

## Seminar 1

(S1.1) Any polyhedron is a convex set.

(S1.2) Prove that

(i) Affine sets are polyhedra.

(ii) Singletons are polyhedra of dimension 0.

(iii) Lines are polyhedra of dimension 1.

(iv) The unit cube  $C_3 = \{x \in \mathbb{R}^n \mid 0 \leq x_i \leq 1 \text{ for all } i = 1, 2, 3\}$  in  $\mathbb{R}^3$  is a full-dimensional polyhedron.

(S1.3) [Farkas lemma - variant] The system  $Ax = b$  has a solution  $x \geq \mathbf{0}$  if and only if  $y^T b \geq 0$  for each  $y \in \mathbb{R}^m$  with  $y^T A \geq \mathbf{0}^T$ .

(S1.4) [Farkas lemma - variant] The system  $Ax \leq b$  has a solution if and only if  $y^T b \geq 0$  for each  $y \geq \mathbf{0}$  with  $y^T A = \mathbf{0}^T$ .