

Serban Basarab

Curriculum vitae.

A) General data.

Born in Bucharest-Romania, 3.03.1940, graduate of the Faculty of Electronics of the Technical University of Bucharest in 1961 and of the Faculty of Mathematics of the University of Bucharest in 1969 with a diplom thesis devoted to the Galois Cohomology, having the late Prof. Ionel Bucur as supervisor. In 1977 I defended the PhD thesis with the title "Arithmetic and model theory" at the Faculty of Mathematics of the University of Bucharest under the supervision of Acad. Octav Onicescu, who replaced Prof. Ionel Bucur after his premature death.

During the period 1979-1982, with some intreruptions, I activated as visiting professor at the Institute of Mathematics of the University of Heidelberg thanks to a two years fellowship granted by the Alexander von Humboldt Foundation. As a member of the research group of Algebra and Number Theory, I have been decisively influenced by the personality of my academic mentor Prof. Peter Roquette.

In 1983 I obtained a four months fellowship to visit the Universities of Firenze and Camerino-Italy, but unfortunately I was prevented from honouring the invitation by the communist Romanian authorities.

In 1993 I visited the Universities of Wales-Bangor, Queen Marry-London, and Oxford Mathematical Institute under a three months fellowship granted by the European Communities.

This year I obtained from the the Alexander von Humboldt Foundation a three months fellowship. The visit in Germany will take place in the period 1.10-31.12.1998, including research stages at the Universities of Heidelerb g(Prof. Peter Roquette), Konstanz (Prof. Alexander Prestel), and MPIM-Bonn.

In the present I am Senior researcher 1 and the scientific secretary of the Institute of Mathematics of the Romanian Academy. I am also associate professor at the University "Ovidius"-Constanta, Romania.

Beginning with 1990 I was accepted as supervisor of doctorands. In June 1995 my doctorand Mihai Caragiu defended his PhD thesis entitled "Applications of finite fields: power distribution, finite semiplanes, arithmetical physics".

I am a member of the editorial board and the scientific secretary of "Revue Roumaine de Mathematiques Pures et Appliquees".

I am a reviewer to "Mathematical Reviews", "Zentralblatt fur Mathematik und ihre Grenzgebiete", "Journal of Symbolic Logic".

I am the delegate for Mathematics in the executive secretariate of

the Humboldt Club Romania founded in 1990.

In 1991 I received the "Gheorghe Lazar" prize for Mathematics of the Romanian Academy.

B) The research activity.

The results of my scientific activity are presented in about 40 papers published in the following mathematical journals: Journal of Algebra, Journal fur reine und angewandte Mathematik, Annals of pure and applied Logic, Journal of Symbolic Logic, Journal of pure and applied Algebra, Communications in Algebra, Comptes Rendus Acad.Sci.Paris, Manuscripta Mathematica, Results of Mathematics, Fundamenta Informaticae, Revue Roumaine de Mathematiques pures et appliquees, Studii si cercetari matematice, etc.

The main contributions are described in the following.

1) Model theoretic Algebra.

1.1) Henselian valued fields.

Having as starting point the fundamental works of James Ax, Simon Kochen and Jurii Ershov from 1965-1966 concerning some diophantian problems over local fields, I developed in my PhD thesis and in a series of papers [4, 12-15] a systematic model theoretic study of the Henselian valued fields of characteristic zero, positive residue characteristic and finite ramification index. The main results are model theoretic classification criteria (elementary equivalence, model-completeness, etc) for such valued fields in terms of some elementary invariants of the value groups and the residue rings. An application to the theory of integrally defined functions on valued fields is given in [15], where an interesting class of valued fields, called prehenselian, is introduced and investigated.

Using algebraic and model theoretic techniques, I proved in [36] a theorem on the relative elimination of quantifiers for Henselian valued fields of characteristic zero, extending the results of Angus MacIntyre, and Prestel-Roquette on quantifier elimination for p -adically closed fields, as well as the results of primitive-recursive nature of V. Weispfenning. Related to I mention the isomorphism criterion for Henselian valued fields, algebraic over a given common valued subfield, presented in the joint paper [41] with F.V. Kuhlmann.

Devoted to the same field of interest, the paper [30] extends a classical theorem of Abraham Robinson on quantifier elimination for algebraically closed valued fields as well as a result from 1973 of Lipshitz and Saracino, and Carson concerning the model completion of the elementary theory of regular (in the sense of von Neumann) commutative rings.

1.2) Formally p -adic fields.

The theory of formally p -adic fields was developed by Simon Kochen and Peter Roquette as a p -adic analogue of the classical theory of formally real fields initiated by Emil Artin and Schreier. An

enlarged framework for this theory is provided in [17] where the main results of the paper "The Nullstellensatz over p -adically closed fields", Journal of the Mathematical Society of Japan, 32(1980), by M. Jarden and P. Roquette, are proved in this more general context. In a prolongation of [17], the paper [18] investigates some situations when certain objects associated to a field extension F/K (places of F/K , the Kochen ring and the holomorphy ring of F/K for a p -adically closed base field K) are obtained by the contraction of the corresponding objects associated to a field extension N/K subject to $F \subset N$. As an application, the existence of some recursive bounds in the theory of fields and the theory of formally p -adic fields is proved.

Devoted to the same field of interest, the paper [21] provides a generalization of the preorders of higher level introduced in 1979 by E. Becker to extend Artin-Schreier theory to arbitrary powers sums in fields. Using Kadison-Dubois representation theorem for Archimedean partially ordered rings, an operator theoretic description of the t -preorders of level n is given, recovering an unpublished result of P. Roquette in the particular case $n=2$.

1.3) Nullstellensätze.

The remarkable fact noticed by A. Robinson, namely the equivalence between Hilbert's Nullstellensatz and the model completeness of the "elementary" theory of algebraically closed fields opened the way for using specific model theoretic concepts and results in the approach of some problems of the algebraic geometry (in particular, the Nullstellensatz) over base fields which are not necessarily algebraically closed, however having suitable arithmetical and model theoretical properties. To this field of interest belong the papers [17, 23, 27, 30] devoted to extensions of some Nullstellensatz over ordered fields, p -adically closed fields, pseudoalgebraically closed fields due to D. Dubois, G. Stengle, M. Jarden, P. Roquette, B. Jacob, K. McKenna.

1.4) Pseudoreal closed fields.

In Ax's fundamental work from Annals of Mathematics (1968) devoted to the "elementary" theory of finite fields, an important class of fields, called later by G. Frey pseudoalgebraically closed fields, is introduced and investigated. An order theoretic analogue of this concept was introduced and studied in [25]. Later A. Prestel extended this concept calling a field pseudoreal closed if it is existentially closed in any regular and totally real field extension; equivalently, in geometric terms, a field K is pseudoreal closed iff every absolutely irreducible affine variety defined over K has a rational point over K whenever it has a simple rational point over any real closure of K . In [24], an alternative proof based on nonstandard arithmetic techniques is given for Prestel's result on the recursive axiomatizability of the class of pseudoreal closed fields.

The paper [26] is devoted to the algebraic and the model theoretic investigation of an important subclass of pseudoreal closed Hilbertian fields. A positive answer to a question raised in [26] is

announced without proof by J. Ershov in a note from Dokladi Akad. Nauk SSSR (1982).

The papers [28] and [31] are devoted to the absolute Galois group of a pseudoreal closed field, while some model theoretic transfer principles for pseudoreal closed fields are proved in [32].

1.5) Abelian groups.

The models of the "elementary" theories of the classes of finite, resp. profinite, resp. torsion Abelian groups are characterized in [8] and [11] in terms of some specific elementary invariants. The paper [8] is the starting point—the Abelian case—of Felgner's works on pseudofinite groups, the group theoretic analogue of Ax's pseudofinite fields.

2) Rings with approximation property.

Using the model theoretic concept of existential completeness, certain types of good approximation in rings are introduced in the joint paper [19], and the general theory is applied to the particular case of rings with approximation property.

3) The p-adic spectrum of a commutative ring and compactification.

By analogy with Zariski's spectrum and the real spectrum of a commutative ring, the concept of a p-adically closed field induces through a natural process of globalization the notion of a p-adic spectrum introduced and investigated in [35]. p-adic analogues of some results of the real algebraic geometry (as Artin-Lang theorem and the finiteness theorem conjectured by Brumfiel) are obtained. Finally these results together with Kuhlmann-Prestel density theorem (Crelle's Journal, 1984) provide a model theoretic unitary approach of the compactification procedure of the affine algebraic varieties defined over local fields of characteristic zero, introduced in 1984 by J.W. Morgan and P. Shalen for the complex and the real case, avoiding in this way the appeal to Hironaka's desingularization.

4) Diophantine problems.

In the paper "Zeros of polynomials over local fields. The Galois action", Journal of Algebra (1970), J. Ax introduced the "diameter of conjugates" as a measure of the "closeness to the base field" of an algebraic element over a given Henselian valued field. This concept is used in [6, 10] to study the "closeness to rationality" of the points of an elliptic curve defined over a local field of characteristic zero and positive residue characteristic.

Some questions concerning the torsion points on elliptic curves defined over local and global fields are discussed in [20], where some extensions of certain results of Demianenco and Hellegouarch are obtained.

The techniques of the nonstandard arithmetic are used in [7, 22] to approach some questions concerning the class field theory and the

diophantian approximation.

Barry Mazur's distributions are powerful tools in the study of some arithmetical problems on cyclotomic fields, modular functions and abelian extensions. A purely algebraic approach of the distributions defined on distributive lattices and profinite groups is developed in [34], where some extensions of certain results of Sinnott, Kubert and Lang are obtained.

5) Arboreal group theory.

In the last twenty years various extensions of the Bass-Serre theory of group actions on simplicial trees have been the subject of much investigation combining elementary geometric considerations with very sophisticated techniques. The variety of topics and applications of the field is well reflected in the proceedings "Arboreal Group Theory", ed. R.C. Alperin, Mathematical Sciences Research Publications 19, Springer-Verlag, 1991.

Reading by chance the paper of J. Morgan and P. Shalen, "Valuations, trees and degenerations of hyperbolic structures. I", Annals of Mathematics, 120 (1984), I became interested in lambda-trees and the combinatorial group theoretic information carried by a group action on a lambda-tree. Stimulated by this paper and also by the paper of R.C. Alperin and H. Bass, "Length functions of group actions on lambda-trees", Annals of Mathematical Studies, 111, Princeton University Press, 1987, I succeeded in [37] to extend to lambda-trees some basic constructions and results contained in the first chapter of Serre's book "Trees", while some model theoretic principles for lambda-trees have been established in [33].

The technique developed in [37] has two complementary aspects: a group theoretic one concerning actions on groupoids, and a metric one concerning Lyndon length functions on groupoids. The simultaneous approach of these two aspects introduced some complications in the logical line of the exposition in [37], so in [38] and [40], I considered more natural to treat them apart and eventually relate them. Moreover, a more general concept of tree, including distributive lattices, lambda-trees where lambda is a lattice ordered group, and Tits' buildings as special cases, is introduced and investigated in [40], while the dual of the category of these general trees is described in [39] using a suitable extension of Stone's representation theorem for distributive lattices. To my surprise I learned later that my general concept of tree was known already from 50's to lattice theorists under the name of median algebra, however totally unknown to group theorists. Thus, having as starting point of my research the geometric point of view of group actions, I rediscovered the significant concept of median algebra and some results concerning it. Fortunately, the geometric motivation of my approach permitted me to obtain also some new results. For instance, in [43] I considered two basic operators on the class of generalized trees assigning to a generalized tree T the generalized tree $\text{Dir}(T)$ of the directions on T , resp. the directed

generalized tree $\text{Fold}(T)$ of the foldings (retractions) of T . The main results of [44] show that the two operators above commute, providing an interpretation of the composite operator $\text{Fold} * \text{Dir} = \text{Dir} * \text{Fold}$ in terms of the so called quasidirections on generalized trees.

Interested in the way in which the theory of structured groupoids interacts with various combinatorial and metric problems, Prof. R. Brown invited me in 1993 to visit the University of Wales in Bangor under an European Communities research fellowship. With this occasion I noticed and investigated some interesting connections between generalized trees and the mathematical objects called racks, well known for their applications in the theory of knots, links and braids, singularity theory and set theory. Some related results are included in [43].

In the last three years I was mainly interested in applying the general concepts and results of the theory of generalized trees to some significant mathematical frameworks. Thus in [42], motivated by some difficult problems concerning the model theory of free groups and free profinite groups, I considered a class of groups called discrete hyperbolic arboreal groups, showing that given a discrete hyperbolic arboreal group G and a suitable family of Abelian discrete hyperbolic arboreal groups which are convex extensions of maximal abelian arboreal subgroups of G , the corresponding amalgamated sum has a canonical structure of discrete hyperbolic arboreal group. The works [45-47] are devoted to a systematic study of the arboreal structure of a class of groups including the free groups, the free Abelian groups and the Coxeter groups whose relations involve only commuting generators. In a work in preparation [48] I study the generalized trees of groups and their applications to SL_2 over global fields, including connexions with Hilbert modular group and Bost-Connes phase transitions with spontaneous symmetry.

6) Other papers concern nonabelian cohomology [3,5], formal Moufang loops [9], logical design of circuits and technical applications [1,2].

Citations of my works occur in papers and books of P. Roquette, J. Ershov, A. Prestel, W. Hodges, D. Popescu, M. Jarden, U. Felgner, E. Becker, F. Pop, V. Weispfenning, F. V. Kuhlmann, D. Haran, F. Delon, L. Belair, G. Georgescu, M. Roller, R. Transier, R. Farre, A. Solian, etc. 16 papers are mentioned in the "Omega Bibliography of Mathematical Logic", vol. 3-Model Theory, Springer-Verlag 1987.

Among the international conferences to which I had the opportunity to participate giving lectures, I mention Oberwolfach sessions on Model Theory, Algebraic Number Theory, p -adic Analysis, Mathematical Logic, Field Arithmetic, Hannover 1979 International Congress on Logic, Philosophie and the Methodology of Science, Firenze 1982 Logic Colloquium, Easter Conferences of Model Theory-Humboldt University Berlin, AMS Conference on Logic, Local Fields and Subanalytic Sets-Amherst 1990, 1991 Banach Semester on Algebraic Methods in Logic and Applications in Informatics, NATO Advanced Study Institute on Semigroups, Formal Languages and Groups-York 1993, 1996 Banach minisemester in the memory of Helena Rasiowa, 1997

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4. Some metamathematical aspects of the theory of Henselian fields (in Romanian) *Studii si Cercetari Matematice* 10 (1973) 1449-1559; MR52#8104, Zbl1295# 12106.
5. Cohomologie des petites categories, *Revue Roumaine de Math.* 5 (1974) 559-575.
6. The closeness to rationality of the points of an elliptic curve defined over a local field (in Romanian) *Studii si Cercetari Matematice* 6 (1974) 783-792.
7. Some remarks concerning nonstandard admissible morphisms, *Revue Roumaine Sciences Sociales, Serie de Philosophie et Logique* (1975) 205-210.
8. The models of the elementary theory of finite Abelian groups (in Romanian) *Studii si Cercetari Matematice* 4 (1975) 381-386; MR53#7770, Zbl335# 02036.
9. Commutative formal Moufang loops (in Romanian) *Studii si Cercetari Matematice* (1976) 259-265.
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17. Towards a general theory of formally p -adic fields, *Manuscripta*

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18.Extension of places and contraction properties for function fields over p -adically closed fields, *J.Reine Angew.Math.*326(1981)54-78; MR82j:03040,Zbl491#12025.

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20.Some remarks concerning the torsion points of elliptic curves, *Revue Roumaine Math.*6(1982)621-642.

21.On a class of preorderings of higher level, *Manuscripta Math.*37(1982) 163-210.

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