

Introduction to the Grothendieck-Teichmüller group

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The goal of this short course is to introduce the Grothendieck-Teichmüller group (GT), one of the most mysterious mathematical objects which appear in recent years. This group, defined by V.G. Drinfeld in 1990, contains the absolute Galois group $\text{Gal}(\overline{\mathbb{Q}}|\mathbb{Q})$, and at the same time appear in the most various situations : quantum deformations of mathematical structures, invariants of knots, mathematical physics, algebraic geometry. . . It is also closely related to the theory of motives and (as such !) motivates deep conjectures in transcendent number theory.

The course will concentrate on the basic tools needed to define and study this group : the braid groups (one can build a third braid by putting one braid above another one), completions and infinitesimal versions of them. We will then introduce *GT* and related objects and show how elements of $\text{Gal}(\overline{\mathbb{Q}}|\mathbb{Q})$ embed in this group of automorphisms of braids.

Preliminaries : Lie algebras, notions of algebraic topology (π_1, H_*, H^*) and differential/analytic geometry, basic group theory.