UNIVERSITY OF TORONTO MISSISSAUGA

NAME: _____

LEC #: _____

TUT #: _____

October 25, 2018

MAT 244 – Fall 2018 – Midterm

INSTRUCTIONS – PLEASE READ

Please turn off your cell phone and put it away.

➪ Please write your name and your lecture and tutorial number right now.

➪ This is a closed book exam. You are NOT allowed to use a calculator or any other electronic device or aid.

➪ Show your work. To receive full credit, your answers must be neatly written and logically organized. If you need more space, write on the back side of the preceding sheet, but be sure to label your work clearly. You do not need to simplify your answers unless explicitly instructed to do so.

➪ Academic integrity is expected of all University of Toronto students at all times, whether in the presence or absence of members of the faculty.

PROBLEM	SCORE
1.	
2.	
3.	
4.	
5.	
6.	
Total	

LEC 1	Tu	5-6pm/Th 3-5pm	DV2074/DV2082	Remus Radu
LEC 2	Tu	3-4pm/Th 1-3pm	DV2082/DV2080	Remus Radu
TUT 1	W	11:00 - 12:00	DV 3131	Sobia Khan
TUT 2	W	12:00 - 13:00	DV 1092A	Sobia Khan
TUT 3	W	14:00 - 15:00	CC 2134	Amjad Mobayed
TUT 4	W	15:00- 16:00	IB 320	Amjad Mobayed
TUT 5	F	10:00 - 11:00	IB 340	Marc DeBenedetti
TUT 6	F	11:00 - 12:00	IB 390	Sobia Khan

Problem 1. (22 points) Find the solution of the given initial value problem

a) $y' - 2y = 0, \quad y(1) = 1.$

b) $y' - 2y = e^x, \quad y(0) = 1.$

Problem 2. (10 points) We know that $y(x) = 0$ is a solution of the following ODE

$$\frac{dy}{dx} = 3x\, y^{3/2}$$

satisfying the initial condition $y(244) = 0$. Are there any other solutions with $y(244) = 0$? Explain why or why not. Can there be solutions with $y(244) = -3$?

Problem 3. (18 points) A second order chemical reaction involves the interaction (collision) of one molecule of a substance P with one molecule of a substance Q to produce one molecule of a new substance X; this is denoted by $P + Q \to X$. Suppose that p and q, where $0 < p < q$, are the initial concentrations of P and Q, respectively, and let $x(t)$ be the concentration of X at time t. Then $p - x(t)$ and $q - x(t)$ are the concentrations of P and Q at time t, and the rate at which the reaction occurs is given by the equation

$$\frac{dx}{dt} = \alpha(p - x)(q - x),$$

where α is a positive constant.

a) Determine the critical points and classify each one as stable or unstable. Draw the phase line and sketch several typical trajectories. If $x(0) = 0$, determine the limiting value of $x(t)$ as $t \to \infty$ without solving the differential equation.

(problem 3 continued)

b) If the substances P and Q are the same, then $p = q$ and the differential equation becomes

$$\frac{dx}{dt} = \alpha(p - x)^2,$$

Determine the critical points and classify each one as stable or unstable. Draw the phase line and sketch several typical trajectories. If $x(0) = 0$, determine the limiting value of $x(t)$ as $t \to \infty$ without solving the differential equation.

Problem 4. (16 points) Determine whether the following equation is exact. If it is exact, find its solutions.

$$(2xy - 1)\frac{dy}{dx} = 3x^2 - y^2$$

Problem 5. (16 points) Find the solution of the initial value problem

$$2y'' - 3y' - 2y = 0, \qquad y(0) = 0, \qquad y'(0) = -1.$$

Problem 6. (18 points)

a) Find the general solution of the homogeneous equation $y^{(4)} + 3y'' = 0$.

b) Find a particular solution of the equation $y^{(4)} + 3y'' = 5x$.