SNS-B COURSES FOR THE ACADEMIC YEAR 2015-2016

FALL SEMESTER

Eugen Mihailescu–Invariant probabilities and applications
Level: PC/M1/M2

The course is suited for Master students and for Preliminary Cycle students. It is mostly an Analysis course. The course requires a background at the level of the usual undergraduate university courses. If some notions will appear less familiar to students, then they will be done in class.

Invariant probabilistic measures are an important notion in Ergodic Theory/Dynamical Systems, Fractal Theory, Probability Theory, Stochastics, Analytic Number Theory, and are used also in Statistical Physics. They provide a better understanding of the geometric properties of invariant sets, and of the long term behaviour of certain systems.

Here is an outline of the chapters of the course:

1) Generalities about invariant probabilistic measures on compact metric spaces.

2) Ergodicity and mixing; decomposition of invariant measures into ergodic components.

3) Iterated function systems (finite and infinite cases). Self-similar measures. Conformal iterated function systems.

3) Invariant probabilities from symbolic dynamics and geometric fractal theory.

4) Inverse limits of measure-preserving endomorphisms on Lebesgue spaces.

5) Entropy of a partition, measure-theoretic entropy, conditional entropy, topological entropy.

6) Lyapunov exponents of invariant measures.

7) Gibbs measures for Hölder continuous potentials.

8) Uniform and non-uniform hyperbolicity conditions. Local product structure.

9) Correspondence between spectral properties of Koopman operator and ergodic properties.

10) Julia sets in complex dynamics, and Green measures.

11) Chaotic behaviour of orbits. Notions of thermodynamical formalism.

Sergiu Moroianu–Differential forms and algebraic topology
Level: PC/M1/M2
Prerequisites: Second year differential geometry course.

We aim to study the topology of manifolds using de Rham cohomology.

Outline:

- Quick review of differential geometry: manifolds, partition of unity, vector fields, differential forms.
- de Rham cohomology, Maier-Vietoris sequence.
- Poincaré lemma, computation of examples.
- Harmonic forms, Hodge theory.
- Poincaré duality.
- Relative and absolute cohomology of compact manifolds with boundary, duality.
- Signature, Euler characteristic.
- Vector bundles, connections, curvature.
- Chern-Weil characteristic classes.
- Hirzebruch's signature, cobordism invariance, L-class.
- Gauss-Bonnet-Chern formula for the Euler characteristic.

Bibliography: we will mainly use the book by Bott and Tu "Differential forms in algebraic topology", as well as Wells' "Differential Analysis on Complex Manifolds".

Traian Pirvu–Stochastic Calculus for Finance
Level: PC/M1/M2

Course Outline:

- General Probability Theory
- Information and Conditioning
- Brownian Motion
- Stochastic Calculus
- Risk Neutral Pricing
- Connections with Partial Differential Equations
- Exotic Options
- American Derivative Securities.

SPRING SEMESTER

Cristian Anghel–Topology of algebraic varieties
Level: PC/M1/M2

We intend to introduce the student to some topological problems associated with algebraic varieties:
- lectures 1-4 / exercises 1- 4 - smooth projective varieties
- lectures 5-9 / exercises 5- 9 - smooth quasi-projective varieties
- lectures 10-12 / exercises 10-12 - a brief intoduction to singularities of algebraic varieties with special emphasis on plane curves.

Cristian Cazacu–Selected topics in Partial Differential Equations
Level: PC/M1/M2

The aim of this course is to present some general tools for solving either linear or nonlinear partial differential equations.

First we will treat variational methods to study the existence (and uniqueness, if it is the case) of solutions to elliptic equations.

Then, making use of the semigroup theory, we analyze the well-posedness issue of standard evolution equations (e.g. heat, wave and Schrödinger equations) in the linear or semi-linear context. Moreover, we will study qualitative aspects of these equations such as local/global existence of the solutions, blow-up alternative, energy estimates, the asymptotic behavior in time, regularity, conservative properties, etc.

Although there is not a continuation of a previous SNS-B course, some of the topics of this proposal are related to the SNS-B course on *Nonlinear Evolution Equations* delivered by L. Ignat and myself in the Spring 2015.

Course Outline

(1) **Elliptic equations.**
   - *Linear equations.* Existence and uniqueness of the solutions via Lax-Milgram and Stampacchia lemmas, maximum principle, comparison methods. Self-adjoint and skew-adjoint operators and their properties.
   - *Nonlinear equations.* The direct method of Calculus of Variations. Non-existence results: multiplier techniques and Pohozaev-type identities. Unique continuation properties. Regularity and bootstrap arguments.

(2) **Evolution equations.**

- *Semigroup theory.* Maximal disipative operators and well-posedness of the abstract Cauchy problem, Hille-Yoshida theory for the homogeneous/nonhomogeneous problem.
- *Wave equation:* multiplier techniques and energy methods. Local/global existence of solutions for nonlinear equations via Banach fix point theorem. Hidden regularity results.
- *Heat and convection-diffusion equations:* asymptotic behavior and time decay rates. Tools: scaling, self-similarity transformations, Kato and Sobolev-type inequalities.
- Local/global existence for linear and nonlinear *Schrödinger* equations, the Fourier transform, local smoothing effect.

Alexandru Gica–Analytic Number Theory
Level: PC/M1
The main purpose of this course is to present the classical results of Analytic Number Theory, which should be part of the culture of every mathematician.

The course outline:

1) Zeta Riemann function. The zero free region. The Grand Riemann Hypothesis.

2) Bernoulli numbers. How to compute $\zeta(2k)$, where k is a positive integer.

3) The Prime Number Theorem.

4) Primes in arithmetic progressions (Dirichlet's theorem).

5) Sieve methods. Brun's theorem (the sum of the reciprocals of the twin primes converges).

6) Circle method. Vinogradov's theorem (every large odd integer is the sum of three primes; recently was proved that the result is true for every odd integer $n \geq 7$).

Serban Stratila–Compact Operators, Unbounded Operators and Integral and Differential Equations
Level: PC/M1/M2
Course outline:

- Compact Operators in Hilbert Spaces
- The duality between the trace class operators, compact operators and all bounded operators
- Compact Operators in Banach spaces
- Application: Integral Equations
- Basic Theory of Closed Unbounded Operators
- Examples: Differential Operators.