SNS-B COURSES FOR THE ACADEMIC YEAR 2017-2018

Professor	Area	Level	Semester
Viviana Ene	Algebra	M1/M2	I
Eugen Mihailescu	Ergodic Theory	CP/M1/M2	I
Liviu Paunescu	Analysis	CP/M1	I
Florin Belgun	Geometry	M1/M2	II
Cezar Joiţa	Analysis	CP/M1	II
Alexandru Popa	Number Theory	CP/M1/M2	II

FALL SEMESTER

Viviana Ene – Binomial ideals and toric algebras
Course Assistant: Claudia Andrei
Level: M1/M2
Course Outline:

In the last twenty years, combinatorial commutative algebra emerged as a new and very active branch of mathematics. It has turned out that methods of commutative algebra applied to multigraded objects, in particular to monomial and binomial ideals, create a fascinating link between combinatorics and algebra. Via this interaction these fields mutually enhance each other and give rise to new theories and surprising applications.

Binomial ideals appear in various areas of commutative algebra, combinatorics, and statistics. Toric ideals are of particular interest. As it was shown by the work of Diaconis and Sturmfels, toric ideals and their Gröbner bases have interesting applications to statistics.

The course is structured as follows.

I. **Basics**
   (1) Short survey on Gröbner basis theory.
   (2) Review of commutative algebra.
   (3) Binomial ideals and toric ideals.

II. **Edge polytopes and edge rings**
   (1) Convex polytopes.
   (2) Edge polytopes of finite graphs.
   (3) Toric ideals of edge rings.

III. **Binomial edge ideals**

   (1) Primary decomposition.
   (2) Koszul binomial edge ideals.
   (3) Syzygies of binomial edge ideals.

IV. **Hibi rings and ideals**
   (1) Review of lattice theory.
   (2) Hibi rings and their canonical modules.
   (3) Syzygies of Hibi rings.

All the lectures will be complemented with exercises with an increasing level of difficulty. One of the main goals of the course is to open the interest of the students in informing themselves about possible developments of the presented subjects. We will also discuss some open questions which could inspire the students in initiating independent research.

References

[1] V. Ene, J. Herzog, *Gröbner bases in commutative algebra*, Grad. Stud. Math. **130**, Amer. Math. Soc., Providence, RI, 2012.
[2] V. Ene, J. Herzog, T. Hibi, *Cohen-Macaulay binomial edge ideals*, Nagoya Math. J. **204** (2011), 57–68.
[3] V. Ene, J. Herzog, S. Saeedi Madani, *A note on the regularity of Hibi rings*, Manuscripta Math. **148** no. 3-4 (2015), 501-506.
[4] J. Herzog, T. Hibi, F. Hreinsdotir, T. Kahle, J. Rauh, *Binomial edge ideals and conditional independence statements*, Adv. Appl. Math. **45** (2010), 317–333.
[5] T. Hibi, *Distributive lattices, affine semigroup rings and algebras with straightening laws*, In: "Commutative Algebra and Combinatorics" (M. Nagata and H. Matsumura, Eds.), Adv. Stud. Pure Math. **11**, North–Holland, Amsterdam, (1987), 93–109.
[6] B. Sturmfels, *Gröbner Bases and Convex Polytopes*, American Mathematical Society, Providence, RI (1996).

Eugen Mihăilescu – Ergodic number theory
Level: CP/M1/M2
Course outline:

This course is suited for Master and Preliminary Cycle students interested in probability theory, number theory, analysis, etc. The prerequisites are the usual University courses from the first 2 years (especially Real Analysis). We will start with basic notions from Measure Theory, and Ergodic Theory, then will apply these methods to Number Theory. Ergodic Number Theory is a field of high interest currently. Some topics of the course are:

1) Introduction to probability spaces and invariant measures.
2) Iterated systems and limit sets.
3) Ergodic measures, mixing measures. Birkhoff Ergodic Theorem.
4) Various types of entropy.
5) Expansions of real numbers in integer bases.
6) Beta-transformations.
7) Lűroth series and generalizations.
8) Continued fractions, and Diophantine approximation.

Liviu Păunescu–The Fourier Transform
Level: CP/M1
Course Outline:

(1) Fourier coefficients, Fourier series, convergence theorems, the isomorphism between $L^2(S^1)$ and $l^2(\mathbb{Z})$;
(2) Fourier transform on $\mathbb{R}$;
(3) Tauberian theorems and applications.

SPRING SEMESTER

Florin Belgun – Gauge theory, characteristic classes and the index theorem

Level: M1/M2

Course outline:

The goal of this lecture series is to describe the background of many geometric structures, defined by gauge theory: principal bundles, connections and their holonomy. For example, special geometries occur in Riemannian geometry as metrics that have special holonomy or are related to special holonomy by standard constructions. More generally, one can describe G-structures on a manifold as reductions of the structure group of the frame bundle. On the other hand, the Chern-Weil theory leads to a differential-geometric approach to characteristic classes, which lead to some applications of the classical index Theorem of Atiyah and Singer.

The course will navigate "diagonally" through many fields of differential geometry, and may serve as an introduction to some of them. More precisely, the visited topics include:

- Vector fields, Flows, the Frobenius Theorem
- Lie Groups and Lie Algebras, homogeneous spaces
- Principal bundles, connections and holonomy
- The curvature tensor, gauge transformations and gauge invariance
- Chern classes and Chern-Weil theory
- The Atiyah-Singer index theorem
- G-structures on manifolds. Special riemannian holonomy, spin geometry.
- Applications

Cezar Joiţa – A second course in Complex Analysis
Level: CP/M1/M2
Course outline:

This course is meant as a sequel to the standard one-semester course included in the Licence program of study at the Department of Mathematics, University of Bucharest.

The following is a preliminary list of topics covered by the course. The final one will depend on the actual background of the students taking the course.

- The $\overline{\partial}$-equation
- Runge's Theorem
- Riemann mapping theorem
- The gamma function and Riemann zeta function
- Analytic continuation
- Picard's Theorem

Alexandru Popa – Lattices, codes and modular forms
Level: CP/M1/M2
Course outline:

The course will follow the book *Lattices and codes* by Wolfgang Ebeling, which is based on notes from a course given by Hirzebruch. This will be supplemented with background on modular forms from *Elliptic modular forms and their applications*, by Don Zagier (in the lecture volume "The 1-2-3 of modular forms"), and with the original article establishing connections between codes and modular forms, by M. Broué, M. Enguehard, *Polynômes des poids de certains codes et fonctions thêta de certains réseaux.* Ann. sci. de l'ÉNS, 5 (1972), 157–181

Among the topics that we aim to cover there are:

- Lattices and codes, how to construct ones from the others
- Weight enumerator for codes, the MacWilliams identity
- Modular forms: Eisenstein series, theta series associated to lattices, the Poisson summation formula
- The connection between codes and modular forms: the theorem of Broué and Enguehard.
- Codes over number fields.

The discussion of codes over number fields will serve as an introduction to algebraic number theory, it time permits.