SNS-B COURSES FOR THE ACADEMIC YEAR 2016-2017

Proposer	Area	Level	Sem.
Eugen Mihailescu	Dynamical Systems	CP/M1/M2	I
Sebastian Burciu	Algebra	CP/M1/M2	I
Daniel Cibotaru	Geometric measure theory	CP/M1/M2	I
Liana David	Differential Geometry	M1/M2	II
Cristian Anghel	Algebraic Geometry	CP/M1/M2	II
Liviu Ignat	Ecuatii	CP/M1/M2	II
Cristodor Ionescu	Algebra	M1/M2	II
Serban Stratila	Algebre de operatori	CP/M1/M2	II

FALL SEMESTER

Eugen Mihailescu–Dynamics on fractals
Level: PC/M1/M2
Course Outline:

This course is suited for Masters and Preliminary Cycle students. It uses, as background only the courses from first and second university years (for eg Real Analysis). Dynamics study the chaotic behaviour of various types of systems on fractals. They attracted a lot of attention in Mathematics in the last decades. We will present some important basic notions, results and examples. Some topics are:

(1) Topological transitivity, non-wandering sets.
(2) Examples of dynamical behaviour.
(3) Stable/unstable differentiable foliations.
(4) Markov partitions.
(5) Smale Horseshoes.
(6) Measure-preserving systems.
(7) 1-sided and 2-sided Bernoullicity.
(8) Topological pressure; variational principle.
(9) Chaotic systems on fractals; equilibrium measures.

Florentiu Daniel Cibotaru – Introduction to Geometric Measure Theory
Level: PC/M1/M2

Course Outline:
Geometric Measure Theory tools have wide applications: from Riemannian Geometry and Geometric Analysis to non-linear PDE and Optimal Transport. The purpose of the course is to familiarize the participants with the basic language and some fundamental facts of this topic. The only (strict) prerequisite is a course in Measure Theory. Knowledge of integration of differential forms (on chains and/or on smooth manifolds) is desirable but not strictly necessary.

(1) Review of basic notions of measure theory: Fubini-Tonelli, Radon measures, Riesz representation theorem, Hausdorff measures.
(2) Lipschitz functions: Rademacher theorem, area and coarea formulas.
(3) Differential forms: exterior derivative, Stokes theorem, homotopy formula, fiber integration.
(4) Currents: weak topology, basic operations, currents representable by integration, the mass and flat topologies, Constancy Theorem.
(5) Rectifiable sets and rectifiable currents with integer multiplicity: various characterizations. Compactness Theorem (sketch), the weak solution of the Plateau Problem.
(6) Slicing of integer multiplicity currents.
(7) Functions of bounded variation, sets of finite perimeter, Gauss-Green Theorem.

References

[1] . B. Folland, *Real analysis. Modern techniques and their applications*
[2] . Evans, R. Gariepy, *Measure theory and fine properties of functions*
[3] . Federer, *Geometric Measure Theory*
[4] . Krantz, H. Parks, *Geometric integration theory*
[5] . M. Lee, *Introduction to smooth manifolds*
[6] . Morgan, *Geometric Measure Theory. A beginners Guide*
[7] . Simon, *Lectures on Geometric measure theory*

Sebastian Burciu – Representation theory of finite groups
Level: PC/M1
Course outline:

The goal of the course is to cover the basics on the representation theory of finite groups, both in characteristic zero and positive characteristic. We will follow some parts of the books "Linear Representations of Finite Groups" by J. P. Serre and "Character theory of finite groups" by I. M. Isaacs.

In the first part we will develop the character theory for finite groups and prove their main properties such as linear independence, integrality and orthogonality relations. Using character theory we will prove Frobenius theorem stating that over the field of complex numbers the dimension of an irreducible representation divides the order of the group.

Next we will introduce the notion of kernel of a character and prove Brauer-Burnside's theorem on faithful characters. The basic results concerning Clifford's theory for representations of finite groups will also be described. A description of the irreducible representations of the symmetric groups in terms of Young tableaux will also be given.

The rest of the course will be devoted to the modular representation theory of groups. We introduce the notions of vertex and source of a representation and plan to prove the Green correspondence for them.

Some familiarity with linear algebra and basic representation theory of finite dimensional algebras will be useful, although all the required prerequisites will be reviewed during the course.

The course will last 10 weeks with 4 hours per week of classes, of which 1 or 2 hours will be devoted to problem sessions.

If the students are motivated, we may also discuss few more advanced topics in representation theory such as Green functors and crossed Burnside rings.

SPRING SEMESTER

Liana David - Conformal geometry and twistor theory
Level: M1/M2

Conformal geometry is one of the well-established areas in differential geometry. It is also the most natural generalization of Riemannian geometry. This course will be self contained and aims to give to the student a good basis in this field. A preliminary outline of the course is the following (this will be adapted during teaching, according to the level of the students):

1) Preliminary material: here we recall (shortly, mostly without proofs), the standard facts from differential geometry which we shall need along the course (e.g. the definition of complex and CR-structures, connections on principal and vector bundles, the Chern connection);

2) Examples of conformal manifolds: the flat model and the Feffermann space;

3) Basic facts on conformal geometry: density line bundles, Weyl connections, their curvature, the Weyl tensor, invariant conformal operators, Weitzenböck formulae;

4) The canonical connection of a conformal manifold and conformal holonomy;

4) Twistor theory in 4-dimensions: the twistor space of a conformal self-dual 4-manifold; the integrability of its almost complex structure; twistor interpretation of Einstein metrics on a self-dual conformal 4-manifold; the Penrose operator associated to a Weyl connection and a Penrose transform for the sections in its kernel; holomorphic structures on the vertical tangent bundle of a twistor fibation;

5) (If time allows): Conformal structures as G-structures; the group of conformal transformations.

Bibliography:

1. H. Baum, A. Juhl: *Conformal Differential Geometry, Q-curvature and Conformal Holonomy*, Birkhauser Verlag, 2010.

2. P. Gauduchon: *Structures de Weyl et theoremes d'annulation sur une variete conforme auto-duale*, Ann. Sc. Norm. Sup di Pisa, serie IV, vol. XVIII, fasc. 4 (1991).

3. P. Gauduchon: *Elements de geometrie twistorielles*, unpublished notes.

4. S. Kobayashi: *Transformation Groups in Differential Geometry*, Springer Verlag, 1972.

Cristian Anghel: Algebraic curves and surfaces by examples
Level: PC/M1/M2

The course is intended primary for the preliminary cycle, but can be adapted for the master level too. The aim of the course is to introduce the student in the world of algebraic varieties through the simplest ones: the curves and the surfaces. The (maximal) outline is the following:

- Affine and projective varieties.
- Divisors and line bundles
- Intersection theory in projective space. Bezout theorem. - Riemann-Roch theorem on curves.
- Intersection theory on surfaces.
- Riemann-Roch theory on surfaces.
- Birational geometry and rational surfaces.
- The geometry of surfaces in P3
- Complex versus algebraic varieties

Liviu Ignat – Harmonic Analysis and Partial Differential Equations

Level: CP/M1/M2

The course presents basic techniques to obtain the well-posedness to some evolution equations: heat, convection-diffusion, Schrödinger and wave equations.

The course is divided in two parts. The first one is focused on the harmonis analysis and give the students a review of the most classical properties of the Fourier transform.

The second part of the course will be dedicated to the study of the well-posedness of various nonlinear PDS's with the aim to understand for example local existence of the solutions to nonlinear problems, blow-up alternative, energy estimates. Finally we will present various results for the nonlinear Schrödingier equation by using advanced methods from the first part of the course.

This course continues at the master level the course that L. Ignat gave at the University of Bucharest to third year students. The course at UB was focused on basic properties of Fourier transform, linear models, Sobolev spaces with Fourier transform. The proposed course is going in the direction of nonlinear models by using advanced tools.

Course outline

(1) **Properties of the Fourier transform.** Oscillatory integrals: Asymptotics, Van der Corput's Lemma, Non degenerate critical points. Singular integrals: L^p inequalities for potentials. Littlewood-Paley Theory and multipliers: Marcinkiewicz multiplier Theorem on $\mathbb{R}$.
(2) **Dispersive equations**: Strichartz estimates for constant coefficient linear Schrödinger and wave equation. Local smoothing effect. Resolvent estimates. Well-posedness for nonlinear models.

Cristodor Ionescu – Homological Methods in Commutative Algebra
Level: M1/M2

The aim of the course is to introduce the students to the use of Homological Algebra in the study of Commutative Rings. This started by the works of M. Auslander, D. Buchsbaum and J.P. Serre in the second half of the last century and nowadays is a widely spread tool in Commutative Algebra.

Prerequisites: Basic Commutative Algebra (noetherian rings and modules, primary decomposition, Krull dimension) and Basic Homological Algebra (projective and injective modules, Tor and Ext).

Course outline

- Grade, depth and projective dimension;
- Koszul complex;
- Regular and Cohen-Macaulay rings;
- Gorenstein rings and Complete Intersection rings;
  Time permitting and depending on the interests of the audience we may also discuss:
- Canonical module; existence, uniqueness.
- Local cohomology; Depth and Krull Dimension via Local Cohomology.
- Applications to Combinatorics.

Serban Stratila – Basic Operator Algebras
Level: PC/M1/M2
Course outline:

The Course will cover the Functional Calculus for normal elements, Basic results in C*-algebra theory and von Neumann algebras (von Neumann bicommutant theoren and Kaplansky continuity and density theorems), Geometry of projections and classification of von Neumann Algebras, The existence of traces on finite von Neumann Algebras, The Coupling element for finite von Neumann Algebras with finite commutant, the Jones index of subfactors.

Prerequisites: The needed Functional Analysis is contained in the Appendix (cca 50p) of my book ”Integrala Lebesgue si Transformarea Fourier”, Theta Foundation, 2015