FMI, CS, Master I
Winter Term 2015/2016
Techniques of Combinatorial Optimization
Laurenţiu Leuştean

14.01.2016

Seminar 3

(S3.1) Figure 1 represents a flow network $N = (D, c, s, t)$.

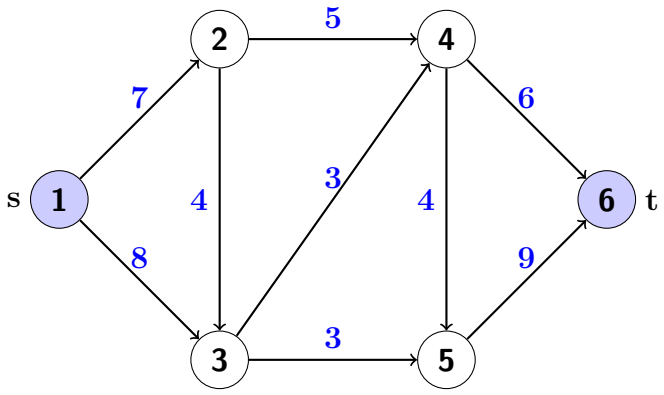

Figure 1: The flow network N

Write the corresponding digraph D and the capacity function c.

(S3.2) Find vectors b, d and a matrix B such that

$$\max\{\text{value}(f) \mid f \text{ is an } s-t \text{ flow for } N\} = \max\{d^T f \mid Bf \leq b\}.$$

(S3.3) Figure 2 represents an s-t flow f for the network N.

(i) Verify that f is an s-t flow. What is the value of f?

(ii) Show that the set $\{(2,4),(3,4),(3,5)\}$ is an s-t cut and compute its capacity.

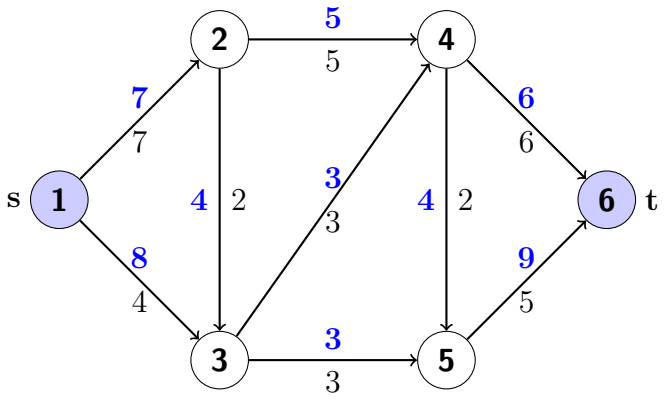

Figure 2: The flow network N with the flow f

(iii) Prove that f is a maximum flow.

(S3.4) Give two iterations of the Ford-Fulkerson algorithm for the flow network N, considering the path $P = 1246$ for the first augmentation and $Q = 1356$ for the second augmentation.

(S3.5) Figure 3 represents a flow network N' and an s-t flow f for N'.

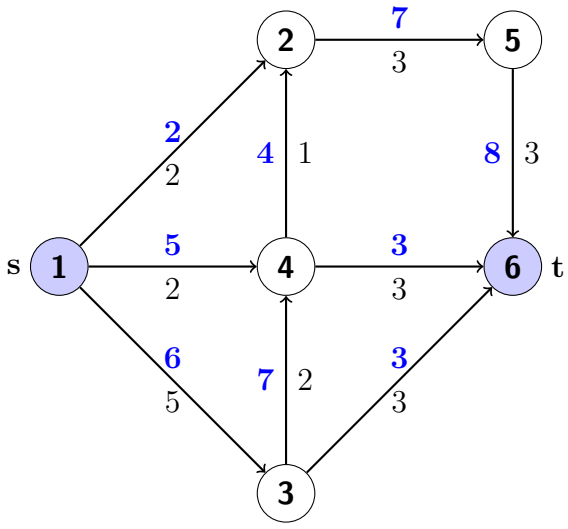

Figure 3: The flow network N' with the flow f

(i) Represent the residual graph D_f and the residual capacities c_f.

(ii) Choose an f-augmenting path P of minimum length and compute the flow $g := f_P^\gamma$, where $\gamma = \min_{e \in A(P)} c_f(e)$.

(iii) Represent the residual graph D_g and the residual capacities c_g. Can you find an s-t path in D_g?

(iv) What is the maximum value of an s-t flow for N'?

(v) Give an example of an s-t cut in N' of minimum capacity.