SNS-B COURSES FOR THE ACADEMIC YEAR 2018-2019

Professor	Title	Level	Sem.
Cristina Benea	Introduction to Harmonic Analsysis	CP/M1/M2	I
Florin Belgun	Spin geometry, Dirac operators and holonomy groups	M1/M2	I
Viviana Ene	Polytopes: Algebra and Combinatorics	M1/M2	I
Eugen Mihailescu	Ergodic Theory and Applications	CP/M1/M2	I
Liviu Ignat	Nonlinear Evolution PDEs	CP/M1/M2	II
Cezar Joiţa	Several Complex Variables	M1/M2	II
Sergiu Moroianu	Pseudodifferential Operators and K-theory	M1/M2	II

FALL SEMESTER

Cristina Benea – Introduction to Harmonic Analsysis
Level: CP/M1/M2
Course Outline:
We introduce some of the main concepts in harmonic analysis.

(1) the Hardy-Littlewood maximal function and covering lemmas
(2) the Hilbert transform and Calderón- Zygmund operators
(3) weak L^p spaces, interpolation theory
(4) the sharp maximal function, the BMO space, interpolation with BMO
(5) Fourier multipliers and Littlewood-Paley theory
(6) introduction to pseudo-differential operators

Pre-requisites: introductory course in real analysis, some knowledge of Fourier analysis

Bibliography:

1. C. Muscalu and Wilhem Schlag, *Classical and multilinear harmonic analysis*, 2013, Cambridge University Press
2. J. Duoandikoetxea, *Fourier analysis*, 2001, Graduate Studies in Mathematics.

Florin Belgun – Spin geometry, Dirac operators and holonomy groups
Level: M1/M2
Course Outline:

The goal of this lecture series is to describe the basic notions of spin geometry in the Riemannian setting: the Clifford Algebra, the spin representations, the Clifford multiplication leading to the definition of the Dirac operator.

More precisely, the visited topics include:

- The Clifford Algebra
- The Spin group
- Spin structures on manifolds
- The Dirac operator
- The twistor operator
- Parallel spinors and holonomy
- Applications

Viviana Ene – Binomial ideals and toric algebras
Course Assistant: Rodica Dinu
Level: M1/M2
Course Outline: The main goal of this course is to cover Chapter 4 and Chapter 5 of the very recent book [2]. The course is structured as follows.

I. **Basics**
   (1) Convex polytopes.
   (2) Faces and f–vectors.
   (3) Simplicial polytopes.

II. **Normal polytopes and unimodular triangulations**
   (1) Integral polytopes and integer decomposition property.
   (2) Normal polytopes.
   (3) Triangulations.

III. **Edge polytopes**
   (1) Edge polytopes of finite graphs.
   (2) Normality of edge polytopes.

References

[1] V. Ene, J. Herzog, *Gröbner bases in commutative algebra*, Grad. Stud. Math. **130**, Amer. Math. Soc., Providence, RI, 2012.
[2] J. Herzog, T. Hibi, H. Ohsugi, *Binomial ideals*, Grad. Stud. Math. **279**, Springer 2018.
[3] B. Sturmfels, *Gröbner Bases and Convex Polytopes*, Amer. Math. Soc., Providence, RI 1996.
[4] G. M. Ziegler, *Lectures on polytopes*, Grad. Stud. Math. **152**, Springer 1995.

Eugen Mihăilescu – Ergodic Theory and Applications
Level: CP/M1/M2
Course Outline: This course is suited for Master and Preliminary Cycle students interested in Analysis, Ergodic Theory, Fractals, Probability Theory, Number Theory, Geometry, etc. The prerequisites are the usual University courses from first 2 years (especially Real Analysis). Ergodic Theory and Dynamical Systems are fields of high current interest, with problems for possible future research. They have applications in several fields of Mathematics, and also in Physics, Chaos Theory.

Some topics of the course are:
1) Introduction to measure theory and probability.
2) Analysis on invariant sets. Invariant measures.
3) Birkhoff Ergodic Theorem.
4) Entropy.
5) Fractals. Classical examples.
6) Expansions in integer bases.
7) Continued fractions, Diophantine approximation.

SPRING SEMESTER

Liviu Ignat – Nonlinear Evolutions PDEs

Level: M1/M2
Course outline:

The course presents basic techniques for systematically analyzing the well-posedness issue of some evolution equations: heat, convection-diffusion, Schrodinger and wave equations. The course is divided into two parts. The first part treats the classical theory with the aim of understanding, for example, local existence of the solutions to nonlinear problems, blow-up alternative, energy estimates. The second part of the course will be dedicated to the study of well-posedness for the nonlinear Schrodinger equation.

Outline

(1) **Classical theory.** Sobolev Spaces, Semigroups, Hille-Yosida theorem, Inhomogeneous equations, compactness methods and monotone operator method.
(2) **Dispersive equations.** Strichartz estimates for constant coefficient linear Schrodinger and wave equation. Local smoothing effect. Resolvent estimates. Well-posedness for nonlinear models.

References

[1] Haim Brezis, *Functional analysis, Sobolev spaces and partial differential equations,* Universitext, Springer, New York, 2011. MR 2759829
[2] T. Cazenave and A. Haraux, *An introduction to semilinear evolution equations.* Transl. by Yvan Martel. Revised ed., Oxford Lecture Series in Mathematics and its Applications. 13. Oxford: Clarendon Press. xiv, 1998.
[3] Doina Cioranescu, Patrizia Donato, and Marian P. Roque, *A*n introduction to second order partial differential equations, World Scientific Publishing Co. Pte. Ltd., Hackensack, NJ, 2018, Classical and variational solutions. MR 3753702
[4] F. Linares and Ponce G., *Introduction to nonlinear dispersive equations.* Springer, 2009.
[5] Songmu Zheng, *Nonlinear evolution equations,* Chapman & Hall/CRC Monographs and Surveys in Pure and Applied Mathematics, vol. 133, Chapman & Hall/CRC, Boca Raton, FL, 2004. MR 2088362

Cezar Joiţa – Several Complex Variables
Level: CP/M1/M2
Course outline:

This course will be an introduction into Several Complex Variables. The main text that I will use is J. P. Demaillys book Complex Analytic and Differential Geometry and the goal is to cover chapters 1 and 2, parts of chapter 4, and chapter 5 of this book. This means:

- Basic properties of holomorphic functions of several complex variables,
- Pseudoconvexity, mainly for domains in $\mathbb{C}^n$,
- Analytic sets,
- Coherent analytic sheaves, including Oka and Cartan Coherence Theorems,
- Complex spaces,
- Cohomology with values in a sheaf,
- Hermitian and holomorphic vector bundles emphasising the case of line bundles,
- Linear connections,
- The first Chern class,
- Universal Vector Bundles.

Sergiu Moroianu– Pseudodifferential Operators and K-theory
Level: M1/M2
Course outline:

This course is an introduction to elliptic theory on compact manifolds. I will introduce the theory of pseudodifferential operators, first on $\mathbb{R}^n$ and then on compact smooth manifolds. In parallel I will construct topological K-theory and study its homological properties. We will prove Bott periodicity for K-theory using pseudodifferential operators. The Fredholm index of elliptic pseudodifferential operators will be shown to depend only on the K-theory class of its principal symbol. We will finally deduce the Atiyah-Singer index formula.

Outline.

(1) Fourier transform, Schwartz functions
(2) Spaces of symbols
(3) Scalar pseudodifferential operators
(4) Composition of ΨDO's
(5) Diffeomorphism invariance
(6) Sobolev spaces; action of ΨDO's on H^s
(7) Vector bundles; connections; homotopy equivalence; K^0
(8) One-point compactification; K^1; link with elliptic operators
(9) Curvature tensors; Chern character; characteristic forms
(10) Push-forward in K-theory; isomorphism between rational K-theory and rational cohomology
(11) Bott periodicity
(12) $\hat{A}$-genus; the Atiyah Singer index theorem.

Bibliography: we will use Atiyah's book on K-theory, and lecture notes by the proposer.