CURSURI SNS-B: CICLUL PREGĂTITOR 2014-2015

Semestrul I

Representation theory of finite groups
Sebastian Burciu

The goal of the course is to cover the basics on the representation theory of finite groups. We will follow some parts of the books "Linear Representations of Finite Groups" by J. P. Serre and "Character theory of finite groups" by I. M. Isaacs.

In the first part we will develop the character theory for finite groups and prove their main properties such as linear independence, integrality and orthogonality relations. Using character theory we will prove Frobenius theorem stating that over the complex numbers the dimension of an irreducible representation divides the order of the group. Next we will introduce the notion of kernel of a character and prove Brauer-Burnside's theorem on faithful characters. The basic results concerning Clifford's theory for representations of finite groups will also be described. A description of the irreducible representations of the symmetric groups in terms of Young tableaux will also be given.

Some familiarity with linear algebra and basic representation theory of finite dimensional algebras will be useful, although all the required prerequisites will be reviewed during the course.

The course will last 12 weeks with 4 hours per week of classes, of which 1 or 2 hours will be devoted to problem sessions. There will be weekly homework and a final written project or oral exam.

If the students are motivated, we may also discuss few more advanced topics in representation theory such as modular representation theory, Green functors and crossed Burnside rings.

Semestrul II

Introduction to differential topology
Dorin Cheptea

The course aims to introduce the readers to the basic objects and properties in Algebraic and Differential Topology, and to prepare them to be able to read advanced courses and graduate-level literature.

1. Homotopy, isotopy, immersions, embeddings, transversality, intersection index, degree of a mapping, Sard's theorem.

2. Introduction to homotopy and homology theory from algebraic and differential viewpoints, cell complexes, cell homology, de Rham cohomology.

3. Morse theory.

4. Time permitting, we can go into more or less detail about characteristic classes, handlebodies, intersection forms, basic ideas in the geometry and topology of 2-, 3- and 4-dimensional manifolds.

Necessary preliminary knowledge: basic analysis (implicit and inverse function theorems, differential forms), basic differential equations (basic existence and uniqueness), basic differential geometry (manifolds, vector fields) and basic general topology. If partition of unity, covering spaces, and the definition of the fundamental group are not known by all readers, they will be introduced.

Main references for this course proposal:

- John Milnor, *Topology from the differentible viewpoint*
- John Milnor, *Morse theory*
- James Munkres, *Elementary differential topology*
- Yukio Matsumoto, *An introduction to Morse theory*
- Raoul Bott, Loring Tu, *Differential forms in algebraic topology*

Additional references:

- Allen Hatcher, *Algebraic Topology*
- Boris Dubrovin, Anatolii Fomenko, Sergei Novikov, *Modern geometry. Methods and applications*

There would be 24 courses of 2 hours each. All but the first and last course would be 1 hour of lecture + 1 hour of problems and discussion.

Group extensions and group cohomology

Phillipe Gille

Given a group G, an extension E of G by another group H is a group E together with a surjective morphism $f : E \to G$ such that $\ker(f)$ is isomorphic to H. For example, there are only two extensions of the symmetric group S_n ($n \geq 4$) by $\mathbb{Z}/2\mathbb{Z}$ (Schur). It is of interest to classify group extensions for many purposes: group theory, Galois theory, topology,...

The theory of group extensions is related to the homology and cohomology of groups. More precisely, one shall define the homology groups $H_i(G, \mathbb{Z})$ and the cohomology groups $H^i(G, \mathbb{Z})$ which are important invariants related in small weights with group extensions of G.

If time permits, we shall sketch the begininning of Galois cohomology and how group extensions of absolute Galois groups are related to the Galois inverse problem.

We shall use Weibel's book "Introduction to Homological Algebra".

Algebraic number theory

Vicentiu Paşol, Alexandru Popa

After reviewing Galois theory for arbitrary field extensions, we will present the study of finite extensions of the rational numbers. Among the topics included will be: ideal factorization in rings of integers, the ideal class group and the unit group, the Dirichlet class number formula. If time permits, at the end we will give an overview of Class field theory. Emphasis will be placed on examples and applying the theory to concrete problems in number theory.

We will not assume any prerequisites other than familiarity with the basic structures of algebra (groups, rings, fields). We will follow the textbook *Number fields* by D. Marcus.