# Romania helps Uganda for the International Mathematical Olympiad

I am Sergiu Moroianu, a former gold medalist at the 1991 IMO. This is the story of supporting Uganda's participation in the 2019 and 2020 International Mathematical Olympiads.

I am writing this report to thank all of you who have been so generously involved in our project so far. It was an adventure of the human mind and of true friendship between nations, in a pragmatic age like ours, more cynical and decadent rather than idealistic or disinterested.

# About the IMO

It all started in Sinaia in 1959 with the first "International Mathematical Olympiad" (IMO). Only eight Eastern countries participated. Since then, the IMO took place every year and has grown steadily. No less than 115 countries competed in the 60th edition in England! The IMO is today the most prestigious scientific event for high-school students around the world, always copied, never surpassed. The IMO model was adopted for other disciplines: physics, computer science, chemistry, etc., and also for regional competitions: the Balkan Mathematical Olympiad, the Central European Olympics, the Pan-African Olympics, etc.

The IMO is an intellectual competition similar in nature to the athletic Olympics. Participants train from an early age - typically 10-11 years old - and dedicate all their free time to preparing for the IMO during 6-7 years. They are trained by professionals, just like performance athletes, they participate in training camps and courses, and work driven by inner motivation. Although for intellectual "sports" there exists no equivalent of the doping from physical activities, we still find doping-type behaviors in preparation for math competitions, most often manifested through memorizing strong results without assimilating in depth their proof or the theory behind them. What distinguishes the IMO from other elementary mathematics competitions is precisely the prominence of creativity over "doping": since IMO-type problems have simple statements but require original ideas, it is neither necessary nor sufficient to know hard theorems by heart.

The IMO evaluates a certain ability of solving elementary-level problems fast. This ability develops through long-term exercises and is not identical to what is needed to become a professional mathematician. While it is true that many IMO participants later become professional mathematicians, many of them choose other professions as well.

Professional mathematicians would stand no chance at the IMO against student participants, just as in sports coaches would not be able to compete directly with their pupils.

IMO is a real Olympics in the traditional sense of the word because it keeps alive the Olympic spirit of fair-play. Competitors fight to outdo themselves, not against other participants. Another nice feature of the IMO is the quasi-absence of cheats. We can count on the fingers of one hand the unpleasant events when participating teams illegally communicated with their leader, obtaining the statements before the contest. These violations were severely punished. Grading at the IMO is done by team leaders, not by a committee. Coordinators, usually former IMO participants, have only the role of verifying the score given by the leader and validating it. If leaders sometimes insist on getting an odd extra point, it is not uncommon for the coordinators to propose a higher grade than the one initially given by the leader, because they find that the solution proposed by the competitor is close to one of the solutions known to the jury.

### IMO and Romania

The IMO is an Eastern European cultural product, to which Romania has made an important contribution. A notable forerunner of the IMO was the yearly competition organized by the Gazeta Matematică, the oldest periodical publication in elementary mathematics in the world, published without interruption since 1890, even during the wars.

The IMO was born in the "second world" - the so-called communist camp - but quickly spread to developed countries. At the first twenty or so Olympiads, Easterners authoritatively dominated the competition, but today countries such as Italy, France, Canada, Germany or Australia compete on an equal footing against Poland, Ukraine, Romania, Bulgaria or Hungary. The explanation lies - in my opinion - in the professional methods of selection and training that I will describe below, initially introduced in the ex-communist countries. These methods yield spectacular results wherever they are adopted. The countries that have stood out at the IMO in recent years are China, the USA and Russia. The last time Romania ranked 1st was in 1996.

### Training and selection for the IMO

In order to compete successfully, Romania selects future IMO participants from all middle schools in the country. Starting with fifth grade, students are offered the opportunity to solve, each month, four Olympiad-type problems from the Gazeta Matematică. The most diligent of them get to participate at the end of the year in the GM solvers contest. The mathematics Olympiad, organized annually, has two mass stages - school and county - where the best students from each class in the country participate. Middle school and high-school teachers receive favorable evaluations if their students qualify for the higher stages of the competition. In each county capital, extracurricular excellence clubs are organized weekly, paid for either by the Ministry of Education or by private foundations. Teachers in top schools and high-schools train their talented students far beyond the official curriculum. Numerous training camps, clubs, online training programs take place every year.

Coaches for the Olympiad are most often former participants in the Olympics and former problem-solvers of Gazeta Matematică.

Through this approach, native talent is selected from a maximal selection base, i.e. all the 200,000 students in a generation. A significant number of talented students are subsequently trained over a long period of time, encouraged and stimulated, including materially. The costs of participating in the Olympiad from school stage to IMO are fully covered by the state. The winners of the Olympiad receive prizes in cash, excursions, automatic admission to the state universities in Romania, but also admission offers and scholarships from universities abroad.

As in sports, the results at the math Olympiads are directly correlated with the material effort invested by society in young competitors.

### Third world countries and the IMO

Developing countries have taken a more sinuous path in adapting to the IMO competitive culture. Some of them - Thailand, Korea, Singapore, but also Peru, Colombia or Brazil - now have a solid tradition, and their rise in IMO rankings follows that of their GDP/capita. Others "missed the train". The most disadvantaged countries in terms of participation - and results - in the IMO are those of sub-Saharan Africa.

Until 2010, no country in "Black Africa" - that is, other than the Maghreb and South Africa - participated in the IMO. Only 11 African countries out of 54 participated in the 2020 International Olympiad organized by Russia, of which 7 were sub-Saharan: Nigeria, Ghana, Kenya, Tanzania, Botswana, Ghana and Uganda.

There are some obvious structural difficulties for a country that is only now starting to attend the IMO in competing against countries that already have a 50-year Olympiad tradition behind them.

- In the 1960s and 1970s, at the beginning of the IMO, Uganda was just gaining independence, and later went through devastating civil wars. As years go by, it becomes more and more difficult for a country without professional coaches, without the experience of the mass Olympiads and without former IMO participants, to obtain good results fast enough in order to justify further participation.
- Other difficulties are of a practical nature, difficult to understand in our privileged countries. The leadership of some poor countries does not value the Olympiads, considers them an unnecessary luxury, and therefore does not finance either the national Olympiad - the natural basis of selection - or the preparation for IMO. Some do not even pay for plane tickets, hotel and visas! Other countries simply cannot afford to pay this money, even if they wanted to. And in some sad cases, countries have prepared their team, have invested in selection and training and did pay for their plane tickets, only to be eventually denied entry visas in the host country. This is the case of the Nigerian team, which in 2019 had a student on whom they placed high hopes for winning at least a bronze medal. For bureaucratic reasons, the team's visas for England were not issued until the first day of the competition, when their plane tickets had already been canceled by non-presentation.
- IMO problems are getting harder every year. As an example, here is the first problem from IMO 1959 (Braşov, Romania): Show that the fraction (21n+4)(14n+3) is irreducible for any natural number n. This problem is today accessible even to a good sixth grade student!

# Why Uganda?

In 2018, the 59th IMO took place in Cluj-Napoca. Along with over 40 other former participants, some from abroad, others from Romania, for three days I was part of the team of coordinators. I spoke to half of the more than 100 team leaders from all over the world. The problem I coordinated was of level 3 - the most difficult - and because of that most of the papers were either empty or did not contain anything worth discussing. According to the protocol however, we held a courtesy meeting with these leaders even if their team did not claim any points. I took the opportunity to discuss with them the situation of the Olympiad in their respective countries. One of these leaders was Jasper Okello of the Ugandan team and the initiator (I found out later) of the idea of participating in the IMO of this country. He left me the impression of being honest and involved. I decided to get involved in helping him (without guessing how much I will actually get involved).

# The mathUganda project

I was surprised to learn that Uganda does not support its students to participate in the IMO. Moreover, the Uganda Math Society has never been able to obtain support for participation from bodies International international such as the Union of Mathematicians (IMU), which are otherwise generous with funding of all kinds. During the following year, together with Jasper we sent funding applications to various charities, which unfortunately remained without result. I wrote directly to some acquaintances in the academia without more success. It seems that funding from rich countries is simply not aimed at talented young people in countries like Uganda. Motivated by these failures, I opened a private list of online donations at http://www.imar.ro/~sergium/mathuganda/new mU/index.html. The total travel cost of the team (2 leaders and 6 students) was estimated about 10,000 euros, of which I anticipated that we would raise 1,000.

I am still amazed by the generous response I received. The donations started to flow in amounts between 5 and 200 euros, from family, friends, colleagues or strangers who found out about the project by accident. I reached my target in just 3 weeks. The SSMR (Romanian Society of Mathematical Sciences) agreed to become involved in the project, and colleagues with solid experience in training for IMO offered online lessons. I received a substantial sponsorship from a private company, Frame Film, whom I thank

again here. By June 2019, I had raised \$8,000, which we transferred in three stages to the Uganda Mathematical Society to buy plane tickets and pay for visas.

Thanks to our help, in 2019 Team Uganda

- presented a complete team of 6 students, unlike 2018, when they could fly only 4 participants;
- benefited from 13 distance training sessions with the help of Radu Bumbăcea, Dragoş Manea, Flavian Georgescu, Liviu Paunescu and Lucian Turea, coaches of the Romanian team;
- was able to purchase plane tickets early enough to be able to submit the visa applications on time, unlike the less fortunate Nigerians who did not receive their visa, as mentioned above.

The results in the 2019 competition were somewhat disappointing. Uganda presented the same team of 3 girls and 3 boys which had won 3 bronze medals at PAMO 2019. But Reagan Yuggu, the informal leader of the team, was ill during the competition. In total, the UGA team scored only 5 points and ranked 102nd out of 112 participating countries. An unfavorable factor was the absence of a level 1 Geometry problem at IMO 2019. Before that year, Uganda had twice received a Honorable Mention in 2016 and 2017, through students who solved the Geometry problem (Joseph Ssekitto and Andrew Tugume, respectively). The best ranking of the team had been reached in 2017 (14.55%). The performance in 2019 (9.01%) was nonetheless the third-best since their first participation in 2012.

It is worth noting that out of the 5 points obtained by Team Uganda, 2 were due to the UGA6 participant, Eva Kakyo, who was initially a reserve and joined the team in April because one of the team members renounced participating for personal reasons. Eva's trip to IMO2019, and implicitly the team's result, are thus largely due to the generosity of our Romanian sponsors.

### Expanding international assistance in 2020

Encouraged by the success of our project, Jasper Okello applied for and succeeded in getting Uganda included in an MIT IMO-preparation program that had already been implemented in Ghana in 2019. In January 2020, a team of three MIT students, including a former gold medalist at IMO 2018, oversaw the annual selection camp for the Uganda team near Kampala. Under the leadership of Michael Ren, the two weeks of camp were devoted to intensive courses and selection tests. Uganda's 2020 team for IMO and PAMO 2020 was selected in this professional way.

### Training course at Kampala

In February 2020 I traveled to Kampala for two weeks at the invitation of the mathematics department of Makerere University (MUK), the most prestigious higher education institution in East Africa. Makerere University is also the headquarters of the Uganda Mathematical Society (UMS), which organizes the local Olympiad and the selection of the IMO team. I paid for my trip, the hotel, the yellow fever vaccine and dinner from my own funds. I donated to the Uganda Mathematical Society 6 used-but-functional laptops that I received from donors in Romania, as well as 15 volumes of high-school mathematics offered by the GIL publishing house in Zalău (these printed books were very highly appreciated by the students).

I trained the IMO + PAMO teams of Uganda for ten days. Preparation began each morning at 8:00AM. At 10:30 we had a short breakfast, and at 13:00 a matoke-based lunch, which was supported by the UMS. The students paid for their own food and daily commute by minibus and boda-boda through the crowded African metropolis.

The training camp was attended, besides by five of the 6 members of the UGA 2020 IMO team, by the three girls who completed Uganda's team for the Pan-African Mathematical Olympiad (PAMO). A few words about this Olympad: the specific regulations require teams to have as many boys and girls. In 2019, Uganda's IMO team already contained 3 girls (they finally got more points than the boys in the IMO; one of them, Hagar, won a bronze medal at PAMO 2019). In 2020, the IMO team consisted only of boys, so the first three girls in the selection tests were additionally selected for the PAMO.

In the training courses I set myself the goal to make the team comfortable with PAMO level problems. I covered:

- synthetic geometry, with special emphasis on similarity, the circle, cyclic quadrilaterals, intersecting secants, polar lines (65% of the time)
- recurrent sequences, a topic often present at PAMO (5%)
- inequalities: Cauchy-Schwarz, AM-GM and applications (10%)
- various other Olympiad-type topics, including number theory (5%)
- functional equations (15%)

• funny physics problems, during one afternoon when they had received their results at the national exams and were too excited for "serious" things. These exams determine their future, insofar as their university admission and scholarship depend on them.

The level of the team was homogeneous among students but unequal across math subjects, due to the fact that they had attended virtually no Olympiad-type training until 2020. Recurrent sequences and functional equations were virtually unknown to them, and they had not seen at all Olympiad-type problems requiring clever reasoning without relying on much theory. About inequalities, they knew about Cauchy-Schwartz and AM-GM but had never solved problems with them. I was impressed that they quickly solved the following problem, which seems impossible to many adults:

# *Construct 3 equilateral triangles with 6 matches, without breaking or overlapping them.*

Geometry is traditionally their strong point. A professor from the Mathematics department at MUK coached the team for their first participations; in time, this led to the performance of completely solving a geometry problem in 2016 and 2017. To achieve excellence on this topic, we thoroughly covered the whole theory starting from the axioms (the three cases of congruence and parallelism). I had already noticed in 2019 that students tend to learn results "by heart", while at the Olympiad all reasoning must be fluent, without "black boxes" and without gaps in proofs, because those can lead to circular reasoning. The standard Ugandan curriculum is also very far from the type of problems required at the Olympiads, so we first had to "un-learn" everything they knew in order to prove again all the results at expert-level. The beginning was slow. We are talking about 17-18 years old students who had learned these things 4-5 years ago. They needed some good minutes until they re-discovered the proofs of the sum of the angles in a triangle, the properties of isosceles triangle and of the parallelogram, concurrence of important lines, similarity, Thales. We continued with the properties of angles inscribed in a circle and with cyclic guadrilaterals. From that point on they took off! We started to solve together various problems in a "IMO training" format as I know it: I would give them a list of 2-3 problems, the student who solved one of them would explain it to the others on the board, and eventually I would rephrase the proof with more details if needed. At the beginning, the lists of problems were at the level of middle-school Olympiads in Romania, then we advanced to problems taken from PAMO, the Balkan Olympiad or even the IMO. There was a moment of catharsis when I first told them (casually) that the problem they had just solved was from some IMO in the 2000s. The light in their eyes was priceless!

My evenings were busy preparing for the next day. Very soon, I found myself in the situation so familiar to IMO coaches, where the lists of 10-20 problems prepared with difficulty on the eve were solved by the students in just 2-3 hours. I started to vary the topics and introduce recurrent sequences, then inequalities and the rest of the material.

### Jesse and the others

I noticed two aspects: the first, as I said above, was the homogeneity of the team. The second was Jesse's faster progress compared to his colleagues. I had learned from Jasper that the best student on the team was Michael, and the most promising was Shafik, who was only in 11th grade. Unfortunately, Shafik could only participate in one day of training because the new management of his high school, the "Light Academy", did not allow him to participate on weekdays. At the IMO 2020, Shafik did not solve the geometry problem that I think he would have solved, had he participated in the training. As for Michael, he was indeed the one who knew the most at the beginning of the camp, and at IMO he got 1 point on Problem 1 but also 1 point on Problem 3, being only the second participant from Uganda to get a nonzero result in any problems of category 3, the hardest. For some reason I do not control, however, during my lessons he guickly relinguished the role of leader to lesse, the one who made the fastest progress. Such reversals of the informal hierarchy often occur in preparation for the Olympiads. Some students simply "explode" at a certain point, surpassing their peers without any apparent explanation.

Most of the students came from middle-class families. The exception was Jesse, who is the son of a former member of Parliament, now in the opposition. While the others traveled to Makerere University by public transportation, Jesse was either riding an Uber or was being driven by a chauffeur. He has only one sister, while everyone else came from families with at least 4 children. But at no time did I detect any conceited attitude from him. In fact, all these students from Uganda left on me a very positive personal impression. During class, the girls sat in the first row along with Michael and Shawal. In the second row were Daniel and Jonathan, who to me looked like twins even though they are of different age (both of them solved Problem 1 at IMO 2020). Jesse sat in row 3, as if he wanted to oversee the whole room. He had the same high-road approach toward mathematics; probably this was what allowed him to make progress so quickly. In geometry he immediately adopted the attitude of a "hunter" of congruent angles arising from cyclic quadrilaterals. In inequalities he exceeded my level pretty soon. He could solve those type of problems faster and more "correctly" than me, in the sense that he found the most natural solution, while myself I would prepare sometimes a more "twisted" solution (these are notions that only those who prepared for the Olympiads can appreciate).

In functional equations we had a rougher start. It was one of the areas where I failed to bring them to my current level. Myself I was juggling with seemingly absurd substitutions like "x = f(x)" or notions like injectivity, surjectivity and symmetry, which they either didn't know or found quite difficult to use. The most interested in the subject was Shawal, who had in the end become skilled enough to successfully tackle competition-level problems. Unfortunately, PAMO was not held in 2020 due to the pandemic, and at the IMO there were no problems with functional equations.

Both Daniel and Jonathan were also efficient at functional equations and inequalities. During the training they used to explain their solutions to each other; this gave them an advantage over the others, who did not systematically talk to their colleagues. They both have a very neat and coherent writing, which in the end saved them from certain errors of reasoning, or at least allowed me to spot their mistakes, if any. At the IMO 2020 they both solved the Geometry problem.

### Uganda team results at IMO 2020

Due to the 2020 pandemic, the Olympiad organized by Russia, which was to take place in St. Petersburg in July 2020, was organized online in September. Jesse was already in the United States, where he is currently enrolled as a freshman at the University of Illinois at Urbana Champaign. The IMO community in the US arranged for him to participate in an accredited competition center in Portland. The 5 other students attended the competition in Uganda. In order to purchase the surveillance equipment, I had to

urgently transfer to the UMS the last \$1,500 I had raised after July 2019. Fortunately, the organization according to the security requirements was successful, and Team Uganda was able to take part in the competition under the conditions imposed by the organizers.

Jesse Ekanya achieved this year the best performance at any IMO in the history of Uganda's participation: Honorable Mention with 7 points for Problem 1 and 2 points for Problem 2, being the first competitor in Uganda ever placed in the second tier of competitors (35, 93%).

Daniel Ayebare and Jonathan Ngabirano solved Problem 1, obtaining also Honorable Mentions. Together, Jesse, Daniel and Jonathan solved 3 problems, surpassing the cumulative performance of Team Uganda from 2012 to 2019. Compared to 2012 and 2013 when the team scored only 2 and 1 point respectively, the leap is truly impressive.

Shafik Kayemba scored 4 points, Michael Sserwanga 2 points and Shawal Mbalire 0 points. It is worth mentioning the role of hazard in reaping these partial-credit points: each competitor has typically a strong field in which he or she is more likely to solve the problem, or at least have an idea worth some points. In Shawal's case, it is likely that he would have gotten some progress on a level 1 problem in functional equations, which did not exist this year. Geometry was also accessible for him, in fact.

The aggregate result was Uganda's 87th place out of 105 participating countries, beating for example Algeria, Morocco, Chile or Costa Rica, countries that are significantly richer and better placed mathematically.

### The Geometry problem

In September 2020 during the IMO I tried to solve myself Problem 1. At first it didn't look like anything familiar, but when I managed to draw an acceptable picture I had a lucky premonition: this problem was approachable with the cyclic quadrilateral methods I had so much insisted upon during the Kampala camp in February! From my direct experience with them, all the 5 students who took part in the training were perfectly able to solve it completely. So the final result in which 3 out of 6 students solved this problem was in line with what we knew about their abilities. I must emphasize this fact in

order to dispel the legitimate questions that may be asked about this performance happening precisely in the year when the IMO took place online.

# What are the explanations for Team Uganda's success at IMO 2020?

- A decisive role was played by the selection camp held in January by the MIT team led by Michael Ren (gold medal at IMO 2018). This camp produced a solid and homogeneous team that already had two weeks of high-level training and contests behind them.
- Another favorable element was the postponement of the IMO until September. In this way, the students had 2 months of additional training. For students from China, Russia or Romania, who prepare intensely starting from the age of 10, these two months no longer really matter. But for Team Uganda, who only came into contact with the Olympiads in January 2020, two months means 1/4 of their total IMO training time.
- I have already described in detail how the training period in February went and how it contributed towards solving the geometry problem. Would the three students have taken the maximum score on the geometry problem without this training? Maybe yes, maybe no. I think that the training probably made it easier for them to progress from 1 to 7 points for Problem 1.
- I have not yet mentioned the role of Andrew Tugume (honorable mention at IMO 2017) in preparing the team. Andrew, currently an Engineering student in Kampala, delivered an excellent Geometry lesson in my presence. He was the team's main coach in the months that followed. All IMO teams benefit from the involvement of former participants. Although Uganda does not count many such former participants, the fact that they have Andrew is precious. I hope he will stay involved in the future.

# Jasper

Without question, the most important success factor in any competition is participating. It is clear that Uganda would not be able to get results at the IMO if they did not participate, and this participation is largely due to Jasper Okello, a mathematics teacher at Nabisunsa Girls 'School. Assisted by Assumpta Kasamba, a teacher at King's College Budo, Jasper has been the heart and driving force of this project for 10 years. The first participations were chilling, as Uganda's team scored between 1 and 2 points. The prohibitive cost of travel was not covered by the state, so sponsors had to be found. But what sponsor from such a poor country would finance the trip to a contest where the team risks getting 0 points?! The fact that Jasper managed to participate in these conditions is admirable. His tenacity was rewarded: first the team started being helped by the "Light Academy" high-school, which hosted the selection camps and supported travel costs to the IMO for two students, even if they did not come from that school. Later on, when the director of the Light Academy was replaced and the new one did not help logistically anymore, there came the good results of Joseph Ssekitto and Andrew Tugume from 2016-2017. The year 2018 was a critical point in Jasper's project. The Light Academy was no longer helping, and at the IMO organized in Cluj-Napoca Uganda could only present a team of 4 students. Visas for Romania cost 100 euros each, and on landing in Otopeni they were surprised to find out that the competition city was over 8 hours away by train! The airfare to Cluj drained their reserves. Had the organizers not paid for meals and accommodation for all participants, the team would have had to sleep in the train station! That was the moment when I met Jasper.

### Lessons

For me, as a former IMO participant, who benefited from the training infrastructure in Romania in the years 1980-1990 (probably the most efficient in the world at that time) Jasper's perseverance was a remarkable example. I know from personal experience that it is extraordinarily motivating to participate in a competition where one's team is the best. But how demoralizing is failure! How easy it is to throw in the towel, to say that you have no chance, that the Europeans or the Chinese are infinitely stronger, that you will never be able to compete with them, that you will make a fool of yourself! Jasper did not care that he would be laughed at, or rather he bravely faced this risk. After ten years of hard work, he managed the incredible performance for his team to surpass at the IMO 2020 countries with a strong mathematical tradition, such as Algeria or Morocco!

The second lesson I received was the generosity of my fellow citizens. From my children and my wife to family, colleagues, former IMO participants, to strangers who wanted to donate anonymously, the response was quick and unconditional. The idea of helping a distant country, poorer than us, for a noble and intelligent purpose, reached a secret chord in the souls of our fellow Romanians. Myself I was driven in this adventure by an unseen force that made me go far beyond the involvement I had originally planned. Eventually I visited the heart of equatorial Africa, a place I would have sworn a year before that I would probably never see. On the plane that took me to Kampala with a stopover in Kigali, in the middle of the African night, I had a moment of doubt. I wondered what I was looking for in there, if I would be safe, if it wasn't crazy what I was doing. Evidently, these questions turned out to be absurd, because Kampala is one of the most peaceful places on the planet. Despite poverty, the inhabitants are calm and courteous, always with a smile on their face. Nature is incredibly green and you can't see anywhere the garbage that my mind inadvertently associates with warm countries. I overcame these preconceptions to discover a young and fresh world, from which I had to learn as much as I could transmit to them my little knowledge of elementary mathematics.

# Epilogue

I returned to Romania via Amsterdam on March 2, 2020, just one week before the borders closed. I don't know when or if another trip to Kampala will be possible. I got in touch with mathematicians who want to organize a joint training network at IMO for East African countries. The project continues.

Bucharest, December 22, 2020

http://www.imar.ro/~sergium/mathuganda/new\_mU/index.html

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