ABSTRACT

- We are developing two new approaches, based on optimal control theory, to important subjects like variational methods for differential equations and fixed domain techniques in variable domain optimization problems (optimal design). Our treatment is new and original and allows both for theoretical and numerical applications. We mention the generalized Naghdi curved rods and shell models as new applications of the control variational technique for differential equations. In shape optimization problems we have considered geometric controllability properties and optimal control approaches for elliptic problems governed by Dirichlet and Neumann boundary conditions.
- In the domain of semilinear elliptic and parabolic problems involving singular or degenerate terms we have considered problems that models real life phenomena such as Gierer-Meinhardt systems arising in the molecular biology or evolution problems arising in magnetic field. Our reseach mainly focused on qualitative properties of classical solutions such as: existence and nonexistence, uniqueness, asymptotic behavior at the boundary and large time behavior for time dependent solutions. We have elaborated one monograph (published by Oxford University Press), 8 research papers published in prestigious mathematics journals with high impact factor, and 6 papers in the press in mathematics journals or volumes published by important international editors. The results have been communicated in several international conferences or in seminars organized by mathematics departments in France, Italy or Slovenia. All these results are concentrated around the theme of this Research Grant and the problems we have studied are at the interplay between Mathematical Physics, Nonlinear Functional Analysis, and Numerical Analysis. These results have opened the path of important collaborations with important international research teams.
- A series of studies on the financial markets show the presence of long-range dependence property. For this reason was proposed the fractional Brownian motion as a substitute for the standard fractional Brownian motion in the corresponding mathematical models. Such a stochastic process is self-similar and has long-range dependence. There were obtained results concerning:
  1. Strong and renormalized variations, Dirichlet property of the sub-fractional Brownian motion (sfBm for short).
  2. Asymptotic behavior of the variation of the stochastic integral with respect to the sfBm and of the multiple fractional stochastic integrals.
  3. Strong and weak approximations of multiple sub-fractional Stratonovich integrals.
  4. Filtering problem for linear stochastic evolution equations driven by fractional Brownian motion.
- We proved a maximum principle and comparison theorem for quasilinear parabolic Stochastic PDEs, similar to the well known results in the deterministic case. The proofs are based on estimates of the positive part of the solution. Moreover we established an existence result and estimates for the Burger's SPDE with Dirichlet condition.
- The problem of H_2-control of a discrete-time linear system subject to Markovian jumping and independent random perturbations was considered. Several kinds of H_2 types of performance criteria (often calls H_2-norms) are introduced and characterized via solutions of some suitable linear equations on the spaces of symmetric matrices. The purpose of such performance criteria is to provide a measure of the effect of additive white noise perturbation over an output of the controlled system. Different aspects specific to the discrete-time framework were

displayed. Concerning the H_2 optimal controller we shown that in the case of the access to the measurements of the full state vector the best performance is achieved by a zero order controller. The corresponding feedback gain of the optimal controller is constructed based on the stabilizing solution of a system of discrete-time generalized Riccati equations. If only an output is available for measurements, the state space realization of the H_2-optimal controller coincides to the stochastic version of the well known Kalman-Bucy filter. In the construction of the optimal controller the stabilizing solutions of two systems of discrete-time coupled Riccati equations are involved. Necessary and sufficient conditions for the existence of the stabilizing solutions of this discrete-time Riccati equations are given in terms of the solvability of some suitable systems of LMIs. An iterative procedure for numerical computation of such solutions is obtained.

- Let H be a self-adjoint operator on a Hilbert space, with an eigenvalue E_0 embedded either in the continuum or at a threshold. The eigenprojection P_0 is assumed to be of finite rank. Let W be a bounded self-adjoint operator and $H(\varepsilon)=H+\varepsilon W$ for ε small. If $P_0 \exp\{-itH(\varepsilon)\}P_0=\exp\{-ith(\varepsilon)\}P_0+\delta(\varepsilon,t)$ with $\delta(\varepsilon,t)$ bounded by $C\varepsilon^p$ uniformly for $t>0$ for some $p>0$, then the effective Hamiltonian $h(\varepsilon)$ is uniquely determined up to a certain order in ε, which depends on the assumptions on $\mathrm{Im}h(\varepsilon)$.
- The quadratic term in the Taylor expansion at the origin of the backscattering transformation in odd dimensions $n\geq 3$ gives rise to a symmetric bilinear operator B_2 on $C_0^\infty(\mathbb{R}^n)\times C_0^\infty(\mathbb{R}^n)$. We prove that B_2 extends to certain Sobolev spaces with weights and show that it improves both regularity and decay. This is joint work with Prof. Anders Melin (Lund University). The most important result obtained is available as preprint (I. Beltita, A. Melin, Analysis of the quadratic term in the backscattering transformation, arXiv 08012230), and will appear in Mathematica Scandinavica.
- The perturbation of the generator of a Borel right process by a signed measure is investigated, using probabilistic and analytic potential theoretical methods. We establish a Feynman-Kac formula associated with measures charging no polar set and belonging to an extended Kato class. A main tool of this approach is the validity of a Khas'minskii Lemma for Stieltjes exponentials of positive left continuous additive functionals.
- In previous articles we have developed a pseudodifferential calculus and a quantization procedure in the presence of a (bounded) variable magnetic field. In this framework, we have proposed a candidate for a quantum relativistic Hamiltonian $H_{\mathbb{A}}$ in a magnetic field B=dA defined in terms of the vector potential A, which is covariant under gauge transformations. In have proved a Cwickel-Lieb-Rosenblum bound for the number of negative eigenvalues of a perturbed Hamiltonian $H(A,V)=H_{\mathbb{A}}+V$, for which it is shown that the essential spectrum is contained in the positive real axis. This result implies Lieb-Thirring-type estimations, useful for problems related to the stability of the relativistic matter. To prove this result we follow Lieb's original proof of the nonrelativistic case without magnetic field, obtaining first a Feynman-Kac formula, and we adapt some arguments of Ichinose and Tamura, which are working with a Hamilton that is not gauge covariant. The most difficult step is to prove a Feynman-Kac formula for the relativistic Hamiltonian without magnetic field. Such a result is published in an article of I. Daubechie, but as far as we know no complete proof exists in the literature; this determined us to make a detailed presentation. The main technical problem is to replace the usual Wiener measure with the measure associated to a process with jumps. To include the magnetic field, we find a 'diamagnetic' inequality and show that the Levy-Khinchine still holds.
- We have proved a Beal's type commutator criterion for an opperator to be a

magnetic pseudodifferential operator and following a proposal by J.M. Bony we have defined a class of magnetic Fourier integral operators and studied their connection with evolution operators.

- We have studied spectral properties of quantum Hamiltonians in curved geometric context. It has been showed the absence of point spectrum for the Dirac operator on manifolds with incomplete gradient-confromal vector fields outside a compact (the results are now published in the Journal of Functional Anlaysis, december 2007). We also stidied in depth the spectrum of magnetic Laplacians and of k-form Laplacians on conformally cusp manifolds. The results are partly published in Annales Henri Poincare (2008) and partly submitted for publication.
- We present a holomorphic representation of the Jacobi algebra $\mathfrak{h}_n \rtimes \mathfrak{sp}(n, \mathbb{R})$ by first order differential operators with polynomial coefficients on the manifold $\mathbb{C}^n \times \mathcal{D}_n$. We construct the Hilbert space of holomorphic functions on which these differential operators act.
- We consider the massless supersymmetric vector multiplet in a purely quantum framework. First order gauge invariance determines uniquely the interaction Lagrangian as in the case of Yang-Mills models. Going to the second order of perturbation theory produces an anomaly which cannot be eliminated. We make the analysis of the model working only with the component fields.
- We consider the massless supersymmetric vector multiplet in a purely quantum framework and propose a power counting formula. Then we prove that the interaction Lagrangian for a massless supersymmetric non-Abelian gauge theory (SUSY-QCD) is uniquely determined by some natural assumptions, as in the case of Yang-Mills models. The result can be easily generalized to the case when massive multiplets are present, but one finds out that the massive and the massless Bosons must be decoupled, in contradiction with the standard model. Going to the second order of perturbation theory produces an anomaly which cannot be eliminated. We make a thorough analysis of the model working only with the component fields.