

## IMAR Monthly Lectures

# Totally geodesic sets and Carathéodory geometry in the polydisk

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**Abstract:** The Carathéodory and Kobayashi metrics are obtained by exporting the hyperbolic metric on the disk to general domains  $\Omega$  in  $\mathbb{C}^n$ , and are important in understanding complex geometry. If  $V$  is a subvariety of  $\Omega$ , it has its own intrinsic Carathéodory and Kobayashi metrics; when do these coincide with the ones inherited from  $\Omega$ ? This is an analogue in complex geometry of asking when a submanifold of a Riemannian manifold is totally geodesic, and it only occurs when there is a tight relationship between  $V$  and  $\Omega$ . For example, if  $\Omega$  is the ball, then  $V$  must be a retract.

We shall examine what happens when  $\Omega$  is the polydisk. It turns out that every such  $V$  is built out of disks and a very special two dimensional set

$$\mathcal{K} = \{(x, y, z) \in \mathbb{D}^3 : x + y + z = xy + yz + xz\}.$$

I shall explain all these notions, and describe how one builds the general  $V$  out of the disk and  $\mathcal{K}$ . I will not assume any knowledge of complex geometry, except for the Schwarz lemma.

This is joint work with Ł. Kosiński.