

Project No.: 006375 Project Acronym: MORIMAR

Project Full Name: The spectrum of geometric operators on manifolds with singularities

Marie Curie Actions

EIF-OIF-IIF-IRG-ERG Final Activity and Management Report

Period covered: from 01/10/2004 to 30/09/2005

Start date of project: 01/10/2004

Project coordinator name: Dr Radu PURICE

Project coordinator organisation name: Institutul de matematica and quot;Simion Stoilow and quot; al Academiei Romane **Date of preparation:** 14/11/2005 **Date of submission (SESAM):**

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GENERAL INFORMATION

Project No.:	006375
Project acronym:	MORIMAR
Project full name:	The spectrum of geometric operators on manifolds with singularities
Period number:	1st
Period covered - start date:	01/10/2004
Period covered - end date:	30/09/2005
Project start date:	01/10/2004
Project duration [months]:	12
Project coordinator name:	Dr Radu PURICE
Project coordinator organisation name:	Institutul de matematica and quot;Simion Stoilow and quot; al Academiei Romane
Date of submission:	14/11/2005

PUBLIC OUTREACH: PUBLISHABLE SUMMARY OF WORK PERFORMED AND RESULTS

Include all main keywords referring to the objectives and results of this project, stressing the most important scientific achievement made

1) Magnetic operators. One of the research subjects has been motivated by objects arising from theoretical Physics, namely magnetic Schroedinger operators have been investigated on certain manifolds with thin ends. These manifolds admit compactifications to smooth manifolds with boundary and the metric is asymptotically conformally cylindrical. In summary, we show that the magnetic Laplacian on a conformally cusp manifold has purely discrete spectrum unless the magnetic field satisfies some integrality condition on the manifold. We obtain examples of compact perturbations of the magnetic field with

long-range effect, in other words, which affect the essential spectrum of the operator. In Euclidean space this is known to be impossible. The result was submitted for publication and can be consulted online at arxiv.org/abs/math.DG/0507443

2) Dirac operators on Riemannian manifolds. Dr. Sergiu Moroianu has continued a scientific collaboration with Dr. Mihai Visinescu from the National Institute of Physics and Nuclear Engineering in Bucharest and Dr. Ion Cotaescu from the West University of Timisoara on "Quantum anomalies for generalized Taub-NUT metrics". The results of this collaboration have been published in J. Phys. A - Math. Gen. 38 (2005), 7005-7019 and the work has continued this year with the study of the axial anomaly on R^4. A preprint has been elaborated http://arxiv.org/pdf/math-ph/0511025. 3) Quantum groups. The quantum permutation group of the set X_n corresponds to a Hopf algebra constructed with generators and relations, known to be infinite dimensional for n>3. We find an explicit representation of this algebra related to Clifford algebras. For n = 4 the representation is faithful in the discrete quantum group sense. Our goal for further research is to extend such results to infinite permutation groups. This paper (by T. Banica and S. Moroianu) will appear in Proc. Am. Mat. Soc.

3) A post-doctoral fellow comming from Cergy University has been recruited for 12 months for research activity on some of the above problems.

4) Five scientists from Europe and USA have benn invited for short scientific visits and cooperation.5) A 4 days workshop has been organized at IMAR, for investigating possible applications of the results to the study of some physical models of interest in nano-technologies.

6) Dr. S. Moroianu is co-editor at the Proceedings of the 7-th International Conference on Differential Geometry and its Applications.

7) Some of the results of this project have been presented by Dr. S. Moroianu in 4 invited talks and by Dr. S. Golenia in 8 communications.

8) Two courses have been delivered by Dr. S. Moroianu for undergraduate students at Bucharest University and "Scoala Normala Superioara - Bucharest".

List of Keywords

magnetic Schroedinger operators; pure point spectrum; eigenvalue asymptotics; magnetic Laplacians; quantum permutation groups; spectral analysis;

Websites where additional information may be found

http://www.imar.ro/~sergium/ http://katedra.fuw.edu.pl/~golenia/ arxiv.org/abs/math.DG/0507443 http://arxiv.org/abs/math.QA/0411576 http://www.imar.ro http://arxiv.org/pdf/math-ph/0511025

REPORT ON WORK PERFORMED AND RESULTS

Please report on the work performed and on the results of the research

a) Accomplishment of research objectives as presented in the original proposal:

- Objective of the research.

1. To prove an asymptotic growth law for the eigenvalues of

the Dirac operator on a manifold with corners, with conformally cusp metric on the interior.

2. To study such asymptotic laws for conformally conical metrics,

including conical singularities of smaller codimension.

- Work performed.

Investigation of Schroedinger operators on conformally cusp manifolds with thin ends. These manifolds admit compactifications to smooth manifolds with boundary and the metric is

asymptotically conformally cylindrical. The type of asymptotics is related to Melrose's cusp metrics. i) We show that the magnetic Laplacian on a conformally cusp manifold has purely discrete spectrum unless the magnetic field satisfies some integrality condition on the manifold X. The simplest setting to explain this condition is for a compactly supported magnetic field B. This is therefore a compactly-supported exact 2-form, which defines a real 2-cohomology class on X that has to be with integer coefficients.

ii) We prove that the Hodge-Laplace operator on differential forms of degree k has purely discrete spectrum if and only if the k and k-1

Betti numbers of the end vanish.

iii) We prove Weyl-type eigenvalue asymptotics for these operators.

Our result has interesting Physical consequences. We obtain examples of compact perturbations of the magnetic field with long-range effect, in other words, which affect the essential spectrum of the operator. In Euclidean space this is known to be impossible.

- Results and degree to which the objectives were met.

The result of our collaboration was submitted for publication and can be found at arxiv.org/abs/math.DG/0507443 It fully meets the stated objective of analyzing "The spectrum of

geometric operators on manifolds with singularities".

- List specific training received on scientific and technical aspects

Dr. Moroianu benefitted of scientific training on spectral analysis for magnetic Schroedinger operators by the members of the mathematical physics group at IMAR (V. Iftimie, M. Mantoiu, G. Nenciu, R. Purice), participating at the weekly seminar of the group.

Dr. S. Golenia benefitted of scientific training on differential geometry and pseudodifferential techniques on manifolds with boundary.

Both Dr. Moroianu and Dr. Golenia participated in a total of 12 scinetific visits and conferences where they learned about developments in their field of research.

- Relevance for basic and applied science and for applications, including industrial links Such links with nanotechnology were investigated in a workshop organized at IMAR in January 2005. The subject of research has direct applications to theoretical Physics.

b) New objectives established during the course of work and new lines of research - Objective

Find a faithful representation of the quantum permutation group

- Work performed

Dr. Moroianu and T. Banica found such a representation in dimension 4. This construction generalizes to higher dimensions but it stops being faithful. It uses among others the techniques of Clifford algebras. We expect this to have a link with Dirac operators.

- Results

A paper called "On the structure of quantum permutation groups" by T. Banica and S. Moroianu will appear in Proc. Am. Mat. Soc.

- List specific training received on scientific and technical aspects Books and journals with this topic have been purchased.

The work on the proposed objectives has been carried out without significant changes (see point a) above). A complementary objective has been addressed, see point b).

A main collaborator (post-doctoral fellow) was recruited at IMAR for a duration of 12 month. He actively participated in the scientific activities at IMAR and co-authored a paper with Dr. Moroianu. He is now at Warsaw University on another post-doctoral position. Besides him, several scientists were invited at IMAR for scientific collaboration on this contract.

1. Prof. R. Litcanu (France), between 11--21 Feb. 2005. He instructed us on the arithmetic of trace

formulas, a subject that Dr. S. Moroianu is actively pursuing in connection with this scientific program.

2. Prof. D. Burghelea (USA) between Feb. 22 -- Mar. 29 2005. Burghelea is a world expert in Geometric Analysis. Dr. Moroianu consulted him on the algebraic topological aspects of his work with Golenia. He also gave a series of talks at IMAR.

3. Prof. T. Jecko (France) between Aug. 31 -- Sep. 5, 2005. He gave a talk at IMAR and worked with Golenia on Mourre estimates.

4. Prof. P. Loya (USA) visited between Sep. 5 -- 19, 2005.

He gave a talk at IMAR. He started with Dr. Moroianu a fruitful collaboration on zeta functions on manifolds with boundary.

5. Dipl. J. Ditsche (Germany) between 15 -- 29 Sep. 2005. Ditsche is a doctoral student of B. Gramsch at Mainz, interested in the so-called zero-calculus. Dr. Moroianu instructed him on Dirac operators in the zero-calculus; they computed some K-theory groups of the 0-calculus and tried to extend a result of Moroianu about absence of eigenvalues of Dirac operators of some \$0\$-metrics.

Dissemination of results

Dr. Moroianu participated in the following international conferences:

i) Conference in honor of Paul Gauduchon, Ecole Polytechnique Palaiseau, May 2005;

ii) Second joint meeting of AMS, DMV, OMV, Mainz, Iun. 2005;

iii) Analysis and Geometric Singularities, Oberwolfach, Aug. 2005;

iv) Seventh International Workshop on Differential Geometry and its Applications, Deva, Sept. 2005.

He gave invited talks at the last three of these conferences. He also visited the University of Potsdam, Germany in June 2005, where he gave a talk.

At the same time, Dr. Golenia participated in the following conferences and scientific visits: i) Cergy, France, 12 -- 29 Mar. 2005;

ii) Caen, May 2005

iii) Orleans, May 2005

iv) Cergy, May 2005

v) Paris, May 2005;

vi) Bucharest, Aug. 2005;

vii) Deva, Sep. 2005;

viii) Oberwolfach Sep. 2005.

Community service

i) Dr. Moroianu co-organized a mini-workshop at IMAR in January 2005, with the title "Mathematical Models for Transport in Macroscopic and Mesoscopic Systems". He invited for this workshop the following scientists: Horia Cornean (Denmark), Claude Pillet (France), Pierre Duclos (France) and Benjamin Ricaud (France).

ii) Dr. Moroianu is an Editor for the Proceedings of the Seventh International Workshop on Differential Geometry and its Applications (Deva, September 2005).

iii) Two courses have been delivered by Dr. S. Moroianu for undergraduate students at Bucharest University (Algebraic topology) and "Scoala Normala Superioara - Bucharest" (K-theory and differential operators).

ASSESSMENT BY THE SCIENTIST IN CHARGE ON THE FELLOW'S WORK DURING THE FELLOWSHIP

Dr. Sergiu Moroianu has been recruited by our Institute starting with ..., on a permanent position. On June 30, 2005 he has been promoted on a superior level (senior researcher of 2-nd degree - one step bellow full professor) by a scientific panel organized by our Institute. He is working in the team on Algebraic and Differential Topology, having collaborations with researchers from the team of Partial Differential Equations and Mathematical Physics. His presence in our Institute has been very stimulating due to his research activity, the international cooperations he has (he has invited several scientists from abroad for conferences and short scientific visits at IMAR), the training programs he has organized for the young researchers (courses, seminars, scientific discussions) and due to his involvement in organizing important scientific meetings with international participation (one workshop in January 2005 and one conference in August 2005).

During this first year of the project, Dr. Sergiu Moroianu has presented several talks at IMAR concerning the results of his research activity concerning spectral analysis on cusp manifolds and has elaborated at least two scientific papers. He is also actively involved in the research activity on two important research projects proposed by IMAR and accepted to be financed by the Romanian Ministery for Education and Research.

This year, in the frame of this Marie Curie ERG project, Dr. Sergiu Moroianu has collaborated with Dr. Sylvain Golenia recruited by IMAR on this purpose with a one year post-doctoral fellowship position supported by this project. This collaboration has been benefiting to both of them and had an important impact on the scientific life at IMAR. In fact, Dr. Sylvain Golenia has brought a number of new ideas concerning differential operators on graphs and spectral analysis, results of his previous work with V. Georgescu at Cergy University) and has benefitted of the training on differential geometry (by S. Moroianu) and on Schroedinger operators with magnetic fields (by M. Mantoiu and R. Purice). The paper ellaborated jointly by S. Golenia and S. Moroianu is a materialization of this fruitful collaboration.

A very important achievement of this program has been the organization of the Workshop "Mathematical Models for Transport in Macroscopic and Mesoscopic Systems" in January 2005. This workshop has generated a fruitful exchange of ideas on some very important scientific problems and leading to interesting cooperation projects.

RESEARCH RESULTS RELATED TO THE FELLOWSHIP

Type of Event	Active participation		Passive participation	
Type of Event	Oral	Poster	Invited (oral + poster)	i assive participation
Conferences	1	0	3	1
Workshops	0	0	2	2
Other Scientific Meetings	0	0	0	0

Participation in conferences and other scientific events

Patents

Type of Patent	Application filed	Pending	Granted
National Patents - Member States and/or Associated States	0	0	0
National Patents - Third Countries - US	0	0	0
National Patents - Third Countries - Japan	0	0	0
National Patents - Third Countries - Other	0	0	0
European Patents (EP number)	0	0	0
International Patents (WO number)	0	0	0

Publications

Type of Publication	As main author	Total	Of which were co-authored with researchers from other institutions
Peer Reviewed - Articles in Journals	4	4	3
Peer Reviewed - Chapters in Books	0	0	0
Peer Reviewed - Articles in Conference Proceedings	0	0	0
Peer Reviewed - Books and Monographs	0	0	0
Non-Peer Reviewed	0	0	0
Submitted	2	2	2
Manuscripts in preparation	1	1	1

Other outcomes

- Dr. Moroianu was awarded the prize of "Fundatia Nationala pentru Stiinta si Arta" for 2004.

- In June 2005 Dr. Moroianu obtained a position of ``Cercetator Stiintific 2" at IMAR, which is 1 level below full professor

- Dr S. Golenia has been recruited on a post-doctoral position at the Warsaw University (the research team of J. Derezinski http://www.fuw.edu.pl/~derezins/), starting with October 1, 2005.

MANAGEMENT REPORT

Please justify any deviations and/or modifications to the initial financial planning of the project

At the begining of the project an agreement was signed with Dr. Moroianu, recruiting him as senior researcher on a permanent position. In June 2005 Dr. Moroianu has participated at a competition and has been promoted on a superior position at IMAR. A temporary recruitment agreement for a 12 months period, as a full-time researcher has been also signed with Dr. S. Golenia who has actively participated at the scientific research of the group on Differential Geometry and Mathematical Physics. Both have benefitted of financial support for a number of participation at conferences, workshops and scientific visits in Romania and abroad (during the period March - September 2005). The second proposed post-doctoral fellow could not be recruited any more due to the delay in signing the contract by the Commission. In this situation, a young researcher working on differential operators and K-Theory has been recruited for one month, benefitting of research training on this field.

The financial management has been done by Mrs. Gabriela Cantaragiu the chief financial officer of our Institute. An independent audit company has done the final financial audit.

The expenses incured by the project have been done in agreement with the proposed objectives. Nevertheless some transfers between the main types of eligible expenses proved to be necessary. In fact, when Dr. Moroianu took his position at IMAR, it become evident that the documentation possibilities offered by our library (while beeng the main mathematical library in Bucharest) where insufficient for the goals of our project and a larger budget was needed for at least the main monographies and journals on the domain. In the same time, the delay in having the first payment by the Commission, made a number of proposed visits and participation in conferences out of object, as well as the recruitment of a second post-doctoral fellow. Let us also mention that the proposed durable equipment could be purchased at a lower price then initially estimated and the existing software at IMAR proved to be sufficient for the needs of the project. In this situation, some funds have been moved between these and the one for documentation.

Attachments	report_jochen.pdf, report_Golenia.pdf, report_Moroianu.pdf
Name	
Date	
Signature	