Convergence

Model with n layers with n increasing⇒Problem (P^n)

$$\frac{\partial p_j^n}{\partial t} + \frac{\partial p_j^n}{\partial a} + \mu_j^n(a, S_j^n(t,y))p_j^n - K_j^n(a)\frac{\partial^2 p_j^n}{\partial y^2} = f_j^n \text{ in } (0,T) \times \Omega_j, \tag{1}$$

$$p_j^n(0,a,y) = p_{0j}^n(a,y) \text{ in } \Omega_j,\ j = 1,2,...,n, \tag{2}$$

$$p_j^n = p_{j+1}^n \text{ on } (0,T) \times \Gamma_{y_j},\ j = 1,2,...,n-1 \tag{3}$$

$$K_j^n(a)\frac{\partial p_j^n}{\partial y} = K_{j+1}^n(a)\frac{\partial p_{j+1}^n}{\partial y} \text{ on } (0,T) \times \Gamma_{y_j},\ j = 1,2,...,n-1, \tag{4}$$

$$K_1^n(a)\frac{\partial p_1^n}{\partial y} = 0 \text{ on } (0,T) \times \Gamma_{y_0}, \tag{5}$$

$$K_n^n(a)\frac{\partial p_n^n}{\partial y} = 0 \text{ on } (0,T)\times\Gamma_{y_L}, \tag{6}$$

$$p_j^n(t,0,y) = \int_0^{a^+} \beta_j^n(a, S_j^n(t,y))p_j^n(a,t,y)da, \text{ for } j = 1,2,...,n. \tag{7}$$

$$S_j^n(t) = \sum_{k=1}^{n}\int_0^{a^+}\int_{y_{k-1}}^{y_k} \gamma_j^n(a,y,z)p_k^n(t,a,z)dadz,\ j = 1,2,...,n.$$

Problem (P)

$$\frac{\partial p}{\partial t}+\frac{\partial p}{\partial a}+\mu(a,y,S(t,y))p-\frac{\partial}{\partial y}(K(a,y)\frac{\partial p}{\partial y})=f \text{ in } (0,T)\times\Omega, \tag{8}$$

$$p(0,a,y)=p(a,y) \text{ in } \Omega, \tag{9}$$

$$K(a,y)\frac{\partial p}{\partial y}=0 \text{ on } (0,T)\times\Gamma_{y_0} \text{ and on } (0,T)\times\Gamma_{y_L} \tag{10}$$

$$p(t,0,y)=\int_0^{a^+}\beta(a,y,S(t,y))p(a,t,y)da, \tag{11}$$

$$S(t,y)=\int_\Omega \gamma(a,y,z)p(t,a,z)dzda. \tag{12}$$

Question: Does the solution to (P^n) approach the solution to a model (P)?

The Cauchy problem (P)

$$\frac{dp}{dt} + Ap = f \text{ a.e. } t \in (0, T) \tag{13}$$

$$p(0) = p_0 \tag{14}$$

The Cauchy problem (P^n)

$$\frac{dp^n}{dt} + A^n p^n = f^n \text{ a.e. } t \in (0, T) \tag{15}$$

$$p^n(0) = p_0^n. \tag{16}$$

Question:

$$(P^n) \to (P) \text{ as } n \to \infty \text{ ?}$$

Theorem. *Let $\mathcal{A}^n$ and $\mathcal{A}$ be quasi $m-$ accretive operators and let $S^n(t)$ and $S(t)$ be the semigroups generated by $-\mathcal{A}^n$ and $-\mathcal{A}$ respectively. If*

$$\lim_{n\to\infty} \mathcal{J}_\lambda^n g = \mathcal{J}_\lambda g$$

for every $g \in \overline{\mathcal{D}}$ and $\lambda > \lambda_0$, where $\overline{\mathcal{D}} = \bigcap_{n\geq 1} \overline{D(\mathcal{A}_N^n)} \cap \overline{D(\mathcal{A}_N)}$, then

$$\lim_{n\to\infty} S^n(t)g = S(t)g$$

for every $g \in \overline{\mathcal{D}}$ and the limit is uniform on bounded intervals for t.

H. F. Trotter, Pacific J. Math. 8 (1958) 887-919.

H. Brezis, A. Pazy, J. Funct. Anal., 9 (1972), 63-74

Existence hypotheses (P_{hyp}) for (P^n) $\Longrightarrow$**Existence hypotheses** for (P)
For each $R > 0$, any $x, \overline{x} \in \mathbf{R}$ with $|x| \leq R$, $|\overline{x}| \leq R$ there exist $L_\mu(R)$, $L_\beta(R) > 0$,

$$|\mu^n(a,y,x) - \mu^n(a,y,\overline{x})| \leq L_\mu(R)\,|x - \overline{x}|\,, \quad \text{uniformly w.r. } a, y \tag{17}$$

$$|\beta^n(a,y,x) - \beta^n(a,y,\overline{x})| \leq L_\beta(R)\,|x - \overline{x}|\,, \quad \text{uniformly w.r. } a, y \tag{18}$$

$$0 \leq \beta^n(a,y,x) \leq \beta_+ \tag{19}$$

$$0 \leq \mu^n(a,y,x) \text{ with } \mu^n(a,y,0) = 0 \tag{20}$$

$$0 \leq \gamma^n(a,y,z) \leq \gamma_\infty, \tag{21}$$

$$0 < K_0 \leq K^n(a,y) \leq K_\infty. \tag{22}$$

Convergence hypotheses $(Conv)$

$$\mu^n(a,y,x) \overset{n\to\infty}{\to} \mu(a,y,x) \text{ uniformly with respect to } a, y \text{ and } x, \tag{23}$$

$$\beta^n(a,y,x) \overset{n\to\infty}{\to} \beta(a,y,x) \text{ uniformly with respect to } a, y \text{ and } x, \tag{24}$$

$$\gamma^n(a,y,z) \overset{n\to\infty}{\to} \gamma(a,y,z) \text{ uniformly with respect to } a, y \text{ and } z, \tag{25}$$

$$K^n(a,y) \overset{n\to\infty}{\to} K(a,y) \text{ uniformly with respect to } a \text{ and } y. \tag{26}$$

Main results

Lemma. *Assume the set of properties* (P_{hyp}). *Then* $\overline{D(A)} = H_\Omega$.

Proposition. *Let* $g \in H_\Omega$ *and assume* (P^n_{hyp}) *and* $(Conv)$. *Then*

$$\lim_{n\to\infty} J^n_\lambda g = J_\lambda g \; in \; H_\Omega.$$

Theorem. *Let* $f^n \in L^2(0,T;H_\Omega)$, $p_0 \in H_\Omega$. *Assume* (P^n_{hyp}), ($Conv$) *and*

$$f^n \to f \; strongly \; in \; L^2(0,T;H_\Omega), \; uniformly \; with \; respect \; to \; a, y, \tag{27}$$

$$p^n_0 \to p_0 \; strongly \; in \; L^2(0,T;H_\Omega), \; uniformly \; with \; respect \; to \; a, y. \tag{28}$$

Then

$$\lim_{n\to\infty} p^n(t) = p(t), \; \forall t \in [0,T]. \tag{29}$$