Reasearch group on *Domain Decomposition Methods for Nonlinear Problems. Applications in Applied Sciences and Engineering*

IMAR Participants: L. Badea.
International Cooperations:
France: Universite de Savoie.
Norway: Bergen University
Workpackages involved: A1, B2, C6.
Doctoral research:

1. Marius Lysaker and Johan Lie, the PhD students of Prof. Xue-Cheng Tai from Department of Mathematics of University of Bergen, Norway, visited IMAR between January 3rd and January 31st, 2003. They worked on the level set methods. M. Lysaker held the conference [5] in the seminar of IMAR. Also, some results partly supported by the contract of IMAR with the European Commission have been included in the paper [8].
2. As we already said, Sylvie Wolf, the PhD student of Prof. I. Ionescu from the Department of Mathematics, Universite de Savoie, France, has been involved in the cooperation concerning the application of the domain decomposition methods to the problems modeling the earthquake dynamics. The results have been published in the papers [9] and [10], as well as in the S. Wolf's PhD thesis (held on December 2nd, 2003).

Scientific Objectives:

1. Results concerning the multilevel methods.
2. Domain decomposition methods.

Main Scientific Results:

1. L. Badea, X.-C. Tai and J. Wang, Convergence rate analysis of a multiplicative Schwarz method for variational inequalities, SIAM J. on Num. Anal, vol. 41, nr. 3, 2003, pp. 1052-1073.
2. L. Badea, Convergence rate of a multiplicative Schwarz method for strongly nonlinear variational inequalities, in Analysis and Optimization of Differential Systems, V.Barbu, I. Lasiecka, D. Tiba and C. Varsan, Eds, Kluwer Academic Publishers, 2003, pp. 31-42.
3. B.-O. Heimsund, T. Chan, T. K. Nilsen and Xue-Cheng Tai, Level sets methods for a parameter identification problem, in Analysis and optimization of differential systems, V.Barbu, I. Lasiecka, D. Tiba and C. Varsan, Eds, Kluwer Academic Publishers, (2003) pp. 189-201.
4. L. Badea, On a multiplicative Schwarz domain decomposition method for variational inequalities, in L. Badea, C. Faciu, L. Gratie, M. Mihailescu-Suliciu, D. Polisevski and N. Simion, Current Topics in Continuum Mechanics, vol. II, Lazar Dragos, Ed., Editura Academiei, 2003, pp. 11-40.
5. L. Badea, Domain decomposition Schwarz method for strongly nonlinear inequalities, Preprint series of the Institute of Mathematics of the Romanian Academy, no. 6/2002.
6. L. Badea, On the Schwarz-Neumann method with an arbitrary number of domains, IMA J. Num. Anal., 24, 2004, pp. 215-238.
7. L. Badea, Convergence rate of a Schwarz multilevel method for the constrained minimization of non-quadratic functionals, SIAM J. Numer. Anal., submitted, 2003.
8. Marius Lysaker, Stanley Osher and Xue-Cheng Tai, Noise removal using smoothed normals and surface fitting, IEEE, submitted, 2003.
9. L. Badea, I. Ionescu and S. Wolf, Domain decomposition method for dynamic faulting under slip-dependent friction, Preprint nr. 03-11c, Laboratoire de Mathematiques, Universite de Savoie, France, 2003.
10. L. Badea, I. Ionescu and S. Wolf, Domain decomposition method for dynamic faulting

under slip-dependent friction, J. of Comptational Physiscs, accepted for publication, 2004.
11. L. Badea, C. Faciu, L. Gratie, M. Mihailescu-Suliciu, D. Polisevski and N. Simion, Current Topics in Continuum Mechanics, vol. II, (Editor Lazar Dragos) Editura Academiei, 2003.
12. L. Badea and P. Daripa, On a boundary control approach to embedding domain method, SIAM J. on Control and Optimization, Vol. 40, No. 2, 2001, pp. 421-449.
13. L. Badea and P. Daripa, A fast algorithm for two-dimensional elliptic problems, Numerical Algorithms, 30, 2002, pp. 199-239.
14. L. Badea and P. Daripa, On a Fourier method of embedding domains using an optimal distributed control, Numerical Algorithms, 32, 2003, pp. 261-273.
15. L. Badea and P. Daripa, A Domain Embedding/Boundary Control Method to Solve Elliptic Problems in Arbitrary Domains, in Proceedings of the 41st IEEE Conference on Decision and Control, December 2002, pp. 3004-3009.
16. L. Badea and P. Daripa, A domain embedding method using the optimal distributed control and a fast algorithm, Numerical Algorithms, 36, 2004, pp. 95-112.

Research Activity:

- The multilevel or multigrid methods can be studied by interpreting them as a Schwarz domain decomposition method. First, the research has been focused on the extension of the known results in the literature on the convergence of the multilevel methods for linear equations to the variational inequalities. A rate of convergence has been found for the one and two level Schawarz methods applied to inequalities coming from the minimization of quadratic functionals. The obtained results extend the ones obtained by L. Badea in 1991, where a variant of the method for variational inequalities is proposed, and uses some techniques recently obtained by X.-C. Tai for the convergence rate in the case of the nonlinear equations. For a subspace correction method applied to variational inequalities in a general Hilbert space, we prove a uniform convergence provided that the convex set verifies a certain assumption. We prove in the following that this assumption holds for the Schwarz method in which the convex set is described by constraints on the function values at the points of the domain. Also, this assumption holds for the one and two-level Schwarz method in the finite element space, and we explicitely write the constants in the error estimation depending on the domain decomposition and mesh parameters. Numerical examples are given to illustrate the convergence of the method with both one and two levels, for the problem of a membrane stretched over an obstacle. As for the case of the equations, the obtained convergence rate of the two-level method for variational inequalities is independent of the mesh. The obtained results have been included in: the paper ii) [1], the talk in an international conference [5], and the talk in seminar [2]. Also, a variant of the method containing a relaxation paramenter has been given in [4].
- The next step in the research activity was to extend the above results to the minimization of the non-quadratic functionals. We prove the convergence and estimate the error of a general algorithm for the minimization of non-quadratic functionals over a convex set in a reflexive Banach space, provided that the convex set verifies a certain assumption. In the case of the Sobolev spaces, our algorithm is exactly a variant of the Schwarz domain decomposition method, and we prove that the introduced assumption holds if the convex set has a general enough property. Also, this assumption holds for the one-, two-level, multi-level multigrid Schwarz methods in the finite element space, and we explicitely write the constants in the error estimation depending on the domain decomposition and mesh parameters. We give some numerical examples concerning the two-obstacle problem of a nonlinear elastic membrane. The obtained estimates show the same dependence of the error on the mesh and overlapping parameters as in the quadratic case. The results have been included in: the papers [2], [5], [7], the talks in international conferences and seminars [1], [2], [4], [6], [7], [9]--[12], and the talks in seminars [3], [6].
- The Schwarz-Neumann method is closely related to both Schwarz method and fictitious domain methods. In the Schwarz-Neumann method as in Schwarz method, we iteratively solve problems

having the same equation on some domains. However, besides the fact that in the Schwarz method the domain is the union of some subdomains and in the Schwarz-Neumann method it is the intersection of some super-domains, the boundary conditions of the problems are taken in different ways in the two methods. We think that the Schwarz-Neumann method can be very efficient, especially for exterior problems, when we are able to give direct solutions on the super-domains. In this case the method consists in the iterative calculation of the boundary data of a problem from the values of the solutions on the other domains. A generalization of the Schwarz-Neumann method to more than two domains is given. We prove the convergence and the numerical stability of the algorithm. The results apply to both bounded and unbounded domains, and are given for the weak solution of an elliptic problem with mixed boundary conditions. Numerical results are given for both bounded and unbounded domains. The results have been included in: the paper [6], the conferences [8] and [13], and the talk in seminar [1].

- Prof. I. Ionescu from Department of Mathematics, Universite de Savoie, France, visited IMAR and gave the talks [7], [8], [9] and [10], with a direct application to the study of the earthquakes. Dr. Lori Badea cooperated with him to the application of the Schwarz method to the solution of the variational inequalities appearing in the problems of contact and friction modeling the earthquake dynamics. Prof. I. Ionescu also involved in this cooperation his PhD student Sylvie Wolf. The results have been published in the papers [9] and [10], as well as in the S. Wolf's PhD thesis.
- The domain embedding methods are very related to the domain decomposition methods. The research activity of the joint research group has been extended (and partly supported by the contract IMAR with European Commission) to these methods by the cooperation between Dr. Lori Badea from IMAR with Prof. Prabir Daripa from the Department of Mathematics of the Texas A&M University, USA. The results concernig the domain embedding methods associated with an optimal boundary or distributed control problem has been published in [12], [13], [14], [15] and [16].
- Finally, the book [11] contains results of some researchers from IMAR which are involved in the EURROMMAT Programme.

Communications:

1. L. Badea, Taux de convergence d'une methode de Schwarz multi-nivaux pour des inequations variationnelles fortement nonlineaires, 6eme Colloque Franco-Roumain de Mathematiques Appliquees, September 2-6, 2002, Perpignan, France.
2. L. Badea, Convergence rate of a multiplicative Schwarz method for strongly nonlinear variational inequalities, Conference on Analysis and Optimization of Differential Systems, September 10-14, 2002, "Ovidius" University, Constanta, Romania.
3. B.-O. Heimsund, T. Chan, T. K. Nilsen and Xue-Cheng Tai, Level sets methods for a parameter identification problem, Conference on Analysis and Optimization of Differential Systems, September 10-14, 2002, "Ovidius" University, Constanta, Romania.
4. L. Badea, On a multilevel Schwarz method for the Constrained Minimization of non-Quadratic Functionals, The 15th International Conference on Domain Decomposition Methods, July 21-25, 2003,
5. Berlin, Germany.
6. X.-C. Tai, Nonlinear Positive Interpolation Operators for Analysis with Multilevel Grids, which has been presented at The 15th International Conference on Domain Decomposition Methods, July 21-25, 2003, Berlin, Germany.
7. L. Badea, On a multilevel method for the minimization of non-quadratic functionals, New trends in continuum mechanics, September 8-12, 2003, Constanta, Romania.
8. L. Badea, On a Schwarz multigrid method for the constraint minimization of non-quadratic functionals, The 5th Congres of Romanian Mathematicians, June 22-28, 2003, Pitesti, Romania.
9. Schwarz-Neumann method with more than two domains, The Second European Finite Element Fair, Berlin, June 4-5, 2004, Germany.
10. Constrained minimization of non quadratic functionals by a multilevel Schwarz method,

IMAR Workshop, June 22-27, 2004, Bucharest, Romania.

11. L. Badea, Taux de convergence d'une methode multi reseau pour des inequations variationnelles fortement nonlineaires, Seminar of the Laboratory of Applied Mathematics, University of Chambery, France, January 29, 2002.
12. L. Badea, Taux de convergence d'une methode de Schwarz multiechelle pour la minimisation de fonctionnelles non quadratique, Institut de Mathematiques, Ecole Polytechnique Federale de Lausanne, Switzerland, November 13, 2003.
13. L. Badea, Methode de Schwarz multiechelle pour la minimisation de fonctionnelles non quadratiques, Laboratoire Jacques-Louis Lions, Pierre et Marie Curie (Paris 6) University, January 19th, 2004, France.
14. L. Badea, On the Schwarz-Neumann method with an arbitrary number of domains, Institut de Mathematiques, Ecole Polytechnique Federale de Lausanne, March 24, 2004, Switzerland.

Talks and Seminars:

1. L. Badea, On the Schwarz-Neumann Method with an Arbitrary Number of Domains, in the seminar of the Department of Mathematics of the University of Bergen, May 9th, 2001.
2. X.-C. Tai, Domain Decomposition and Multigrid Methods for Nonlinear Monotone Problems and Variational Inequalities, in the seminar of IMAR, on June 18th, 2001.
3. L. Badea, Convergence rate of a one and two level Schwarz method for strongly nonlinear variational inequalities, Seminar of Applied Mathematics, University of Bucharest, Faculty of Mathematics, April 15th, 2002.
4. Xue-Cheng Tai, Level set methods for inverse problems, Joint seminar of IMAR and Faculty of Mathematics, January 23rd, 2003.
5. Marius Lysaker, Noise removal using a fourth order PDE with application to medical images in space and time, Seminar of IMAR, January 28th, 2003.
6. L. Badea, Multigrid methods for variational problems, Joint seminar of IMAR and Faculty of Mathematics, University of Bucharest, October 9th, 2003 (first part), October 23th, 2003 (second part).
7. I. Ionescu, Initiation of instability on a fault system under slip dependent friction, Seminar of IMAR, January 9th, 2003.
8. I. Ionescu, Existence results for dynamic problems with friction, Seminar of IMAR, January 14th, 2003.
9. I. Ionescu, The blocking of an inhomogeneous Bingham fluid. Applications to landslides, Seminar of IMAR, January 16th, 2003.
10. I. Ionescu, Sufficient condition of non-uniqueness for the Coulomb friction problem, Seminar of IMAR, January 21th, 2003.

Lecture Series:

1. Ioan Romeo Ionescu (Chambery University): 4 lectures on *Contact and friction problems in seismology: mathematical models*, at IMAR in December 2002 – January 2003.