FMI, CS, Master I
Winter Term 2015/2016
Techniques of Combinatorial Optimization
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07.01.2016

Seminar 2

(S2.1) Let A be the incidence matrix of a cycle of length 5. Prove that A is a square matrix and compute its determinant.

(S2.2) Verify if the following matrices are totally unimodular:

$$A = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix}$$

(S2.3) Let $A \in \mathbb{R}^{m\times n}, b \in \mathbb{R}^m$ and $c \in \mathbb{R}^n$. Then

$$\max\{c^T x \mid x \geq \mathbf{0}, Ax \leq b\} = \min\{b^T y \mid y \geq \mathbf{0}, y^T A \geq c^T\}.$$

(assuming both sets are nonempty).
(This is Proposition 2.1.1.)

(S2.4) Let G be a graph. Prove that

$$\max\{|M| \mid M \text{ is a matching of } G\} \leq \min\{|S| \mid S \text{ is a vertex cover of } G\}.$$

Show that the complete graph K_3 is an example of a graph where strict inequality holds.