

## Research group on *Potential Theory*

**IMAR Participants:** L. Beznea, N. Boboc, Gh. Bucur, A. Bonciocat.

**Romanian Cooperations:** Bucharest University, The “Gh. Mihoc – C. Iacob” Institute for Mathematical Statistics and Applied Mathematics of the Romanian Academy (through a common scientific seminar organized by IMAR and Bucharest University).

**International Cooperations:**

France: Universities of Evry, Toulouse

Germany: Universities of Bielefeld, Bonn, Eichstaedt

**Workpackages involved:** A1, B1, B2, C4.

**Doctoral research:**

A. Bonciocat from IMAR, a former student of Prof. L. Beznea (IMAR) is preparing her PhD thesis in Bonn with Prof. K.T. Sturm and her doctoral research has largely benefitted from the EURROMMAT Programme.

**Scientific Objectives:**

1. To develop the potential theory associated with the general Markov right processes;
2. Dirichlet structures and stochastic analysis;
3. Geometric aspects.

**Main Scientific Results:**

1. A. I. Bonciocat: On Beurling-Deny formula for quasi-regular Dirichlet forms, *Math. Reports* **4**(54) (2002), no. 2, 143-146 .
2. A. Cornea: Solution of the Dirichlet problem on Riemannian manifolds by integration on the unit sphere in the tangent space, *Rev. Roum. Math Pures et Appl.* **47** (2002).
3. N. Boboc, Gh. Bucur: Dilation operators on the Dirichlet spaces, *Rev. Roum. Math Pures et Appl.* **47** (2002), 571-597.
4. N. Boboc, Gh. Bucur: Aspects of Number Theory in Terms of Potential Theory. In *Advances in Mathematics Research, Volume 2*, pp. 115-149. Nova Science Publish. Inc, New York 2003.
5. L. Beznea, N. Boboc: Sub-Markovian resolvents under weak duality hypothesis, *Probab. Th. Rel. Fields* **126** (2003), 339-363.
6. L. Beznea, N. Boboc: Fine densities for excessive measures and the Revuz correspondence,
7. *Potential Analysis* **20** (2004), 61-83.
8. L. Beznea, N. Boboc: *Potential Theory and Right Processes* (Mathematics and its Applications Vol. **572**), Kluwer/Springer 2004.

**Research Activity of the Potential Theory Group**

- One research direction followed in this research project was *to deepen the potential theory associated with a Markov process on a general state space*, with particular interest for the Dirichlet space context. The natural frame in which this theory is settled is given by a sub-Markovian resolvent of kernels on a Radon measurable space. After a possible extension of the space, such a resolvent becomes that one associated with a right process on a Radon topological space, not necessary locally compact and without existing a reference measure. Basic tools of the theory are: the Ray topology and compactification, the fine carrier and the reduction operation on measurable sets. One takes advantage of the cone of potentials structure for both excessive functions and measures. The results obtained in the joint works of L. Beznea and N. Boboc are related to this theory. There are probabilistic interpretations (in terms of homogeneous random measures and additive functionals of a right process) which have been recently underlined in a paper of P.J. Fitzsimmons and R.K. Gettoor. An important part of this theory is the so called *Revuz correspondence* which substitutes the correspondence between measures and their associated Green potentials from the classical potential theory. A main tool is the proof of the

existence of a fine continuous version of an excessive measure with respect to a second one, extending a technique of G. Mokobodzki, and deepen by D. Feyel. They also studied of the weak duality between two sub-Markovian resolvents on a common Lusin topological space, with respect to a given measure. The context covers the probabilistic frame of two (Borel) right processes in weak duality (initiated by E.B. Dynkin and R.K. Gettoor - M. Sharpe). A special case is given by a sub-Markovian resolvent of kernels satisfying the strong sector condition (or equivalently by the resolvent of kernels of a quasi-regular semi-Dirichlet form). Results on the *Dirichlet structures* have been also obtained by A.I. Bonciocat, proving (by analytical methods) that the Beurling-Deny formula extends to the quasi-regular symmetric Dirichlet space (on a Lusin topological space). A notable part of the research was the study of the *subordination* for sub-Markovian resolvents, that is to investigate the sub-Markovian resolvents which are dominated by a given one. This operation extends the localization on finely open sets (which in the classical case means to pass from the superharmonicity on the whole space to the superharmonicity on a finely open subset), and also the subordination generated by a measure  $m$  (which means to pass from the Laplace operator  $D$  to the operator  $D-m$ ). In terms of Markov processes, the subordinate resolvents are those associated with the subprocesses (induced by multiplicative functionals) of a given right process. We notice the results obtained by N. Boboc and Gh. Bucur on the subordination (dilation) operators in the context of abstract Dirichlet spaces (on a real Hilbert ordered space). A. Cornea gave a representation of the solution of the Dirichlet problem on Riemannian manifold by integration on the unit sphere in the tangent space. Partially obtained results have been presented and discussed in the scientific seminars of the cooperation EU partners and in the mathematical meetings we organized. We mention the valuable comments of K.T. Sturm in Bonn, A. Cornea, H.-P. Blatt, R. Felix in Eichstaett, D. Feyel, F. Hirsch, A. de La Pradelle, G. Mokobodzki in Paris, M. Roeckner, W. Hansen and V. Metz in Bielefeld, J. Bliedtner in Frankfurt, D. Bakry in Toulouse, and of all of them on the occasion of their visits in Bucharest.

- The intensive scientific cooperation with Prof. Karl Th. Sturm from Bonn University received further developments in a doctoral programme. Anca Iuliana Bonciocat, a junior member of the Romanian team, succeeded to obtain a DAAD PhD fellowship in Bonn, the main research theme being the investigation of the diffusions processes and heat conduction on quasi one dimensional objects like quantum wires and nanotubes. A paper by A. Bonciocat on the subject is in preparation (and available as a preprint).
- The cooperation developed in the EURROMMAT frame with Michael Roeckner from Bielefeld (and also with the groups in Bonn, Frankfurt and Eichstaett) led to the initiation of a new international research project entitled *Potential theoretical methods for the analysis of infinite dimensional processes*, supported (4 years, starting with 2004) by the Romanian Academy and the German Science Foundation (DFG), co-directors being L. Beznea (IMAR) and M. Roeckner (Bielefeld Univ.). The aim of this project is to develop certain potential theoretical methods in the study of infinite dimensional processes.

### **Conferences, talks, seminars:**

1. *Beurling-Deny formula for quasi-regular Dirichlet forms*, talk given by A.I. Bonciocat in the "The IX-th Romanian-Finnish Seminar" (International Conf. on Complex Analysis and Related Topics), Brasov, August 2001.
2. *Harmonic map heat flow generated by Markovian semigroups*; plenary talk given by K.T. Sturm from Bonn Univ., Germany, in the "The IX-th Romanian-Finnish Seminar" (International Conf. on Complex Analysis and Related Topics), Brasov, August 2001.
3. *Coupling and regularity*, two lectures by K.T. Sturm from Bonn Univ., Germany, given in the Summer School "Connections between Potential Theory, Geometry and Probability", August-September 2001, Brasov, Romania.
4. *Perturbation de resolventes par des mesures*, lecture given by L. Beznea (IMAR) in "Seminaire de Theorie du Potentiel", Universite Paris 6, January 2002.
5. *On G-invariant differential operators*, talk given at IMAR (in the Potential Theory Seminar) by R. Felix from Eichstaett-Ingolstadt Univ., Germany, in April 2002..

6. *Equilibrium distribution, discrepancy and approximation*, conference given at IMAR (in the Potential Theory Seminar) by Hans-Peter Blatt from Eichstaett-Ingolstadt Univ., Germany, in June 2002.
7. *Strongly supermedian kernels and homogeneous random measures*, talk given by L. Beznea (IMAR) at the University of Bielefeld, Germany, in July 2003.
8. *On the strongly supermedian functions and kernels*, talk given by N. Boboc (Univ. of Bucharest) at the 5th Congress of the Romanian Mathematicians, Pitesti, Romania, June 2003.
9. *Measurable metrics, intrinsic metrics and Lipschitz functions*, two talks given at IMAR (in the Potential Theory Seminar) by F. Hirsch from Evry University, France, in March 2004.
10. *Mesures aleatoires homogenes et leurs potentiels*, talk given by L. Beznea (IMAR) at the Univ. P. Sabatier, Toulouse, France, in June 2004.
11. *Equation de Monge-Ampere en dimension infinie; Integration sur des chemins non rectifiables*; talks at IMAR in the frame of the 2004 IMAR-Workshop by D. Feyel from Evry University, France, in June 2004.
12. *The continuity of generalized contractions in Dirichlet spaces*, talk given by Gh. Bucur (IMAR) at the University of Rome Tor Vergata, Italy, in June 2004.
13. *Maps preserving potentials*, talk given by Gh. Bucur (IMAR) in the *5th International Conference on Functional Analysis and Approximation Theory*, Maratea, Italy, June 2004.

#### **Organization of:**

- **The 2nd IMAR Workshop - Potential Theory**, Bucharest September 9 - 29, 2002.

(<http://www.imar.ro/~eurrommat/PotTh.html>)

The Potential Theory Group at IMAR has organized a three weeks workshop in Bucharest, with the participation of its collaborators from Europe. The main purpose of this meeting was to summarize the most recent activities and results of each partner and to work together on some new interesting problems. The main topic of the meeting has been:

1. Geometric aspects in potential theory;
2. Dirichlet structures and stochastic analysis;
3. Potential theory and Markov processes.

#### ***Scientific Programme:***

- D. Bakry (Univ. Toulouse III, France): *Entropy decay in linear and non-linear evolution equations: links with functional inequalities and geometric properties of diffusion operators*
- (two lectures).
- J. Bliedtner (Univ. Frankfurt, Germany): *1. A simple proof of the Lebesgue Differentiation Theorem. 2. The condenser problem.*
- A. Cornea (Univ. Eichstaett, Germany): *1. Applications of controlled convergence. 2. Solution of the Dirichlet problem on Riemannian manifolds by integration on the unit sphere.*
- F. Di Biase (Univ. Pescara, Italy): *A potential theoretic approach to the McMillan Twist Theorem* (three lectures).
- B. Fuglede (Univ. Copenhagen, Denmark): *Harmonic maps from Riemannian polyhedra to geodesic spaces, I: Hoelder continuity of harmonic maps; II: The Dirichlet problem for harmonic maps.*
- K. Gowri Sankaran (Univ. McGill, Montreal, Canada): *Tangential limits of potential on homogeneous trees* (two lectures).
- W. Hansen (Univ. Bielefeld, Germany): *1. Intrinsic Hoelder continuity of harmonic functions*
- *2. Normalized solutions of Schroedinger equations with potentially bounded measures.*
- F. Hirsch (Univ. Evry, France): *Measurable metrics, intrinsic metrics and Lipschitz*

*functions* (two lectures).

- K. Janssen (Univ. Duesseldorf, Germany): *Choquet-type integral representations in potential theory, I: Integral representation for space-time excessive functions; II: Integral representation for separately invariant functions.*
- T. J. Lyons (Univ. Oxford, Great Britain): *1. The signature of a rough control. 2. Cubature on Wiener space.*
- M. Roeckner (Univ. Bielefeld, Germany): *1. Strong Feller properties for distorted Brownian motion and applications to finite particle systems with singular interactions. 2.  $L^p$ -analysis for the Kolmogorov operators of stochastic Burgers and Navier-Stokes equations.*
- I. L. Stoica (Univ. Bucharest and IMAR, Romania): *Backward stochastic differential equations associated to a symmetric Markov process* (two lectures).
- K.-T. Sturm (Univ. Bonn, Germany): *Martingales in metric spaces* (two lectures).

Two supplementary lectures have been presented:

- E. Popa (Univ. Iasi, Romania): *Resolvents associated with certain semi-dynamical systems.*
- M. Bezzarga (Univ. Tunis, Tunisia): *Subordination for semidynamical systems.*

• **Potential Theory Conference**, Bucharest, September 23-27, 2003.

(<http://www.imar.ro/~eurrommat/PotTh-2.html>)

This meeting was a continuation of *The 2nd IMAR Workshop-Potential Theory*, Bucharest, September 9-29, 2002, organized in the same frame.

**Scientific programme:**

- I. Bachar (Univ. Tunis, Tunisia): *Estimations de la fonction de Green et existence de solutions positives d'equations polyharmoniques non lineaire avec des conditions de Navier a la frontiere.*
- V. Bally (Univ. Marne la Vale, France): *Lower bounds for the density of the law of locally elliptic Ito processes.*
- D. Bakry (Univ. Toulouse, France): *Elementary methods for volume comparison theorems.*
- L. Beznea (IMAR, Romania): *On the strongly supermedian functions and kernels.*
- M. Bezzarga (Univ. Tunis, Tunisia): *Potentials of additive functionals in unstable semidynamical systems.*
- M. Biroli (Univ. Milan, Italy): *Capacity results for Dirichlet functionals.*
- Gh. Bucur (Univ. Bucharest, Romania): *Shih's theorem on the hitting distributions domination-an analytic approach.*
- A. Cornea (Univ. Eichstaett, Germany): *Volume mean value property for solutions of the heat equation.*
- D. Feyel (Univ. Every, France): *Monge-Kantorovitch problem for the Wiener space.*
- B. Fuglede (Univ. Copenhagen, Denmark): *The Dirichlet problem for harmonic maps on regular sets.*
- S. Gardiner (Univ. College Dublin, Ireland): *Poisson kernels, sets of determination, and approximation on the boundary.*
- K. GowriSankaran (Univ de Montreal, Canada): *Minimal fine limit at the Martin boundary of trees.*
- K. Janssen (Univ. Duesseldorf, Germany): *Factorization of excessive kernels.*
- A. de La Pradelle (Univ. Paris 6, France): *Integration along rough paths.*
- J. Lukes (Charles Univ. Prague, Czech Rep.): *Pointwise approximation of the Dirichlet solutions.*
- V. Metz (Univ. Bielefeld, Germany): *Nonlinear Perron-Frobenius theory for  $P$ -concave maps.*

- G. Mokobodzki (Univ. Paris 6, France): *Fonctions de type positif pour une forme de Dirichlet.*
- I. Netuka (Charles Univ. Prague, Czech Rep.): *Uniform harmonic approximation.*
- E. Popa (Univ. Iassy, Romania): *Resolvents associated with semi-dynamical systems in duality.*
- E. Popescu: *Pseudo differential operators in the context of Feller semigroups and Dirichlet forms.*
- M. Simunkova (Univ. Liberec, Czech Rep.): *Kelvin type transformations for elliptic PDO's.*
- I.L. Stoica (Univ. of Bucharest, Romania): *Maximum principle for parabolic quasilinear SPDE's.*
- K.T. Sturm (Univ. Bonn, Germany): *Transport inequalities, entropy and curvature.*
- C. Udrea (Univ. of Pitesti, Romania): *Resolvents and nonlinear potential theory.*

**A proceedings volume of these two meetings is in preparation and will be published by Theta Foundation Bucharest (the editors are: D. Bakry, L. Beznea, Gh. Bucur and M. Roeckner).**

On the occasion of their visits in Toulouse and Bielefeld, L. Beznea and Gh. Bucur also worked together with D. Bakry and M. Roeckner to the preparation of the proceedings.

• **Potential Theory Meeting**, Bucharest, May 25th, 2004

This meeting completed the potential theory workpackage activities organized by IMAR in the framework of the EURROMMAT Programme. In the same time this was the first activity of a new scientific cooperation project between IMAR and University of Bielefeld, supported by the Romanian Academy and Deutsche Forschungsgemeinschaft, Germany.

**Scientific Programme:**

- N. Boboc (Univ. Bucharest, Romania): *Sub-Markovian resolvents and right processes; the associated capacities.*
- A. Cornea (Univ. Eichstaett, Germany): *On the normal derivative of potentials.*
- M. Roeckner (Univ. Bielefeld, Germany): *Potential theory of the stochastic porous medium equation.*

**Lecture Series at IMAR (in the Potential Theory Seminar):**

1. *Classical Potential Theory*, by A. Cornea from Eichstaett-Ingolstadt University, Germany.
2. *Analysis on fractal sets*, by V. Metz from TU Graz, Austria, and Bielefeld Univ., Germany.
3. *Self-similar sets*, by Ch. Sabot, from Laboratoire de Probabilites, Univ. Paris 6, France.