Research Group on *Harmonic Analysis*

IMAR Participants: N. Popa, V. Lie
Romanian Cooperations: Bucharest University
International Cooperations:
        Spain: Universitat de Barcelona
        Sweden: Universities of Karlstad, Linkoeping, Lulea
Workpackages involved: B2, C1.
Post-doctoral fellows at IMAR: Dr. S. Barza (Karlstad Univ.) has collaborated with Prof. N. Popa (IMAR) and his PhD student V. Lie (IMAR) on matrix analogues of some function spaces.
Doctoral research: V. Lie (IMAR) has been the master student and then PhD student of N. Popa and is preparing his thesis.

Scientific Objectives:
1. The study of the matrix analogues of some function spaces.

Main Scientific Results:
1. Sorina Barza, Lars-Erik Persson and Nicolae Popa: *A matriceal analogue of Fejer's theory*, Math. Nachr. 260, 14-20 (2003).
2. Sorina Barza, Victor Lie and Nicolae Popa: *Matriceal pseudomeasures concentrated at a point and an extension of Haar's theorem for infinite matrices*, submitted to Arkiv for Mat.

Research Activity:
Starting from the similarity between the Fourier coefficients of a periodical function on the one dimensional torus and the diagonal submatrices of an infinite matrix we introduced some matrix-analogues of different function spaces like: $C(T)$, $L_1(T)$, $L_\infty(T)$, $A(T)$, $PM(T)$, $M(T)$ (i.e. the space of all continuous functions on the torus, the space of all Lebesgue integrable functions, the space of all essentially-bounded Lebesgue measurable functions, the Wiener algebra of all functions with an absolutely convergent Fourier series, the space of all pseudo-measures on the torus and the space of all bounded Borel measures on T). See *A matriceal analogue of Fejer's theory.*

- We denoted these spaces by $C(l_2)$, $L_1(l_2)$, etc. A notion of the *support of a matrix* from $PM(l_2)$ may be introduced (see another paper submitted to Arkiv for Mat.) and we remarked that the matriceal Schur multipliers may be regarded as the anlogues of bounded Borel measures. Different as in the classical case there are matriceal pseudo-measures (matrices from the space $PM(l_2)$ concentrated at the origin which can not be Schur multipliers. We got there a partial answer to the following problem: 'give sufficient (or necessary and sufficient) conditions on a matrix A in $PM(l_2)$ concentrated at the origin in order that A be a Schur multiplier'.
- Also we obtained an extension of the known Haar theorem about the development of a continuous function in a series with respect to Haar functions.
- So we obtained that large classes of infinite matrices including the class of Hilbert-Schmidt operators, have the property that they can be developed in the operator-norm convergence with respect to Toeplitz matrices associated to Haar functions multiplied in a special way by matrices concentrated at the origin.
- In this aim we introduced a new Banach space of infinite matrices whose intersection with the class of all Toeplitz matrices coincides with the class of all uniform limit of dyadic step functions on the torus. It seems that this space deserves more attention in the future as some other concepts which extend the classical ones as, for instance, a new, commutative, product of infinite matrices.

All these results have been obtained during the mutual visits of S. Barza at IMAR and of N. Popa and V. Lie at Linkoeping, Lulea and Karlstad.

Conferences, talks, seminars:

1. Nicolae Popa - Matriceal Harmonic analysis - lecture given at Univ. of Linkoeping, May 2000.
2. Nicolae Popa - Fejer's theory for infinite matrices - lecture given at Univ. of Karlstad, May 2000.
3. Nicolae Popa - Matriceal Harmonic analysis - lecture given at Technical University of Athens Greece August 2000, at an International Conference of Mathematical Analysis and its Applications.
4. Nicolae Popa - About some Banach spaces of infinite matrices - lecture given at Univ. of Karlstad, September 2001.
5. Nicolae Popa - About matriceal pseudomeasures and some Lebesgue spaces of infinite matrices - lecture given at Chalmers University Goeteborg, August 2002.
6. Nicolae Popa - Some new results and problems in Matriceal Harmonic analysis - lecture given at Univ. of Pitesti Romania at the 5th Congress of all Romanian mathematicians June 2003.
7. Sorina Barza - About some new Banach spaces of infinite matrices - lecture given at Univ. of Lulea, November, 2001.
8. Sorina Barza - Fourier Series and Coefficients in Classical Banach Spaces - Karlstad University, February, 2002.
9. Sorina Barza - Duality results over cones of decreasing functions in higher dimensions, General Inequalities 8, Eger, Hungary, 15-21 September 2002.
10. Sorina Barza - Some spaces of infinite matrices, The 5th Congress of Romanian Mathematicians, Pitesti, 2003.

Between 2001 and 2003 the team leaded by Nicolae Popa attended a scientific seminar at IMAR, entitled *Matriceal Harmonic analysis.*

Also many foreign visitors gave lectures in this seminar.

1. May 9, 2001 at 10:00 *Spectral Synthesis in Function Spaces*; Prof. Lars Inge Hedberg, Linkoeping University at the "Simion Stoilow" Institute of Mathematics in Bucharest.
2. March 13, 2002 at 12:00 *Spectral Type, cotype and convexity Properties of some Banach spaces*; Prof. Lech Maligranda, Lulea University at the "Simion Stoilow" Institute of Mathematics in Bucharest.
3. June 5, 2002 at 12:00 *Weighted Lorentz Spaces and Hardy's Inequalities*; Prof. Javier Soria, Universita Barcelona at the "Simion Stoilow" Institute of Mathematics in Bucharest.
4. November 5 2003 at 12:00 *Multivariate Approximation and Spaces of Functions with Dominating Mixed Smoothness;* Prof. Hans-Juergen Schmeisser at the "Simion Stoilow" Institute of Mathematics in Bucharest.
5. November 12 2003 at 12:00 *Last Developments on Extrapolation Theory;* Prof. Maria Carro at the "Simion Stoilow" Institute of Mathematics in Bucharest.
6. *February 20, 2004: Real interpolation of vector-valued spaces and interpolation of smooth functional spaces;* Prof. Natan Kruglyak at IMAR.