CURSURI SNS-B: CICLUL MASTERAL 2014-2015

Semestrul I

Invariant probabilistic measures and applications
Eugen Mihăilescu

The course is suited for Master students, but also for advanced Preliminary Cycle students. It would be suited for students starting to work in Analysis, and more particularly in Measure Theory, Probability, Analysis/Global Analysis, Functional Analysis, Statistics, etc.

It requires a background at the level of the usual undergraduate university courses. If some notions will appear less familiar to students, then they will be done in class.

Probabilistic measures are an important tool in fractal theory, ergodic theory, dynamics, stochastics, etc., and are used also in statistical physics. They also provide a better understanding of geometric properties of invariant sets, and of long term behaviour of dynamical systems. Here is an outline of the chapters of the course:

1) Generalities about invariant probabilistic measures on compact metric spaces.

2) Ergodicity and mixing; decomposition of invariant measures into ergodic components.

3) Iterated function systems (finite and infinite cases). Self-similar measures. Conformal iterated function systems.

3) Invariant probabilities from symbolic dynamics and geometric fractal theory.

4) Inverse limits of measure-preserving endomorphisms on Lebesgue spaces.

5) Entropy of a partition, measure-theoretic entropy, conditional entropy, topological entropy.

6) Lyapunov exponents of invariant measures.

7) Gibbs measures for Hölder continuous potentials.

8) Uniform and non-uniform hyperbolicity conditions. Local product structure.

9) Correspondence between spectral properties of Koopman operator and ergodic properties.

10) Julia sets in complex dynamics, and Green measures.

11) Chaotic behaviour of orbits. Thermodynamical formalism. Equations involving pressure functions.

The Selberg trace formula
Sergiu Moroianu

The goal of the course is to present a proof of the Selberg trace formula for the Laplacian on compact hyperbolic surfaces. It aims to motivate both hyperbolic geometry and the theory of elliptic operators on compact manifolds.

I will introduce the hyperbolic plane and its isometries, geodesics, distance function. I will recall the fundamental group, and show that closed geodesics on a complete hyperbolic surface are in one-to-one correspondence with conjugacy classes of hyperbolic elements in π_1. I will introduce the Laplace operator Δ on functions and show briefly that its spectrum on a compact surface M consists of positive eigenvalues accumulating to infinity.

If $f : \mathbb{R} \to \mathbb{R}$ is a real function decreasing at infinity sufficiently fast (with some additional properties), so that the sum of $f(\lambda_j)$ over the eigenvalues of Δ is absolutely convergent, I will show that this sum can be computed in terms of the lengths of closed geodesics on M by the explicit trace formula due to A. Selberg.

One nice application is the Selberg zeta function, its functional properties and the study of its zeros.

If the students are motivated, we may also include the trace formula on the modular surface (which is neither compact nor smooth) and display its link to the Riemann zeta function.

Selected topics in statistical mechanics of classical particle systems in continuum
Diana Puțan

One of the main aims of statistical mechanics is to describe the macroscopic behaviour of a system of particles knowing what happens at the microscopic level. The theory is very well developed for lattice systems, i.e. when the particles lie on the vertices of a lattice. However, we will be concerned with the continuous case, in the sense that the particles are represented by points in $\mathbb{R}^d$. With this in mind, we introduce the space of configurations Γ as the set of locally finite subsets of $\mathbb{R}^d$. We proceed by studying measures on Γ, namely the Lebesgue-Poisson measure, the Poisson measure and Gibbsian perturbations of the Poisson measure, which represent the macroscopic equilibrium states of the system. A further topic of interest will be combinatorial harmonic analysis on configuration spaces. We introduce the so-called K-transform, which can be seen as a combinatorial equivalent of the Fourier transform. This will be useful in the definition of correlation measures for given states (i.e. probability measures).

Prerequisites: Basic notions of measure theory, probability and functional analysis.

Semestrul II

Nonlinear Evolution Equations

Liviu I. Ignat, Cristian Cazacu

The course presents basic techniques for systematically analyzing the well-posedness issue of some evolution equations: heat, convection-diffusion, Schrödinger and wave equations. The course is divided into two parts. The first part treats the classical theory as in Cazenave-Haraux [2] with the aim of understanding, for example, local existence of the solutions to nonlinear problems, blow-up alternative, energy estimates. The second part of the course will be dedicated to the study of well-posedness for the nonlinear Schrödinger equation. This part will contain recent tools from harmonic analysis. This course continues at the master level the course that L. Ignat gives at the University of Bucharest for third year students. The course at UB is focused on basic properties of the Fourier transform, linear models, Sobolev spaces with the Fourier transform. This course runs through the properties of nonlinear models by using advanced mathematical tools.

References:

[1] H. Brezis, Functional analysis. *Theory and applications. (Analyse fonctionnelle. Théorie et applications)*, Collection Mathématiques Appliquées pour la Maîtrise. Paris: Masson, 1994.

[2] T. Cazenave and A. Haraux, An introduction to semilinear evolution equations. Transl. by Yvan Martel. Revised ed., Oxford Lecture Series in Mathematics and its Applications. 13. Oxford: Clarendon Press. xiv, 1998.

[3] L. Grafakos, *Classical and modern Fourier analysis*, Pearson Education, Prentice Hall, Upper Saddle River, NJ, 2004.

[4] M. Keel and T. Tao, *Endpoint Strichartz estimates.*, Am. J. Math. 120 (1998), no. 5, 955–980.

[5] F. Linares and G. Ponce, *Introduction to nonlinear dispersive equations*, Universitext, Springer, New York, 2009.

[6] E.M. Stein, *Singular integrals and differentiability properties of functions*, Princeton Mathematical Series, No. 30, Princeton University Press, N.J., 1973.

[7] E.M. Stein, *Harmonic analysis: Real-variable methods, orthogonality, and oscillatory integrals*, Princeton Mathematical Series. 43. Princeton, NJ: Princeton University Press, 1993.

Groups, actions and von Neumann algebras

Mihăiță Berbec

The classification of type II_1 von Neumann algebras has seen spectacular progress during the last 10-15 years, thanks to Sorin Popa's deformation-rigidity theory. The purpose of this course is to offer a gentle introduction to this theory, mainly by studying examples. In the first part of the course we study von Neumann algebras associated to countable discrete groups and to measure preserving actions of countable groups on probability measure spaces. We begin by introducing these two constructions and studying their basic properties. Then we study relevant approximation properties for groups and actions, such as amenability and property (T), and see how they translate to von Neumann algebras and why is relevant to consider them in this context. In the second part of the course, we move towards classification problems and structural properties of II_1 factors and see a few striking results this theory led to, focussing on concrete examples of groups and actions (free groups, hyperbolic groups, Bernoulli actions, etc.)

Course Outline: (4h for each topic)

- Fundamentals on von Neumann algebras
- Von Neumann algebras associated to countable discrete groups
- Von Neumann algebras associated to measure preserving actions of groups on probability spaces
- Amenability and property (T) for groups and von Neumann algebras
- Other approximation properties for groups and von Neumann algebras (Haagerup property, weak amenability, etc.)
- Towards classification of II_1 factors and structural properties (depending of the level of the students)