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RESEARCH INTERESTS

Currently Tudor Ratiu is Head of the Chair of Geometric Anaylsis in the Mathematics Sectioin of the Ecole Polytechnique Federale de Lausanne (Swiss Frederal Institute of Technology Lausanne). He is also the director of the Bernoulli Center, a mathematics research institute.

Most of Tudor Ratiu's research centers on geometric mechanics (both classical and continuum) and nonlinear global analysis.

1. Noncanonical Hamiltonian structures, nonlinear stability, and bifurcation theory. Conservative systems of ordinary and partial differential equations appearing in mathematical physics and as examples of completely integrable systems are naturally Hamiltonian in a noncanonical structure. These Poisson structures are intimately tied to the theory of Lie algebras, symplectic geometry, and the theory of connections on fiber bundles. The interaction between these fields of pure mathematics and questions arising naturally from applied mathematics and physics is a very fertile ground of research. For example, these Poisson structures form the geometric framework in which new criteria for nonlinear stability of relative equilibria can be proven. One obtains in this way bounds guaranteeing the stability of specific solutions. By violating these bounds, bifurcation phenomena appear that can be treated by the same geometric techniques. Furthermore, perturbing the system by the addition of dissipation in the internal variables results in instability of the relative equilibrium, thereby showing that the stability criteria are sharp. The same geometric setup also enables the computation of additional phases in the dynamics of these systems, a phenomenon appearing in both classical and quantum mechanics.

2. Complete integrability. In the theory of conservative systems of ordinary and partial differential equations there are special cases that can be explicitly solved. This is a rare occurrence and is intimately related to the underlying differential geometry of the problem. Such systems have a high degree of symmetry that enables one to closely tie their dynamic behavior to the theory of Lie groups and Lie algebras. The symmetry of these systems implies the existence of a large number of conserved quantities, equal, in fact, to the number of degrees of freedom. The collection of these functions has remarkable convexity properties that

are in turn related to Lie group actions on symplectic manifolds. The interplay between dynamics, geometry, and convexity is an area of great current interest. For example, results in convex analysis give unexpected insight into the Lie theory of diffeomorphism groups and the nature of solutions of certain highly nonlinear partial differential equations.

3. Infinite-dimensional manifolds and Lie groups. In geometry and mathematical physics one often deals with symmetry groups that are not Lie groups in the usual sense. Such groups are usually formed by diffeomorphisms or operators of finite differentiability class (for example, groups formed by diffeomorphisms preserving a special structure, or groups of Fourier integral operators on a compact manifold). These groups turn out to have the structure of a topological group and of a smooth infinite-dimensional manifold. However, only right translation is smooth. Taking the inverse limit with respect to the differentiability class, a Lie group–like object is obtained. It is of great interest to study these groups from the point of view of Lie theory, to explore their structure and its relationship to the topology of the underlying manifold, or to consider actions of such groups on infinite-dimensional manifolds appearing naturally in mathematical physics or geometry.

Selected Publications

Marsden, J.E., T.S. Ratiu, and S. Shkoller [1999] *The geometry and analysis of the averaged Euler equations with normal boundary conditions, Geometric and Functional Analysis.*

Castrillón-Lopez, M., T.S. Ratiu, and S. Shkoller [1998] *Reduction in principal fiber bundles: covariant Euler-Poincaré equations*, Proc. Amer.Math. Soc.

Ortega, J.-P. and T.S. Ratiu [1998] *Singular reduction of Poisson manifolds*, Lett. Math. Phys., 46, 359-372.

Ortega, J.-P. and T.S. Ratiu [1999] *Stability of Hamiltonian relative equilibria, Nonlinearity*

Marsden,. J.E., G. Misiolek, M. Perlmutter, and T.S. Ratiu [1998] *Symplectic reduction for semidirect products and central extensions*, Diff. Geom. and its Appl., 9, 173-212.

Fassò, F. and T.S. Ratiu [1998] *Compatibility of symplectic structures adapted to noncommutatively integrable systems*, Journ. Geom. and Physics, 27, 199-220.

Holm, D.D., J.E. Marsden, and T.S. Ratiu [1998] *The Euler-Poincaré equations and semidirect products with applications to continuum theories*, Advances in Math., 137, 1-81. .

Michor, P. and T.S. Ratiu [1998] *On the geometry of the Bott-Virasoro group*, Journal of Lie Theory, 8. 293-309.

Holm, D. D., J. E. Marsden and T. S. Ratiu [1998*] Euler-Poincaré models of ideal fluids with nonlinear dispersion*, Phys. Rev. Lett., 349, 4173-4177.

Derks, G. and T.S. Ratiu [1998] *Attracting curves in Navier-Stokes and reduced magnetohydrodynamics*, The Royal Society. Proceedings:Math., Phys. and Eng. Sci., Series A, 454, 1407-1444.

Ortega, J.-P. and T.S. Ratiu [1997] *Persistence and differentiability of critical relative elements in Hamiltonian systems with symmetry*, C.R. Acad.Sci. Paris 325, Série I, 1107-1111.

Bao, D. and T.S. Ratiu [1997] *A maximal torus for the symplectomorphism group of the annulus*, Diff. Geom. and Appl. 7, 193-210.

Hoppe, J. and T.S. Ratiu [1997] *Diffeomorphism invariant formulation of field theories*, Classical and Quantum Gravity 14, L45-L48.

Lewis, D.K. and T.S. Ratiu [1996] *Rotating n-gon/kn-gon vortex configurations*, Journ. Nonlinear Science 6, 385-414.

Flaschka, H. and T.S. Ratiu [1996] *A convexity theorem for Poisson actions of compact Lie groups*, Ann. École Normale Supérieure 29,787-811.

Bloch, A.M., P.S. Krishnaprasad, J. Marsden, and T.S. Ratiu [1996] *The Euler-Poincaré equations and double bracket dissipation*, Comm. Math. Phys. 175, 1-42.

Derks, G., D.K. Lewis, and T.S. Ratiu [1995] *Approximations with curves of relative equilibria in Hamiltonian systems with dissipation*, Nonlinearity 8, 1087-1113.

M arsden, J.E., T.S. Ratiu, and G. Raugel [1995] *Équations d'Euler dans une coque sphérique mince* (Euler equations on a thin spherical shell), C.R. Acad. Sci. Paris 321, Série I, 1201-1206.

Bloch, A.M., P.S. Krishnaprasad, J.E. Marsden and T.S. Ratiu [1994] *Dissipation induced instabilities*, Ann. Inst. H. Poincaré, Analyse Nonlinéaire 11, 37-90.

Bloch, A.M., H. Flaschka, and T.S. Ratiu [1993] *A Schur-Horn-Kostant convexity theorem for the diffeomorphism group of the annulus*, Invent. Math. 113, 511-529.