

Research group on *Operator Theory*

IMAR Participants: A. Gheondea, D. Timotin, C. Ambrozie, D. Beltita, B. Prunaru.

Romanian Cooperations: Bucharest University and SNS-Bucharest, West University in Timisoara (collaboration in organization of the OT International Conference Series).

International Cooperations:

Czech Republic: Institute of Mathematics of the Czech Academy of Sciences

France: University Lille 1, University of Bordeaux, University of Lyon

Germany: Humboldt University Berlin, Darmstadt University of Technology

Poland: AGH Technical University Krakow

USA: Georgia State University

Workpackages involved: A2, B1, B2, C2.

Post-doctoral fellows at IMAR: I. Chalendar (Lyon University) worked in collaboration with D. Timotin (IMAR) on two scientific subjects:

1. a generalization of some results by Rochberg and Martin Smith concerning the Paley-Wiener space;
2. factorisation of a positive function from L^1 through a C_0 contraction.

A common paper is under elaboration.

Doctoral research: Kamila Klis (Inst. Of Mathematics, University of Agriculture, Crakow) had a 3 months doctoral fellowship at IMAR in the frame of workpackage B2. During her stay at IMAR she participated to the activity of the Operator Theory group and had discussions with Dr. Aurelian Gheondea from IMAR, she gave a talk in the seminar of the Operator Theory group in IMAR and she worked on n -hyperreflexivity of finite generated subspaces of operators acting on an infinite Hilbert space.

Scientific Objectives:

1. Multivariable operator theory in connection with interpolation problems;
2. Spectral theory of linear operators and its applications.

Main Scientific Results:

1. D. Beltita, *On Banach-Lie algebras, spectral decompositions and complex polarizations*, in Proceedings of the 19th International Conference on Operator Theory, (Timisoara, June 27-July 2, 2002), Oper. Theory Adv. Appl., Birkhäuser Verlag, Basel (to appear).
2. D. Beltita, *Spectra for solvable Lie algebras of bundle endomorphisms*, Mathematische Annalen 324 (2002), no. 2, 405-429.
3. P. Cojuhari, A. Gheondea: *On lifting of operators to Hilbert spaces induced by positive selfadjoint operators*, submitted.
4. A. Gheondea, S. Gudder, P. Jonas: *On the infimum problem*, preprint.
5. F.-H. Vasilescu: *Spectral measure and moment problems*, in the volume "Spectral Theory and Its Applications", Theta 2003, pp. 173-215.
6. Bebe Prunaru: *Strongly reductive algebras are selfadjoint*, J. Operator Theory 48 (2002), no. 3, suppl., 615—619.
7. C.-Gr. Ambrozie, V. Muller: *Invariant subspaces for polynomially bounded operators on Banach spaces*, Journal of Funct. Analysis, accepted.
8. C.-Gr. Ambrozie, D. Timotin: *On an intertwining lifting theorem for certain reproducing kernel Hilbert spaces*, Integral Equations Operator Theory 42 (2002), no. 4, 373—384.
9. C.-Gr. Ambrozie, D. Timotin: *A von Neumann type inequality for certain domains in C^n* , Proc. Amer. Math. Soc. 131 (2003), no. 3, 859—869.
10. M. Bakonyi, D. Timotin: *The intertwining lifting theorem for ordered groups*, J. Funct. Anal. 199 (2003), no. 2, 411—426.
11. I. Chalendar, E. Fricain, D. Timotin: *Functional models and asymptotically orthonormal sequences*, Ann. Inst. Fourier (Grenoble) 53 (2003), no. 5, 1527--1549.

Research Activity:

- *Multivariable operator theory in connection with interpolation problems.* The research was conducted by C.-Gr. Ambrozie and D. Timotin (IMAR). It investigated the possibility of extending interpolation problems of Nevanlinna-Pick type (which classically were related to the Hardy space of unit disk or of the half-space) to other spaces of functions defined on domains in \mathbb{C}^n . For reproducing kernel spaces of Pick type, an intertwining lifting theorem has been proved. In another direction, a representation formula for functions defined on certain domains and dominated in an operatorial space by the defining function of the domain have been obtained, leading to a Nevanlinna-Pick type formula. For these domains, the investigation has been continued further by Ambrozie and Eschmeier (University of Saarbrücken), who have extended the interpolation formula to an intertwining lifting result.
In a slightly different direction, D. Timotin together with M. Bakonyi (Georgia State University) have obtained a general intertwining lifting theorem for discrete ordered groups, which includes several previous partial results of other authors. In particular, lexicographic groups as well as discrete subgroups of \mathbb{R} are cases covered by this general theorem.
- *Functional model spaces.* The research was conducted by D. Timotin (IMAR), I. Chalendar and E. Fricain (University of Lyon). The functional model spaces considered are invariant subspaces of the backward shift acting on the Hardy space H^2 . These have been the object of extensive research, especially by the school of N.K. Nikolskii. In connection with the problem of characterizing, for these spaces, the Riesz bases of reproducing kernels, the research has been extended to other special cases of families of reproducing kernels. In particular, a complete characterization has been obtained for asymptotically orthonormal bases. In another direction, a research still in progress has been initiated during the visit in Bucharest of Isabelle Chalendar; it concerns the generalized Toeplitz operators on functional model spaces, whose properties (boundedness, compactness, etc) have been up to now studied only in the particular case of the Paley-Wiener space.
- *Spectral theory of linear operators and its applications.* This research was conducted by P. Cojuhari (AGH Technical University, Krakow, Poland) and A. Gheondea (IMAR) and it refers mainly to the generalization of the notion of induced Hilbert spaces to unbounded operators in such a way that it shows that the energy spaces associated to several classical partial differential operators are of this type. The main result is a generalization of the Krein-Reid lifting theorem to this unbounded case and it was indicated how it can be used to estimate the spectra of bounded operators with respect to energy spaces.
- *Infimum problem.* This problem refers to the characterization of pairs of positive bounded operators that admit infimum over the cone of positive operators in a Hilbert space. This problem was earlier studied in different contexts by R. Kadison, S. Gudder, M. Moreland, and T. Ando. The research was conducted by the group formed by A. Gheondea (IMAR), S. Gudder (University of Denver), and P. Jonas (Technische Universität Berlin, Germany) and they obtained positive results using parametrizations of 2×2 block operators as well as counter-examples that shows that there is a gap between the solution of T. Ando by means of generalized shorted operators and the solution of S. Gudder and M. Moreland by means of minimization with orthogonal projections.
- *Invariant subspaces.* The research was carried by C.-Gr. Ambrozie (IMAR) and V. Müller (Institute of Mathematics of the Czech Republic). The problem studied is the existence of the invariant subspaces for polynomially bounded operators T on a complex Banach space. It has been proved that any such operator, acting on a reflexive Banach space and whose spectrum contains the unit circle, has nontrivial closed linear invariant subspaces. In the case of a general Banach space, the existence of the invariant subspaces holds as well, but for the adjoint of the given operator. The main techniques introduced to this aim are certain approximation procedures in the predual of the dual algebra generated by the operator, on the line initiated by Scott Brown. These techniques also proved to be very useful for obtaining reflexivity results for polynomially bounded operators. In this way have been generalized several results obtained by Brown, Chevreau and Percy concerning the case of the contractive operators on Hilbert spaces.

- *Operator algebras.* The research was carried out by Bebe Prunaru (IMAR). The first topic was related to the study of strongly reductive operator algebras. These are norm closed algebras of Hilbert space operators for which every sequence of almost invariant projections is almost reducing. These algebras were introduced in 1976 by C. Apostol, C. Foias and D. Voiculescu who proved that every commutative strongly reductive algebra is selfadjoint. The main result of Prunaru in this area is that all strongly reductive (commutative or not) algebras are selfadjoint. The proof was based on the non-commutative Weyl von Neumann theorem of Voiculescu together with a certain result by V. Lomonosov and some other results on reductive algebras by Nordgren and Rosenthal.

The second topic of research was related to the representations of biduals of operator algebras. In the case of a C^* -algebra, the bidual is isomorphic with the enveloping von Neumann algebra and it plays a fundamental role in the representation theory of C^* -algebras. It is important to find faithful representations of the bidual, because, at least in the case of C^* -algebras they are universal, which means that they essentially encode all the representation theory of that C^* -algebra. Given an operator algebra A , Prunaru associates to each representation of A on a Hilbert space H a certain weak* continuous representation of the bidual algebra A^{**} on some ultrapower of H . The main result is that the associated representation of A^{**} is faithful whenever the initial representation is faithful. In this way a new class of universal representations is obtained. An interesting application is the case of group C^* -algebras, which have an intrinsic faithful representation.

- *Spectral properties of Lie algebras.* The research was carried out by Daniel Beltita (IMAR). It investigates spectral properties of the Lie algebras corresponding to the symmetry groups of certain flags of vector bundles over a compact space. Under natural hypotheses, such Lie algebras are solvable, being in general infinite dimensional. The spectral theory of finite-dimensional solvable Lie algebras of operators is extended to this natural class of infinite-dimensional solvable Lie algebras. The discussion uses the language of continuous fields of C^* -algebras. The flag manifolds in C^* -algebraic framework are naturally involved here, providing the basic method for obtaining flags of vector bundles.

In another direction, Complex Kähler polarizations are constructed for a class of real Banach-Lie algebras that are not necessarily L^* -algebras but include all the real compact L^* -algebras. The approach is based on the theory of spectral decompositions of Banach space operators, and particularly on Dunford scalar operators. The main results are illustrated by means of a family of examples that are constructed starting from the Schatten-von Neumann classes of Hilbert space operators.

Conferences, talks, seminars:

1. *Hardy Inequalities*, two talks at IMAR by P. Cojuhari, from AGH University, Krakow, Poland, 2001.
2. *Contraction semigroups, Hankel operators and Carleson measures*, talk by J. Partington from Leeds University, at IMAR, 2001.
3. *The Causality Paradox and the Smirnov class of functions*, talk by J. Partington from Leeds University, at IMAR, 2001.
4. *Strongly reductive algebras are selfadjoint*, talk by Bebe Prunaru (IMAR) at the Operator Algebras & Mathematical Physics International Conference I, Constanta 2001.
5. *Jumps for operator - valued functions*, talk by V. Muller from Institute of Mathematics of Czech Academy, at IMAR, 2001.
6. *Invariant subspaces for unbounded operators*, talk by F.-H. Vasilescu from Universite Lille 1, at IMAR, 2002.
7. *The intertwining lifting theorem for ordered groups*, talk by D. Timotin at the 19th International Conference on Operator Theory, Timisoara, 2002.
8. *Interpolation formulas on some domains in C^n* , talk by D. Timotin at Journées d'espaces de fonctions et théorie des opérateurs, Luminy, 2002.
9. *Operatorial measures and moment problems*, two talks by F.-H. Vasilescu from Université de Lille 1, at IMAR, 2003.

10. *Invariant subspaces for polynomially bounded operators*, talk by C.-Gr. Ambrozie at the 5-th Congress of Romanian Mathematicians, Pitesti, 2003.
11. *Asymptotically orthonormal sequences of reproducing kernels*, talk by D. Timotin at the 5-th Congress of Romanian Mathematicians, Pitesti, 2003.
12. *On spectral decompositions of selfadjoint operators in Krein spaces*, talk by P. Jonas from Technische Universitaet Berlin, at IMAR, 2004.
13. *On the selfadjoint extensions of a nonnegative operator of defect one in Krein space*, talk by P. Jonas from Technische Universitaet Berlin, at IMAR, 2004.
14. *Perturbation theory for selfadjoint operators in Krein spaces and applications*, talk by P. Jonas from Technische Universitaet Berlin, at IMAR, 2004.
15. *Elementary spectral measures and applications*, talk by Isabelle Chalendar from Universite Lyon 1, at IMAR, 2004.
16. *Minimal vectors and invariant subspaces*, talk by Isabelle Chalendar from Universite Lyon 1, at IMAR, 2004.
17. *Interpolation of functions and operator theory*, talk by D. Timotin at the North British Functional Analysis Seminar, Edinburgh. 2004.
18. *On the results of Ambrozie-Muller on invariant subspaces*, talk given by Bernard Chevreau (Bordeaux University) at the 2004 IMAR Workshop (June 2004).

Conference The 19-th Edition of the *International Conference on Operator Theory - OT 19*, Timisoara June 27 - July 2, 2002.

The International Conference on Operator Theory - OT 19 has been organized by IMAR and the West University in Timisoara, as part of the EURROMMAT Programme, workpackage A2. The conference has been supported jointly by the EC (under the EURROMMAT Programme), IMAR (under the CERES contract No. 152/2001) and the West University of Timisoara (under a CNCSIS grant). The conference has been attended by 114 participants. The invited speakers were: A. Atzmon (Israel), L.G. Brown (USA), G. Cassier (France), E. Christensen (Denmark), C. D'Antoni (Italy), J. Esterle (France), I. Gohberg (Israel), P. Goldstein (Croatia), J. Janas (Poland), Y. Kawahigashi (Japan), L. Kerchy (Hungary), D. Larson (USA), V. Muller (Czech Republic), R. Nest (Denmark), J. Partington (Great Britain), G. Pedersen (Denmark), Gelu Popescu (USA), Th. Schlumprecht (Germany), F.-H. Vasilescu (France), G. Weiss (USA), L. Zsido (Italy). There have also been more than 60 communications of the participants.

Abstracts of the talks can be found at <http://at.yorku.ca/cgi-bin/amca/caiz-01>.