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REZUMATUL TEZEI DE DOCTORAT

Extreme value theory for branching processes

Extreme pentru procese de ramificare

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1 OVERVIEW

This extended summary (approximately ten pages, excluding table of contents and references) presents the core ideas, techniques, and results of the thesis *Extreme Value Theory for Branching Processes*.

A centerpiece of the thesis is a *transfer principle and/or the potential theoretic approach* on configuration spaces that identifies the law of the total mass process of a non-local spatial branching model with that of a *pure* continuous-time Galton–Watson (GW) process. As a consequence, classical extinction/survival dichotomies and Malthusian growth theorems from GW theory immediately translate to the mass of the spatial system.

Complementarily, the thesis proposes and discusses some aspects of *EVT picture* for maxima of branching populations, emphasizing how tail behavior of offspring distributions determines Gumbel- or Fréchet-type limits in supercritical regimes while maxima remain a.s. finite in subcritical/critical regimes.

2 CHAPTER 1 — INTRODUCTION

Motivation and scope

Branching processes model populations in which individuals reproduce independently according to a common law. Classical GW chains describe discrete generations; continuous-time analogues allow individuals to reproduce after exponentially distributed lifetimes. The thesis elevates the *mass process* $|X_t|$ to a primary object: even when particles move in space and branching is non-local (offspring may be created at different spatial locations), the *total* number of particles can often be described by low-dimensional, analytically tractable dynamics.

Aims and guiding questions

The work addresses three questions:

- (*Identification*) When does the total mass of a spatial/non-local branching process behave exactly like a pure GW process?
- (*Methodology*) Which analytic structures (semigroups, resolvents, potential theory) allow one to pass from configuration-space dynamics to one-dimensional mass dynamics?

- (*Extremes*) How do the maximum population sizes evolve, and which extreme-value domains of attraction arise in the branching setting?

Contributions in brief

The thesis proves an *equivalence-in-law* between the mass process for a class of spatial non-local branching models and a continuous-time GW process, via a generator-level intertwining (the transfer principle). It also formulates a regime-wise EVT program for maxima in branching, clarifying normalizations and tail-based limit laws in supercritical regimes.

3 CHAPTER 2 — BASICS OF BRANCHING PROCESSES

Discrete-time GW: regimes, generating functions, extinction

Let $(Z_n)_{n\geq 0}$ be a GW process started from $Z_0 \in \mathbb{N}$ with i.i.d. offspring ξ and generating function $G(s) = \mathbb{E}[s^\xi]$. The basic recursion

$$Z_{n+1} = \sum_{i=1}^{Z_n} \xi_{n,i}, \qquad n \geq 0,$$

leads to the well-known trichotomy in terms of $m = \mathbb{E}[\xi]$:

- **Subcritical** $m < 1$: extinction occurs a.s.; $\mathbb{P}(Z_n \to 0) = 1$.
- **Critical** $m = 1$ (with $(\xi) > 0$): extinction still occurs a.s., but with heavy tails for extinction time.
- **Supercritical** $m > 1$: survival with positive probability $1 - q$, where q is the smallest root of $G(s) = s$ in $[0, 1]$.

In the supercritical case, $W_n := Z_n/m^n$ is a nonnegative martingale converging a.s. to W, a key random variable in growth and extremes.

Continuous-time branching and Malthusian scaling

In continuous time, each individual lives an exponential time (rate $a > 0$) before reproduction with the same offspring law. The Malthusian parameter $\alpha = a(m-1)$ yields the normalization $W(t) = Z(t)e^{-\alpha t} \to W$ a.s. Many fine properties (e.g., L^1-convergence, nondegeneracy of W) depend on moment assumptions on ξ.

Configuration spaces and multiplicativity

To connect spatial models with scalar mass dynamics, the thesis uses the configuration space $\widehat{E}$ of finite point measures on a Lusin space E (including the empty measure). Multiplicative functionals on $\widehat{E}$—those satisfying $F(\mu+\nu) = F(\mu)F(\nu)$—characterize the branching property and lead to explicit formulas for transition kernels. The *mass map* $\mu \mapsto \mu, 1$ provides a canonical projection onto $\mathbb{N}$ (or $\mathbb{R}_+$ in measure-valued settings).

Semigroups of contractions and generators

Analytically, the chapter reviews C_0-semigroups $(T_t)_{t\geq 0}$ on Banach spaces, their generators L, and the resolvent family $R_\lambda = (\lambda - L)^{-1}$. These objects appear twice: (i) as linear-operator tools for evolution equations associated with branching; and (ii) as probabilistic semigroups of Markov processes, via Hille–Yosida/Phillips theorems.

4 CHAPTER 3 — GENERAL BRANCHING PROCESSES

Potential theory for right processes

Working on a measurable/Lusin space E, let $X = (X_t)_{t\geq 0}$ be a right Markov process with transition semigroup $(T_t f)(x) = \mathbb{E}_x[f(X_t)]$. The resolvent $U_\alpha f(x) = \int_0^\infty e^{-\alpha t} T_t f(x)\, dt$ defines the cone of U-excessive functions (limits of $\alpha U_\alpha f$ as $\alpha \to \infty$) and induces the *fine topology*, the coarsest making all excessive functions continuous. These notions allow subtle measurability and boundary arguments essential for branching kernels and absorption phenomena.

Non-local branching on finite configurations

On the configuration space $\widehat{E}$, a non-local branching mechanism permits offspring to be placed at random spatial locations according to a kernel. The chapter develops:

- construction of the process via multiplicative functionals and branching kernels;
- analysis of absorbing sets (e.g., extinction) and invariant classes;
- examples, including trivial spatial motion (pure branching) and spatially constant mechanisms.

Continuous-state branching and total mass

When scaling limits lead from $\mathbb{N}$-valued to $\mathbb{R}_+$-valued masses, one obtains continuous-state branching (CB) processes with Laplace exponents solving generalized Riccati equations. For spatially constant branching, the *total mass* $|X_t|$ is shown to have the law of a pure branching process (GW or CB, depending on the setting). In particular, extinction/survival and growth of $|X_t|$ are governed by the mean offspring parameter m and the associated branching mechanism.

5 CHAPTER 4 — A SECOND APPROACH TO THE MASS PROCESS

Statement of the main result (informal)

Consider a spatial, non-local branching process $(X_t)_{t\geq 0}$ on $\widehat{E}$ whose transition operators (P_t) form a C_0-semigroup on a suitable function space and interact multiplicatively with configuration observables. Then the *total mass* process

$$M_t := |X_t| = X_t, 1 \in \mathbb{N}$$

is a *pure continuous-time Galton–Watson process* with offspring law determined by the local branching mechanism. Consequently,

- the extinction probability of M_t equals the GW root q of $G(s) = s$;
- if $m > 1$ then $e^{-\alpha t} M_t \to W$ a.s. for α the Malthusian rate and the same martingale W as in the corresponding GW;
- criteria and tail bounds from GW theory immediately transfer to M_t.

Transfer principle (generator intertwining)

The proof centers on an intertwining between generators on function classes stable under multiplicative lifts. In a schematic form:

$$L\big(F \circ \Phi\big) = \big(KF\big) \circ \Phi, \qquad \Phi : \widehat{E} \to \mathbb{N}, \;\; \Phi(\mu) = \mu, 1,$$

where L is the generator for (X_t) and K is the GW generator on $\mathbb{N}$. When (P_t) is a C_0-semigroup and the Cauchy problem $u'(t) = Lu(t)$ is well posed on the chosen core, the image process $M_t = \Phi(X_t)$ is Markov with semigroup generated by K. This *transfer principle* bypasses heavy pathwise constructions and yields the law of M_t directly from operator relations.

Assumptions, scope, and examples

The C_0 property ensures strong continuity needed for the generator calculus and uniqueness. Typical examples include branching with trivial spatial motion (pure branching), spatially constant mechanisms, and certain jump-diffusions where the spatial motion and branching kernel satisfy regularity conditions making multiplicative cores dense. The chapter also discusses how similar ideas extend, with care, to measure-valued superprocesses.

Extensions and a Schwartz-space route

To broaden applicability, the thesis explores semigroups on the Schwartz space $\mathcal{S}(\mathbb{R}^n)$ and its dual, illustrating how classical PDE generators (e.g., the Laplacian) fit into the picture and how linearization techniques can treat certain nonlinear semigroups. This opens a pathway to infinite-dimensional state spaces and more general motions.

6 CHAPTER 5 — EXTREME VALUES IN BRANCHING PROCESSES

Setup and quantities of interest

Let (Z_n) be a discrete-time GW chain and $Z(t)$ its continuous-time counterpart. Define running maxima

$$M_n = \max_{0 \le k \le n} Z_k, \qquad M(t) = \sup_{0 \le s \le t} Z(s).$$

The chapter discusses some aspects related to $M_n, M(t)$ and their limit laws under various regimes.

Subcritical and critical regimes

When $m \le 1$, extinction occurs a.s.; consequently, M_n and $M(t)$ are a.s. finite. The focus is thus on distributional bounds for M_n and the tail of the extinction time, rather than classical EVT scaling limits (which typically address unbounded maxima).

Continuous time and parallels

The same qualitative picture holds in continuous time, with the Malthusian rate α playing the role of $\log m$. Normalizations inherit W from $Z(t)e^{-\alpha t} \to W$, and the domain-of-attraction classification mirrors that of discrete time.

Techniques

Beyond generating functions and classical EVT (Fisher–Tippett–Gnedenko), the chapter leverages martingale techniques, renewal-type arguments, and comparison principles (e.g., sandwiching M_n between monotone transforms of Z_n). The analysis also clarifies when Weibull-type limits are irrelevant for population maxima.

7 CONCLUDING REMARKS

The thesis advances a coherent operator-theoretic program for branching systems: *identify* the mass process via generator intertwining under C_0-assumptions, and *analyze* extremes by importing EVT with care for the random scaling intrinsic to branching. Methodologically, the work demonstrates how potential theory (excessive functions, fine topology), semigroups/generators, and multiplicative structures on configuration spaces can be combined to deduce low-dimensional laws from high-dimensional particle models. Substantively, it shows that a large class of spatial/non-local branching models inherit GW laws at the level of total mass, and it systematizes the extreme-value behavior of their maxima across regimes and tail classes.

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