Reasearch group on *Mathematical Physics*

IMAR Participants: I. Beltita, V. Iftimie, M. Mantoiu, Gh. Nenciu, R. Purice
Romanian Cooperations: Bucharest University, Department of Theoretical Physics of the National Institute of Physics and Nuclear Engineering and the National Institute for Laser, Plasma and Radiation Physics (through a common seminar on Mathematical Physics and 3 national basic research programs under the CERES National Program).
International Cooperations:

Participant at the Groupe de Recherche Europeen MΦQ organized by CNRS France.
Czech Republic: Institue of Nuclear Physics at Rez (Prague), Doppler Institute (Prague)
Denmark: University of Aalborg and MaPhySto Institute (Aarhus)
France: Theoretical Physics Centre – Luminy, Marseille, University Cergy-Pontoise
Sweden: University of Lund
Switzerland: University of Geneva, ETH Zurich
USA: Rutgers University.

Workpackages involved: A1, B1, B2, C2.
Post-doctoral fellows at IMAR: D. Krejcirik (Prague Univ.). He has continued his scientific research working for 1 month at IMAR with V. Iftimie, M. Mantoiu and R. Purice. The results of this research activity form part of his paper published in common with R. Tiedra de Aldecoa (Univ. of Geneva).
Doctoral research:

1. V. Moldoveanu has been the PhD student of Gh. Nenciu (IMAR and Bucharest Univ.) and F. Bentosela (Univ. Marseille 2) under common supervision. He presented his thesis at Univ. Marseille 2 on May 24, 2004 obtaining his PhD. A large part of the results in his thesis have benefitted of the research activity on the EURROMMAT Programme. Gh. Nenciu and R. Purice have participated in his thesis jury in Marseille.
2. I. Beltita has continued her doctoral research on inverse scattering under the supervision of V. Iftimie (IMAR and Bucharest Univ.) and collaborating with A. Melin (Lund Univ.).
3. M. Mantoiu (IMAR) collaborated with Serge Richard, a PhD student of Prof. Werner Amrein at the University of Geneva and participated in his thesis jury in Geneva.
4. R. Purice (IMAR) was invited in the thesis jury of H. Kovarik, PhD student of P. Exner at Charles University Prague.

Scientific Objectives:

1. Spectral and propagation properties of some quantum systems of interest in nanomaterials and electronic devices.
2. The observables algebra of a quantum system in a non-homogeneous magnetic field.

Main Scientific Results:

1. D. Krejcirik, R. Tiedra de Aldecoa: *The nature of the essential spectrum in curved quantum waveguides.*ArXiv math-ph/0307019.
2. W. Amrein, M. Mantoiu, R. Purice: *Propagation properties for Schrödinger operators affiliated with certain C*-algebras.* Ann. Henri Poincaré **3** (2002), no. 6, 1215--1232.
3. M. Mantoiu and R. Purice: *The magnetic Weyl calculus*, Journal of Mathematical Physics, **45** (2004), no. 4, 1394-1417.
4. M. Mantoiu, R. Purice, S. Richard: *Twisted crossed products and magnetic pseudodifferential calculus,* ArXiv math-ph/0403016 (accepted in the Proceedings of the 2-nd International Conference on Operator Algebras and Mathematical Physics to be published by THETA Foundation, Romania).
5. G. Nenciu: *On the smoothness of gap boundaries for generalized Harper operators;* preprint ArXiv math-ph/0309009.
6. G. Nenciu, V. Moldoveanu; F. Bentosela, P. Duclos: *Zener tunneling in one-dimensional*

crystals. The rsults of this preprint are also part of the PhD thesis of V. Moldoveanu defended at the University Marseille 2.

7. I. Beltita, F. Bentosela, M. Tater: *Absence of propagation of electronic waves through some periodic structures.* (in preparation),

8. I. Beltita: *Inverse scattering in a layered medium.* Comm. Partial Differential Equations **26** (2001), no. 9-10, 1739--1786.

The PhD thesis *Rigorous results concerning transport phenomena through solids* by V. Moldoveanu under common supervision of Gh. Nenciu (IMAR and Bucharest University) and F. Bentosela (Univ. Marseille 2).

Research Activity:

- *The study of the evolution associated to the Stark-Wannier Hamiltonian in one dimension.* This Hamiltonian describes one quantum electron in one dimension, in a superposition of a periodic potential and a constant electric field, thus being of interest in understanding electric conductivity. Important mathematical problems in this sense are to describe the spectral properties of this Hamiltonian and the propagation properties of the evolution that it generates. Considering singular periodic potentials is interesting from the physical point of view and generates important difficulties for the spectral analysis. This problem has been adressed in collaboration by G. Nenciu from IMAR and his PhD student V. Moldoveanu and the team in CPT-Luminy (France) formed by F. Bentosela (Universite de Marseille) and P. Duclos (Universite du Toulon et du Var). This subject represent an imposrtant part of the PhD thesis that V. Moldoveanu has ellaborated under the common supervision of F. Bentosela and G. Nenciu and has defended in Universite de Marseille 2, on June 24, 2004. The main results obtained on this problem are:
  - the rigorous proof of the existence of the 'propagation front' for a large class of periodic potentials;
  - the proof of the existence of a continuous part of the spectrum of the Hamiltonian for periodic potentials of any class more regular then the so-called 'Dirac-comb' for which it is known that the spectrum must have a much complex structure.
- *The study of the observables algebra of a quantum particle in a non-homogeneous magnetic field.*The main idea behind this problem is to generalize the Weyl non-commutative calculus to the case of a general magnetic field (non-constant). This procedure may allow for the spectral analysis of large classes of quantum Hamiltonians with magnetic field and for describing the associated evolutions. In the same time, this new functional calculus, with a much higher degree of non-commutativity provides an interesting class of pseudodifferential operators, connected with a special class of C*-algebras, the twisted crossed-products. This research has been carried on by F. Boca, M. Mantoiu, G. Nenciu, M. Pascu and R. Purice from IMAR in cooperation with V. Georgescu and A. Iftimovici (Universite Cergy-Pontoise) and S. Richard (Universite de Geneve). In fact, the problem of spectral analysis of quantum Hamiltonians with magnetic fields has been approached with several methods by the different members of our team (perturbation theory in Hilbert spaces, C*-algebraic methods, twisted Weyl calculus, usual pseudodifferential calculus), putting into evidence some very interesting connections and some stil open problems. The important results obtained up to now are:
  - 'almost' Lipschitz continuity of the spectral boundaries for a large class of integral operators of 'magnetic type' (G. Nenciu);
  - definition of the magnetic Weyl calculus and the magnetic Moyal algebra (M. Mantoiu, R. Purice);
  - affiliation of magnetic Hamiltonians to a special twisted crossed-product C*-algebra (M. Mantoiu, R. Purice, S. Richard).
- *Inverse scattering problems.* This is the research topic considered by I. Beltita (IMAR) for her PhD thesis under the direction of V. Iftimie (IMAR) and has highly benefitted from scientific contacts with A. Melin (Centre for Mathematical Sciences, Lund Institute of Technology.):
  - Inverse scattering problems for Scroedinger operators with magnetic fields. We improve

results of J. Ralston and G. Eskin, by reformulating their method and heavily using the radiation conditions satisfied by the resolvent. We show that the scattering matrix at a fixed energy uniquely determines the magnetic and electric field, under conditions of exponential decay on the electric and magnetic field, and also under some regularity conditions, weaker than in.

- Inverse scattering for the Helmholtz equation on the line. For the case where no regularity is required for the sound speed (and this is the case in most of the applications), any of the existing methods of cannot be applied, since, due to the possible singularities of the sound speed the solutions of the Helmholtz are highly oscillatory. We treat the problem using a layer stripping approach due to J. Sylvester, T. Winebrenner and F. Gyles-Colwell, and results from the theory of analytic functions on a half-plane. We have obtained only the fact that the entire scattering matrix and the sound of the speed are uniquely determined by the reflection coefficient, and some estimates that might be useful in the characterization of the reflection coefficient.
- Backscattering for Schroedinger operators. This is still in progress, joint work with Anders~Melin (Centre for Mathematical Sciences, Lund Institute of Technology.) Results for backscattering problem are still few, though the problem is of great practical interest (for instance, in radar technology and seismology). We use the methods introduced by A. Melin. The results are valid for odd dimensions. We obtain estimates between Sobolev spaces for the real backscattering transformation, improving thus results of G. Eskin and J. Ralston, and G. Uhlmann.

- *The nature of the essential spectrum in curved quantum waveguides.*Continuing the interst for the conjugate operator method in spectral analysis, the team in Bucharest has collaborated with D. Krejcirik (Prague University) in applying this method to the study of curved quantum waveguides. D. Krejcirik has spent one month at IMAR on a post-doctoral position supported under the EURROMMAT Programme and has studied this research problem. He has continued his research at the University of Geneva and has concluded it in collaboration with R. Tiedra de Aldecoa, ellaborating a preprint in common.

- Other important research activities and scientific cooperations on which we have some partial results are:
  - Scientific cooperation of M. Mantoiu and R. Purice with F. Bentosela (Marseille Univ.) and P. Duclos (Toulon-Var Univ.) on presence of currents in perturbed periodic media in tha presence of a constant magnetic field and a large class of confining potentials, by using the virial theorem; the one-dimensional case has been treated and some technical difficulties concerning the two-dimensional case have been isolated.
  - Scientific cooperation of R. Purice (IMAR) with P. Duclos (Toulon-Var Univ.) on estimating the imaginary part of perturbed eigenvalues imbedded into the continuous spectrum of onedimensional two-component Hamiltonians.
  - Joint work of Marius Mantoiu, Radu Purice (IMAR) and Werner Amrein (Geneva University) on operator algebraic methods in studying spectral and propagation properties of Quantum Systems. The significance of some ideals of the observables algebra for propagation properties has been put into evidence.
  - R. Purice has collaborated with V. Georgescu studying the regularity at the border for the rezolvents of a large class of quantum Hamiltonians; progress has been made considering the case when the commutator with a conjugate operator belongs to the commutant algebra of the Hamiltonian.
  - Joint research by V. Iftimie (IMAR), F. Bentosela and Y. Dermenjian (Marseille Univ.) concerning propagation properties in stratified media.
  - Joint research by G. Nenciu (IMAR) and A. Jensen (Aalborg University) on the study of the behaviour of threshold resonances for Shcroedinger Hamiltonians.
  - Research by I. Beltita (IMAR), F. Bentosela (CPT-Luminy) and M. Tater (Institute for Nuclear Physics, Rez, Czech Republic) on *Point interactions.* One studies the scattering

properties of a monomolecular layer, trying to identify a energy region of total reflection, corresponding to the spectral gap in the case of a 3-dimensional crystal with the same structure as the one of the layer. The same problem for a layer of a periodic structure of balls

- with Neumann boundary conditions was also considered. Numerical computations due to M. Tater have shown that a energy region of total reflection is more likely absent.

- Last year, we have initiated a research seminar at IMAR on *Quantum Informatics*. The notes of the seminar talks will be multiplied in the IMAR Seminar Notes Series and will form the basis for developing a research team on this subject.

Conferences, talks, seminars:

1. *Leaky Quantum Graphs;* talk at IMAR by Pavel Exner from University of Prague, 2001,
2. *Delta-systems: Wannier, Stark and inverse Klauder;* talk given at the Operator Algebras & Mathematical Physics International Conference I, Constanta 2001, by Pavel Exner from University of Prague.
3. *On Differential Galois Theory;* talk by B. Malgrange (J. Fourier Inst. Grenoble) at the Bucharest University, 2001,
4. *Hardy Inequalities and Eigenfunction Decay;* talk by R. Purice (IMAR) at Munchen University, 2001,
5. *Reflexion totale pour des energies dans une lacune spectrale;* talk by R. Purice (IMAR) at CPT-Marseille, 2001,
6. *Exponential decay for eigenfunctions of perturbed periodic Schroedinger operators;* talk by R. Purice (IMAR) at the International Conference *Operator Theory and its Applications in Mathematical Physics* - Bedlewo (Poland) 2002,
7. *Anisotropic Differential Operators: The Essential Spectrum and Localisation Properties;*talk by M. Mantoiu (IMAR) at the International Conference *Operator Theory and its Applications in Mathematical Physics* - Bedlewo (Poland) 2002,
8. *Deformation quantization of Kaehler manifolds*, talk at IMAR by Martin Schlichenmaier (University of Mannheim), 2002,
9. *Localization and delocalization for differential operators*, talk at IMAR by Andreas M. Hinz (Technical University Muenchen), 2002,
10. *Scattering theory for a class of long-range magnetic fields*, talk at IMAR by Horia Cornean (Aalborg University), 2002,
11. *Inverse scattering for the Helmholtz equation on the line*, seminar by I. Beltita at the Centre for Mathematics and Informatics, Chateau Gombert, Marsilia, November 2002,
12. *Almost invariant subspaces for quantum dynamics: semiclassical limit and time dependent perturbation theory*, by G. Nenciu, International Conference "Multiscale methods in Quantum Mechanics; Theory and Experiment", Roma, December 2002.
13. *The algebra of observables in a mgnetic field*, talk by R. Purice at the "Semiclassical Meetings" Nantes, January 2003,
14. *Scattering in magnetic fields*, talk at IMAR by D. Yafaev (Rennes University), 2003,
15. *Operator Algebras and Mathematical Physics*, talk at IMAR by D. E. Evans (Cardiff University), 2003,
16. *C*-algebras associated with quantum Hamiltonians*, talk by M. Mantoiu (IMAR) at the *Operator Algebras and Mathematical Physics Workshop*, IMAR- 2003,
17. *Asymptotically independent abelian C*-algebras*, talk by M. Mantoiu (IMAR) at the *International Conference Operator Algebras and Mathematical Physics II,* Sinaia 2003,
18. *Functional calculus for observables of quantum systems in magnetic field*, talk by R. Purice (IMAR) at the *International Conference Operator Algebras and Mathematical Physics II*, Sinaia 2003,
19. *On the smoothness of gap boundaries for generalized Harper operators*, talk by G. Nenciu (IMAR) at the *International Conference Operator Algebras and Mathematical Physics II*, Sinaia 2003,
20. *Spectral properties of certain classes of operators in rotation algebras*, talk by F. Boca (IMAR)

at the *International Conference Operator Algebras and Mathematical Physics II*, Sinaia 2003,

21. *High-energy and smoothness asymptotic expansion of the scattering ampliutude*, conference by D. Yafaev (Rennes Univ.) - 5-th Congress of Romanian Mathematicians, Pitesti, 2003,
22. *Almost invariant subspaces for quantum evolutions*, conference by G. Nenciu (IMAR) - 5-th Congress of Romanian Mathematicians, Pitesti,2003,
23. *The Weyl calculus in a magnetic field*, conference by M. Mantoiu (IMAR) - 5-th Congress of Romanian Mathematicians, Pitesti, 2003,
24. *Do bosons condense in a homogenuous magnetic field*, conference by H. Cornean (Aalborg Univ.) - 5-th Congress of Romanian Mathematicians, Pitesti, 2003,
25. *The Weyl calculus in a magnetic field*, talk by M. Mantoiu at the *Conference on Mathematical Problems in Quantum Mechanics*, a Satellite Meeting of the *IX-th International Congress on Mathematical Physics* in Lisbon, July 2003,
26. *New results and analytic methods in the study of the time-dependent Scroedinger equation*, conference given at IMAR by Ovidiu Costin (Rutgers University), on February 9, 2004.
27. *Steady state non-equilibrium states and transport phenomena*, two talks given by Claude-Alain Pillet (Toulon and Var University) at IMAR at the 2004 IMAR Workshop (June 2004).
28. *Fock spaces and Laplace operators on tree graphs*, talk given by Sylvain Golenia (Cergy-Pontoise University) at the 2004 IMAR Workshop (June 2004).

Workshop ***"Quantum Hamiltonians with Magnetic Fields",*** Bucharest September 8 – 14, 2002. **(http://pompeiu.imar.ro/~eurrommat/QH-MF-02.html)**

The Mathematical Physics Group at IMAR has organized a one week meeting in Bucharest of some of its collaborators from Europe. The main purpose of this meeting was to summarize the most recent activities and results of each partner and to work together on some new interesting problems. The main topic of the meeting has been *the study of quantum systems in magnetic fields*.

Scientific Programme:

1. F. Bentosela (Marsilia): *Total reflection on surface potentials*
2. D. Krejcirik (Reims): *Bound states in curved quantum layers*
3. A. Jensen (Aarlborg): *he Nelson model with less than two photons*
4. P. Exner (Prague): *Transport in two-dimensional magnetic systems*
5. M. Mantoiu (Bucharest): *Observable algebra of a quantum system in a magnetic field*
6. R. Brummelhuis (London):
7. S. Richard (Geneva): *Scattering theory for a class of anizotropic Schroedinger operators*
8. Y. Dermenjian (Marsilia):
9. V. Iftimie (Bucharest): *Densite d'etats integree pour des operateurs de Schroedinger avec des champs magnetiques tres singuliers*
10. I. Beltita (Bucharest): *Inverse scattering for Schroedinger operators with magnetic fields*

Lecture Series at IMAR:

1. *Groupoides de Lie et theorie de Galois differentielle,* by B. Malgrange from J. Fourier Institute Grenoble.
2. *Stationary Phase Methods for Weakly Singular Phases and Microfunctions with holomorphic parameters,* by Otto Liess (Bologna University).
3. *Quantum and supersymetric gauge theories,* by G. Scharf (Zurich University)
4. *Inverse scattering,* by A. Melin (Lund University)
5. *Weyl Calculus,* by V. Iftimie (IMAR)