

IMAR Monthly Lectures

Totally geodesic sets and Carathéodory geometry in the polydisk

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Abstract: The Carathéodory and Kobayashi metrics are obtained by exporting the hyperbolic metric on the disk to general domains Ω in \mathbb{C}^n , and are important in understanding complex geometry. If V is a subvariety of Ω , it has its own intrinsic Carathéodory and Kobayashi metrics; when do these coincide with the ones inherited from Ω ? This is an analogue in complex geometry of asking when a submanifold of a Riemannian manifold is totally geodesic, and it only occurs when there is a tight relationship between V and Ω . For example, if Ω is the ball, then V must be a retract.

We shall examine what happens when Ω is the polydisk. It turns out that every such V is built out of disks and a very special two dimensional set

$$\mathcal{K} = \{(x, y, z) \in \mathbb{D}^3 : x + y + z = xy + yz + xz\}.$$

I shall explain all these notions, and describe how one builds the general V out of the disk and \mathcal{K} . I will not assume any knowledge of complex geometry, except for the Schwarz lemma.

This is joint work with Ł. Kosiński.