take the pneumatic tyre material relations as operator equations:
$\sigma_{ij} = \check{F}_{ij}(\tilde{\varepsilon}, \tilde{\chi}, T, \vec{x}),$

In our work we use the spatial setting of the problem. (We proceed from setting the problem of the elasticity theory.)

The stress-strain state of the pneumatic tyre govern by a non-linear equations.
Following [1] we take the pneumatic tyre material relations as operator equations:
$\sigma_{ij} = \check{F}_{ij}(\tilde{\varepsilon}, \tilde{\chi}, T, \vec{x})$,

where $\tilde{\chi}$ is destruction tensor.
1. B. E. Pobedria. Mechanics of Composite Materials (Moscow: MGU, 1984) (russian).

This means that for any given moment of time t six independent components of the symmetrical stress tensor components $\tilde{\sigma}$ can be expressed through the six independent symmetrical strain tensor $\tilde{\varepsilon}$ not only at the fixed time t but also at any moment τ preceding this moment $0 \leq \tau \leq t$.

This means that for any given moment of time t six independent components of the symmetrical stress tensor components $\tilde{\sigma}$ can be expressed through the six independent symmetrical strain tensor $\tilde{\varepsilon}$ not only at the fixed time t but also at any moment τ preceding this moment $0 \leq \tau \leq t$.

Thus the stress tensor is determined by all the strain history. The radius-vector $\vec{x}$ expresses the dependence of the operator on the material points coordinates (i.e. the non-homogeneity of the homogenous material and discontinuity of the material functions).

Moreover in order to take into account the "destructions"accumulated in the material under tyre exploitation and interaction with the environment we use the macro-object $\tilde{\chi}$ "Defect"introduced by A.A. Il'ushin in [2].

2. A. A. Ilushin and B. E. Pobedria. Mathematical fundamentals of thermoviscoelastic theory. Moscow: Nauka, 1970. (russian).

This object is the operator reflecting stress process
$\chi_{ij} = \check{\Phi}_{ij}(\tilde{\sigma}, T, \vec{x})$.
Now using the strain tensor instead of stress one we obtain
$\chi_{ij} = \check{\chi}_{ij}(\tilde{\varepsilon}, T, \vec{x})$.
Assume also that the defining relations can be resolved with respect to strain tensor components:
$\varepsilon_{ij} = \check{K}_{ij}(\tilde{\sigma}, \tilde{\chi}, T, \vec{x})$.
Here $\check{K}_{ij}$ is tensor mutually inverse with $\check{F}_{ij}$.

So, the material structure senescence is important for this mathematical model.

There exist several models used for phenomenological description of this senescence [3-4].
It seems natural to introduce the moment strains μ^{ij} in addition to the usual Piola ones for the polar or Cosserat [5] medium.

3. A.I. Lur'e. Nonlinear Theory of Elasticity. Moscow: Nauka, 1980.

4. B.E. Pobedria. On the structural change recognition in composite mechanics // Composite material construction. 2000. n.2. pp. 19-25. (russian).

5. E. Cosserat, F. Cosserat. Theorie descorpes deformables. Paris: Herman, 1909.

Kinematics of such a medium demands for an inclusion of the rotation vector $\vec{\omega} = \omega^i e_i$,
the spin-vector [6] $\vec{M} = \frac{d\vec{\omega}}{dt}$.

6. W. Nowacki. Teoria niesymetrycznej sprezystosci. Warszawa: PWN, 1981.

Kinematics of such a medium demands for an inclusion of the rotation vector $\vec{\omega} = \omega^i e_i$,
the spin-vector [6] $\vec{M} = \frac{d\vec{\omega}}{dt}$.

6. W. Nowacki. Teoria niesymetrycznej sprezystosci. Warszawa: PWN, 1981.

and rotation vector gradient (bending-torsion tensor) [7] κ_{ij}.
The motion moment and kinetic moment postulates imply the continuum motion equations:

$$\rho \frac{d^2 u^j}{dt^2} = \rho f^j + \nabla_i p^{ij},$$
$$J \frac{d^2 \omega^j}{dt^2} = J M^j + \nabla_i \mu^{ij} + \epsilon^{jkl} p_{kl} / \sqrt{g}.$$

7. R. de Wit. Continuum Theory of Dislocations. Moscow: Mir, 1977. (russian).

The thermodynamics laws at the same time imply the heat equation, which is the seventh equation of the system:

$$\rho c_p \frac{dT}{dt} = \rho q + w^* + \Lambda^{ij} T_j \nabla_i (\Lambda^{ij} T_{,j} \nabla_j T) - T \frac{d}{dt}[\alpha_{ij} p^{ij}(f) + \beta_{ij} \mu^{ij}(f)].$$

The thermodynamics laws at the same time imply the heat equation, which is the seventh equation of the system:

$$\rho c_p \frac{dT}{dt} = \rho q + w^* + \Lambda^{ij} T_j \nabla_i (\Lambda^{ij} T_{,j} \nabla_j T) - T \frac{d}{dt}[\alpha_{ij} p^{ij}(f) + \beta_{ij} \mu^{ij}(f)].$$

The formulation of the problem

The thermodynamics laws at the same time imply the heat equation, which is the seventh equation of the system:

$$\rho c_p \frac{dT}{dt} = \rho q + w^* + \Lambda^{ij} T_j \nabla_i (\Lambda^{ij} T_{,j} \nabla_j T) - T \frac{d}{dt}[\alpha_{ij} p^{ij}(f) + \beta_{ij} \mu^{ij}(f)].$$

The formulation of the problem

also includes the initial conditions for $t = t_0$:

$$T = T^0, \quad u^j = U_0^j, \quad \frac{du^j}{dt} = V_0^j, \quad \omega^j = \Omega_0^j, \quad \varphi^j = \Phi_0^j;$$

and boundary conditions at the surface of the tyre $\sum = \sum_1 \bigcup \sum_2$:

$u^j|_{\Sigma_1} = u_0^j, \quad p^{ij}|_{\Sigma_2} = s_0^j,$

and boundary conditions at the surface of the tyre $\sum = \sum_1 \bigcup \sum_2$:

$$u^j|_{\Sigma_1} = u_0^j, \quad p^{ij}|_{\Sigma_2} = s_0^j,$$

$$\omega^j|_{\Sigma_1} = \omega_0^j, \quad \mu^{ij}|_{\Sigma_2} = S_0^j,$$

and boundary conditions at the surface of the tyre $\sum = \sum_1 \bigcup \sum_2$:
$u^j|_{\Sigma_1} = u_0^j, \quad p^{ij}|_{\Sigma_2} = s_0^j,$

$$\omega^j|_{\Sigma_1} = \omega_0^j, \quad \mu^{ij}|_{\Sigma_2} = S_0^j,$$

$\Lambda^{ij} \nabla_j T n_i|_\Sigma = \eta(T|_\Sigma - T_c^0).$
Hence, we reduce the problem to solution of seven differential equations under the given initial and boundary conditions.

Note that the material functions are discontinuous coordinate functions.

Thus we can find only the generalized solution of the given problem.

Note that the material functions are discontinuous coordinate functions.

Thus we can find only the generalized solution of the given problem.

We introduce the geometrical scaling parameter α
and "fast"coordinates ς^i
which we use together with the usual Lagrange "slow"coordinates ξ^i.

This allows us to reduce the solution of the given problem to solution of two recurrent sequence of problems.

The first of these sequences consists of the problems for homogeneous medium with the effective characteristics.
The second sequences contains problems for heterogeneous medium for the composite structural element domain.

This allows us to reduce the solution of the given problem to solution of two recurrent sequence of problems.

The first of these sequences consists of the problems for homogeneous medium with the effective characteristics.
The second sequences contains problems for heterogeneous medium for the composite structural element domain.

Note that only the zero approximation element of each of these sequences is a non-linear problem. All the successive elements of both of the sequences are linear.

We obtain the average tyre material characteristics after we meet the compatibility conditions while passing from one element to another.

We obtain the average tyre material characteristics after we meet the compatibility conditions while passing from one element to another.

Unlike the effective modulus theory, zero-approximation theory allows us to describe discontinuities in the tangential stresses and heat flows.

We obtain the average tyre material characteristics after we meet the compatibility conditions while passing from one element to another.

Unlike the effective modulus theory, zero-approximation theory allows us to describe discontinuities in the tangential stresses and heat flows.

Thanks you for your kind attention!

We obtain the average tyre material characteristics after we meet the compatibility conditions while passing from one element to another.

Unlike the effective modulus theory, zero-approximation theory allows us to describe discontinuities in the tangential stresses and heat flows.

Thanks you for your kind attention!