MAT 351 Differential Equations: Dynamics & Chaos

SPRING 2016

ASSIGNMENT 3

Due Thursday, **February 25**, in class.

Problem 1: Sketch the vector field and some typical trajectories for the following linear systems. Determine whether the equilibrium is stable, asymptotically stable, or unstable.

a) $\dot{x} = x$, $\dot{y} = x + y$.

b) $\dot{x} = -x + y$, $\dot{y} = -5x + y$.

Problem 2: Sketch the phase portrait and classify the fixed point $x^* = 0$ of the following linear systems. Specify if the system is hyperbolic or not.

a) $\dot{x} = -3x + 2y$, $\dot{y} = x - 2y$.

b) $\dot{x} = y$, $\dot{y} = -x - ay$, where $-2 < a < 2$.

Problem 3: The motion of a damped harmonic oscillator is described by $m\ddot{x} + b\dot{x} + kx = 0$, where $b > 0$ is the damping coefficient. The constants $m, k > 0$.

a) Rewrite the equation as a two-dimensional linear system.

b) Classify the fixed point at the origin and sketch the phase portrait in the case when the system is: underdamped ($b^2 < 4mk$), critically damped ($b^2 = 4mk$), or overdamped ($b^2 > 4mk$).

Problem 4: Consider the system

$$\dot{x} = g(x),\ x \in \mathbb{R}^2, \quad \text{where} \quad g\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 3x_1^2 + 7x_1x_2 + x_1 + 2x_2^2 - x_2 \\ -12x_1^2 - 16x_1x_2 - 3x_1 - x_2 \end{pmatrix}.$$

a) Compute the Jacobian matrix at $x^* = (0, 0)$ and find its eigenvalues. Use this information to classify this fixed point and determine its stability.

b) Sketch the phase portrait in a neighborhood of x^*.

c) (EXTRA CREDIT - 3p) Sketch a plausible phase portrait for the whole system.