

IMAR Invited Talk

Potential Theory Seminar

Carleson operator hierarchies and the LGC methodology

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Tuesday, December 16, 2025, 14:00

IMAR, *Miron Nicolescu* Amphitheater

Abstract: This general audience talk consists of two parts:

- *in the first part*, in the increasing order of complexity, for $n \in \{1, 2, \dots\}$ and $\vec{\alpha} := (\alpha_1, \dots, \alpha_n) \in (0, \infty)^n$, we build a hierarchical structure within which we elaborate on the deep and subtle interplay between the following two classes of operators:

– 1st class: the $(n-1)$ -linear $\vec{\alpha}$ -Carleson operator $C_{n-1, \vec{\alpha}}$, defined as¹

$$(1) \quad C_{n-1, \vec{\alpha}}(f_1, \dots, f_{n-1})(x) := \sup_{\lambda \in \mathbb{R}} \left| \text{p.v.} \int_{\mathbb{R}} \prod_{j=1}^{n-1} f_j(x - t^{\alpha_j}) \frac{e^{i\lambda t^{\alpha_n}}}{t} dt \right|;$$

¹Here, for $n=1$ we use the convention $\prod_{j=1}^0 f_j \equiv 1$.

– 2nd class: the n -linear $\vec{\alpha}$ -Hilbert transform $H_{n,\vec{\alpha}}$, defined as

$$(2) \quad H_{n,\vec{\alpha}}(f_1, \dots, f_n)(x) := \text{p.v.} \int_{\mathbb{R}} \prod_{j=1}^n f_j(x - t^{\alpha_j}) \frac{dt}{t}.$$

- *in the second part* of my talk we address the topic of Carleson–Radon transforms, which, generically speaking, are given by²

$$(3) \quad CR_{d,n}f(x, y) := \sup_{P \in \mathcal{Q}_{d,n}} \left| \int_{\mathbb{R}^n} f(x - t, y - |t|^2) e^{iP(t)} K(t) dt \right|, \quad (x, y) \in \mathbb{R}^n \times \mathbb{R},$$

where here $d, n \in \mathbb{N}$ with $\mathcal{Q}_{d,n}$ the set of all real-coefficient polynomials in n variables and of degree less than or equal to d and K a Calderón–Zygmund kernel on $\mathbb{R}^n$.

The crux of this presentation will be to show that all of the above classes of operators (1), (2) and (3) can be treated via the LGC method in a large variety of situations which include non-resonant (non-zero curvature) and degree one resonant (zero curvature) cases.

Finally, we emphasize that this talk will only focus on the intuition/bird eye view of these topics and will avoid as much as possible technical components/proof details.

²Here we are choosing the Polynomial Carleson-Radon transform along the paraboloid formulation.