MAT324: Real Analysis – Fall 2016

Assignment 7

Due Thursday, **November 10**, in class.

Problem 1: Which of the following statements are true and which are false? Explain.

a) $L^1(\mathbb{R}) \subset L^2(\mathbb{R})$

b) $L^2(\mathbb{R}) \subset L^1(\mathbb{R})$

c) $L^1[3,5] \subset L^2[3,5]$

d) $L^2[3,5] \subset L^1[3,5]$

Problem 2: Let f be a positive measurable function defined on a measurable set $E \subset \mathbb{R}$ with $m(E) < \infty$. Prove that

$$\left(\int_E f\, dm\right)\left(\int_E \frac{1}{f}\, dm\right) \geq m(E)^2.$$

Hint: Apply Cauchy-Schwarz inequality.

Problem 3: Let $f_n \in L^1(0,1) \cap L^2(0,1)$ for all $n \geq 1$. Prove or disprove the following:

a) If $\|f_n\|_1 \to 0$ then $\|f_n\|_2 \to 0$.

b) If $\|f_n\|_2 \to 0$ then $\|f_n\|_1 \to 0$.

Problem 4: Show that it is impossible to define an inner product on the space $\mathcal{C}([0,1])$ of continuous function $f : [0,1] \to \mathbb{R}$ which will induce the sup norm $\|f\|_{\text{sup}} = \sup\{|f(x)| : x \in [0,1]\}$.

Problem 5: Decide whether each of the following is Cauchy as a sequence in $L^2(0,\infty)$.

a) $f_n = \dfrac{1}{n^2}\chi_{[0,\frac{1}{n^3}]}$

b) $f_n = \dfrac{1}{x^2}\chi_{(n,\infty)}$.