

# Riemann and Kähler geometry 2019 - IMAR

## Titles and abstracts

Nicolas Ginoux and Sergiu Moroianu

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### 1 Florin Belgun (IMAR)

**Title:** Torus actions and leaf counting on locally conformally symplectic manifolds

**Abstract:** A Vaisman manifold is a locally conformally Kähler (lcK) manifold with parallel, in particular holomorphic, (anti)-Lee vector field. However, until now, the only compact examples of lcK metrics with holomorphic Lee vector field are on Vaisman manifolds. We provide a method to construct new examples, also on non-Vaisman manifolds. The method also provides necessary

conditions for the existence of such metrics (also in the more general locally conformally symplectic - lcs - setting, where we call these structures  $J$ -holomorphic lcs structures). For Vaisman manifolds, and more generally for  $J$ -holomorphic lcs structures, we obtain an upper bound for the number of closed Lee orbits, and characterize the cohomology of the equality case as being the one of a Hopf manifold.

## 2 Liana David (IMAR)

**Title:** Generalized connections, spinors and integrability of generalized structures on Courant algebroids.

**Abstract:** I will present a characterization, in terms of torsion-free generalized connections, for the integrability of various generalized structures (generalized almost complex structures, generalized almost hypercomplex structures, generalized almost Hermitian structures and generalized almost hyper-Hermitian structures) defined on Courant algebroids. Then I will briefly revise the theory of Dirac generating operators for regular Courant algebroids. As an application I will provide a criterion for the integrability of generalized almost Hermitian structures and generalized almost hyper-Hermitian structures defined on a regular Courant algebroid  $E$ , in terms of canonically defined differential operators on spinor bundles associated to  $E$ . This work is joint with Vicente Cortés.

## 3 Paul Gauduchon (École Polytechnique)

**Title:** The Taub-NUT ambitoric structure

**Abstract:** A four-dimensional ambitoric structure is formed of the following pieces: (i) An oriented, four-dimensional (positive definite) conformal manifold  $(M, c)$ ; (ii) two  $c$ -orthogonal complex structures,  $J_+$  and  $J_-$ , on  $M$ , inducing opposite orientations; (iii) two Riemannian metrics,  $g_+$  and  $g_-$ , in the conformal class  $c$ , Kähler with respect of  $J_+$  and  $J_-$  respectively; (iv) an action of the two dimensional torus  $\mathbb{T}^2$ , preserving the two Kähler structures and Hamiltonian with respect to the corresponding Kähler forms. Such structures may appear in some unexpected situation. In this talk, it will be shown that the well-known Taub-NUT metric defined on  $\mathbb{R}^4$  can be linked with one of the complete Bochner-flat Kähler metric constructed by Robert Bryant on  $\mathbb{C}^2$  to form an ambitoric pair of parabolic type.

## 4 Nicolas Ginoux (IECL, Université de Lorraine)

**Title:** Obata-type characterisations of doubly-warped product Kähler manifolds

**Abstract:** We show how to characterise whole families of Kähler manifolds with a single real-valued function satisfying an Obata-like second-order-PDE involving its Hessian. This is joint work with Georges Habib, Mihaela Pilca and Uwe Semmelmann.

## 5 Georges Habib (Université Libanaise)

**Title:** The Bochner formula for Riemannian flows

**Abstract:** We consider a Riemannian manifold  $(M, g)$  endowed with a Riemannian flow and we study the curvature term in the Bochner-Weitzenböck formula of the basic Laplacian on  $M$ . We prove that this term splits into two parts. The first part depends mainly on the curvature operator of the underlying manifold  $M$  and the second part is expressed in terms of the O'Neill tensor of the flow. After getting a lower bound for this term, depending on these two parts, we establish an eigenvalue estimate of the basic Laplacian on basic forms. We then discuss the limiting case of the estimate and prove that when equality occurs the manifold  $M$  is a local product. This work is joint with Fida El Chami.

## 6 Nicolina Istrati (Tel Aviv University)

**Title:** On LCK metrics with positive potential

**Abstract:** Locally conformally Kähler (LCK) geometry is a conformal version of Kähler geometry: a metric  $g$  is LCK if around every point of the manifold,  $g$  is conformal to a local Kähler metric. On the universal cover of the manifold, the corresponding Kähler metrics glue to a globally defined metric. When this metric admits a positive potential on which the deck group acts by homotheties,  $g$  is called with positive potential. A compact manifold which admits such LCK metrics embeds into a Hopf manifold, by a result of Ornea-Verbitsky. In this talk I will discuss the problem of existence of LCK metrics with positive potential and show that any toric LCK metric on a compact manifold admits a positive potential.

## 7 Christophe Margerin (École Polytechnique)

**Title:** The punctured elliptic curve that is a sibling of  $\mathbb{P}^1 \setminus \{0, 1, \infty\}$

**Abstract:** I'll give several constructions, both topological and (complex) geometrical, of the curve in the title and discuss the looming 3-cover that relates them.

## 8 Andrei Moroianu (Université Paris Sud)

**Titre:** Variétés localement conformément kähleriennes à champ de Lee holomorphe

**Résumé:** Une variété localement conformément kählerienne (lck) est une variété hermitienne compacte  $(M, g, J)$  dont la 2-forme fondamentale  $\omega := g(J \cdot, \cdot)$  vérifie  $d\omega = \theta \wedge \omega$  pour une certaine 1-forme fermée  $\theta$  appelée forme de Lee. Nous étudions ici les variétés lck dont le champ de Lee (le dual métrique de la forme de Lee) est holomorphe. Nous allons montrer que si sa norme est constante ou s'il est de divergence nulle, alors la métrique est de Vaisman, c'est-à-dire que la

forme de Lee est parallèle par rapport à la connexion de Levi-Civita de  $g$ . Nous allons ensuite donner des exemples de structures lck non-Vaisman à champ de Lee holomorphe, puis nous allons classifier toutes les structures lck à champ de Lee holomorphe qui admettent un potentiel sur les variétés de type Vaisman. Ces résultats ont été obtenus en collaboration avec F. Madani, S. Moroianu, L. Ornea et M. Pilca.

## 9 Paul-Andi Nagy (Murcia, București)

**Title:** Complex Riemannian foliations of open Kähler manifolds

**Abstract:** Classification results for complex Riemannian foliations are obtained. For open subsets of irreducible Hermitian symmetric spaces of compact type, where one has explicit control over the curvature tensor, we completely classify such foliations by studying the infinitesimal model associated with the canonical connection. We also establish results for symmetric spaces of non-compact type and a general rigidity result for any irreducible Kähler manifold. This is joint work with T. Murphy.

## 10 Alexandra Otiman (Università degli Studi di Roma Tre)

**Title:** Dolbeault cohomology of Oeljeklaus-Toma manifolds

**Abstract:** Oeljeklaus-Toma (OT-) manifolds are a higher-dimensional generalization of Inoue-Bombieri surfaces and were introduced by K. Oeljeklaus and M. Toma in 2005. For any positive natural numbers  $s$  and  $t$ , OT manifolds of type  $(s, t)$  are quotients of  $\mathbb{H}^s \times \mathbb{C}^t$  by discrete groups of affine transformations arising from a number field  $K$  and a particular choice of a subgroup of units  $U$  of  $K$ . They are complex compact non-Kähler manifolds and a subclass is known to carry locally conformally Kähler metrics.

In this talk we compute their Dolbeault cohomology by using the Leray-Serre spectral sequence and by relating their construction to certain domains contained in Cousin groups defined by lattices satisfying a strong dispersiveness condition. In particular, we obtain a new way of computing the Dolbeault cohomology of Inoue-Bombieri surfaces and we show that the Hodge decomposition holds for their de Rham cohomology. These results are joint work with Matei Toma.

## 11 Mihaela Pilca (Universität Regensburg)

**Title:** Special Locally Conformally Kähler Metrics

**Abstract:** A locally conformally Kähler (lcK) metric is defined as a Hermitian metric on a complex manifold, such that in the neighbourhood of each point it is conformal to a Kähler metric. I will talk about lcK metrics with reduced holonomy group. In particular, the existence of Einstein lcK metrics will be

discussed, as well as the geometric description of conformal classes on compact manifolds containing two non-homothetic Kähler metrics. The talk is based on joint work with Farid Madani and Andrei Moroianu.

## 12 Miron Stanciu (IMAR)

**Title:** Locally conformally symplectic cotangent bundle reduction

**Abstract:** We present a generalization of symplectic reduction for locally conformally symplectic manifolds with respect to any regular value of the momentum mapping. This procedure is, under certain conditions, compatible with the existence of additional structures on the manifold (*e.g.* a complex or Vaisman structure). We then prove a theorem characterizing the reduced spaces of cotangent bundles

## 13 Victor Vuletescu (Universitatea București)

**Title:** The analytic structure of LCK 3-folds with algebraic dimension 2

**Abstract:** The analytic structure of (minimal) LCK surfaces  $S$  with  $a(S) = 1$  is well-understood today: they are “elliptic quasibundles”, that is, elliptic fibrations over a curve having all fibers isomorphic (when given the reduced structure). On the other hand, in any dimension  $n$ , any compact complex manifold  $X$  of dimension  $n$  having algebraic dimension  $a(X) = n - 1$  is bimeromorphic to an elliptic fibration over a smooth projective manifold  $B$  (of dimension  $n - 1$ ). We show that all LCK 3-folds  $X$  with  $a(X) = 2$  are obtained as blow-ups of elliptic quasibundles over projective surfaces, under convenient blow-ups. The results are part of a work in progress, joint with D. Angella, L. Ornea and M. Parton.