MAT 303 - HW7 - Additional Problems

Exercise 1. [Reduction of Order] Show that if y_1 is a solution of the differential equation

$$y''' + p_1(t)y'' + p_2(t)y' + p_3(t)y = 0$$

then $y_2 = y_1 v$ is a new solution of the differential equation above, provided that v satisfies the following second order equation in v':

$$y_1 v''' + (3y_1' + p_1 y_1)v'' + (3y_1'' + 2p_1 y_1' + p_2 y_1)v' = 0$$

Exercise 2. Consider the differential equation

$$t^2(t+3)y''' - 3t(t+2)y'' + 6(1+t)y' - 6y = 0, \quad t > 0$$

Assume that a solution of this equation is already known, $y_1(t) = t^2$.

a) Use the method of reduction of order to find a fundamental set of solutions for the differential equations (that is, three linearly independent solutions).

b) Check that the solutions obtained at part a) are linearly independent in two ways: by using the definition of linear independence, and by using the Wronskian test.

c) Find the general solution of the differential equation.

Exercise 3. Consider the differential equation

$$y''' - 3y'' + 3y' - y = 0$$

which has characteristic polynomial $p(r) = (r-1)^3$. By the general theory of equations with constant coefficients, we know that $y_1(t) = e^t$ is a solution of this differential equation. Use the method of reduction of order to find a fundamental set of solutions for this differential equation.

If you need help with 3x3 determinants, you can browse through the following lecture notes:
http://ksuweb.kennesaw.edu/~plaval//math3260/det1.pdf
http://ksuweb.kennesaw.edu/~plaval//math3260/det2.pdf