MAT 351 Differential Equations: Dynamics & Chaos

Spring 2016

Assignment 2

Due Thursday, **February 18**, in class.

Problem 1: Show that the following system

$$\dot{x} = \lambda + \frac{1}{2}x - \frac{x}{x+1}$$

undergoes a saddle-node bifurcation at a critical value of λ, to be determined. Sketch all the qualitatively different vector fields that occur as λ is varied. Sketch the bifurcation diagram of fixed points x^* versus λ.

Problem 2: Show that the system $\dot{x} = x(1-x^2) - 3(1-e^{-\lambda x})$ undergoes a transcritical bifurcation at $x = 0$. Find the critical value of λ for which this occurs. Find an approximate formula for the fixed point that bifurcates from $x = 0$.

Problem 3: For the following equations, find the value of λ at which bifurcations occur, and classify those as saddle-node, transcritical, pitchfork (supercritical or subcritical). Sketch the bifurcation diagram of x^* vs. λ.

a) $\dot{x} = \dfrac{\lambda - x^2}{1+x^2}x$

b) $\dot{x} = x + \tanh(\lambda x)$

Problem 4: Consider the system $\dot{x} = \lambda x - \sin(x)$, for $-4\pi \leq x \leq 4\pi$.

a) Show that for $\lambda > 1$ there is only one fixed point. Describe its stability.

b) Draw a phase portrait and a bifurcation diagram for $\frac{1}{2} \leq \lambda < \infty$. Indicate the stability of the various branches of fixed points.

c) What happens in the interval $0 < \lambda < \frac{1}{2}$? Classify all the bifurcations that occur. (You are not asked to find the exact value of λ at which bifurcations occur.)