EU- Noncommutative Geometry
SIBIU 2007

International Workshop on:

Free Probabilities, Operator Spaces and von Neumann Algebras

Abstracts

Sergio Doplicher

Geometry of quantum spacetime.

We investigate the interplay between the universal differential calculus and other known algebraic structures, like Hochschild boundary on one side, and the C* structure on the other.

The latter provides natural norms one can evaluate on forms; in the specific case of the algebra of Quantum Spacetime, one finds that, while the Algebra itself is fully translation and Lorentz invariant, the four dimensional Euclidean distance is a positive operator bounded below by a constant of order one in Planck units; the area operator and the four volume operator are normal operators, the latter being a Lorentz invariant operator with pure point spectrum, whose moduli are also bounded below by a constant of order one. While the spectrum of the 3 volume operator includes zero.

If time permits we will comment on the perfect agreement of these findings with the physical intuition suggested by the Spacetime Uncertainty Relations which are implemented by the Algebra of Quantum Spacetime.

(Joint works in progress with Klaus Fredenhagen).

Shoichiro Sakai

Recent Topics on $C*$-algebras (Consistency ond Independency) and Related Problems.

The author (1968[1],1971[2]) proved that any derivation on a simple C*-algebra is induced by an element of its multiplier C*-algebra and in particular,any derivation on a unital simple C*-algebra is always inner. On the other hand,any *-automorphism on a separable simple C*-algebra is induced by a untary element of its multiplier C*-algebra ,if and only if it is *-isomorphic to the C*-algebra of all compact linear operators on a separable Hilbert space and in particular, any*-automorphism on a separable unital simple C*-algebra is always inner if and only if it is *-isomorphic to a finite dimensional full matrix algebra [3]. In the paper [3], the author asked whether one can extend this result to non separable cases.and mentioned three outstanding problems of Naimark, Conne and Brown-Douglas- Filmore as related problems. Recently. all of these problems have been solved in three different ways (Consistency,within ZFC and Independency).([4],[5] ,and [6]and[7]).

Since Naimark's problem was negatively solved by Akemann-Weaver [4],the author[8] raised a new problem which might substitute Naimark's problem .In this talk,the author,at first, explains briefly that Phillips-Weaver[6] and Farah[7] imply that the statement of " The Calkin algebra has an outer *-automorphism" is undecidable within ZFC. Then the author discusses three other outstanding problems of Kadison-Singer, Conne's.embedding and Stone-Weierstrass ,which might be interested from the view-point of C*-algebras and Set Theory.

References.

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- 5 A.Ioana,J.Peterson and S.Popa,Amalgamatedproductofw-Rigid Factors and calculation of their symmetry group,arxiv.math.OA/0505589v2,29May2005
- 6 N.C.Phillips andN. Weaver,The Calkin Algebra has outer outmorphisms,Duke Math. Journal (2006) to appear
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Ionel Popescu

Free Talagrand Inequality, a simple proof.

We discuss the equivalent of the Talagrand inequality from the classical probability in the free probability context. This inequality was proved by Biane and Voiculescu using machinery from complex analysis. It was reproved by Ueda and Petz using random matrix approximations and by Ledoux using some duality properties. We discovered an elementary proof based on convexity argument more in the spirit of Talagrand original proof for the classical case.

Jean Renault

A Hausdorff-Young inequality for measured groupoids.

The classical Hausdorff-Young inequality for abelian locally compact abelian groups states that, for $1 \leq p \leq 2$, the L^p-norm of a function dominates the L^q-norm of its Fourier transform, where $1/p + 1/q = 1$. By using the theory of non-commutative L^p-spaces and by reinterpreting the Fourier transform, R. Kunze (1958) [resp. M. Terp (1980)] extended this inequality to unimodular [resp. non-unimodular] groups. The analysis of the L^p-spaces of the von Neumann algebra of a measured groupoid provides a further extension of the Hausdorff-Young inequality to measured groupoids. This is a report of a joint work with P. Boivin.

Franco Fagnola

CYCLE DECOMPOSITION OF QUANTUM MARKOV SEMIGROUPS

A classical Markov system with discrete state space E and evolution given by a semigroup $(T_t)_{t\geq 0}$ on $\ell^\infty(E)$ is in a stationary regime if there exists a probability distribution π on E which is T_t-invariant i.e. $\sum_{j\in E} \pi_j (T_t(f))_j = \sum_{j\in E} \pi_j f(j)$ for all $f \in \ell^\infty(E)$. If the system is irreducible, then π is strictly positive and unique.

The system is in *equilibrium* if π satisfies the detailed balance condition, namely the classical reversibility condition $\pi q_{ij} = \pi_j q_{ji}$, where $(q_{ij})_{i,j}$ denotes the transition matrix associated with the generator of $(T_t)_{t\geq 0}$. This condition is equivalent to self-adjointness of the operators T_t and of the generator in the Hilbert space $L^2(E,\pi)$ determined by the invariant measure.

In a non-equilibrium system the non-zero difference $\pi_i q_{ij} - \pi_j q_{ji}$ represents a current between the states i and j. The system continuously performs some cycles (transitions $i_1 \to i_2 \to \dots i_{n-1} \to i_n = i_1$) and the difference $\pi_i q_{ij} - \pi_j q_{ji}$, between the $L^2(E, \pi)$ generator and its adjoint, can be decomposed as

$$\pi_i q_{ij} - \pi_j q_{ji} \sum_{c \in \mathcal{C}} w_c \left(J_c(i,j) - J_{c^-}(i,j) \right), \tag{1}$$

where the sum is over a family $\mathcal{C}$ of cycles c, c^- is the reverse c of the cycle c, J_c is the unitary passage matrix associated with the cycle c and $(w_c)_{c \in \mathcal{C}}$ is a family of positive constants (currents).

The evolution of a quantum open systems is usually described by a Quantum Markov Semigroup (QMS) on the algebra $\mathcal{B}(\mathsf{h})$ (h finite dimensional) with generator

$$\mathcal{L}(x) = -\frac{1}{2} \sum_{\ell} \left(L_\ell^* L_\ell x - 2 L_\ell^* x L_\ell + x L_\ell^* L_\ell \right) - i[H, x].$$

In this talk we study the structure of the adjoint $\widetilde{\mathcal{L}}$ of $\mathcal{L}$ in the L^2 space of a faithful normal invariant state ρ and discuss the non-commutative generalisation of (1) for QMSs.

The unitary passage matrix J_c defines a normal $*$-automorphism on the algebra $\ell^\infty(E)$. Therefore a natural generalisation of (1) consists in decomposing the difference $\mathcal{L} - \widetilde{\mathcal{L}} - 2i[H, \cdot]$ as

$$\mathcal{L}(x) - \widetilde{\mathcal{L}}(x) - 2i[H, x] \sum_{c \in \mathcal{C}} w_c \rho^{-1/2} \left(U_c^* x U_c - U_c x U_c^* \right) \rho^{-1/2},$$

where $\mathcal{C}$ and $(w_c)_{c \in \mathcal{C}}$ are families of cycles and positive constants respectively, and each U_c is an unitary operator associated to the cycle c.

We show that such a decomposition is usually impossible because we need exactly n unitary operators in order to describe a cycle of length n. We can prove, however, a slightly more complex decomposition formula for QMS of the so-called generic type ([1]).

References.

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[3] D.Q. Jiang, M. Qian, M.P. Qian: *Mathematical theory of nonequilibrium steady states: on the frontier of probability and dynamical systems.* Lecture notes in Mathematics 1833. Springer-Verlag, Berlin 2004.

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Francesco Fidaleo

New topics in nocommutative ergodic theory.

We discuss the very strong ergodic properties of free shifts. Among them, we mention the shifts on the reduced C*–algebra of RD–groups, including the free group on infinitely many generators, and on the amalgamated free product C*–algebra. Then we deal with the ergodic properties of dynamical systems based on the C*–algebra consisting of all the compact operators acting on a Hilbert space. The ergodic properties of the above mentioned dynamical systems are completely understood.

J. Martin Lindsay

KMS-symmetry and Noncommutative Dirichlet Forms.

A Markov semigroup which is symmetric with respect to a measure on the underlying state space is, on the one hand a selfadjoint contraction semigroup on the Hilbert space L^2 which therefore has a (closed densely defined) quadratic form generator, and on the other hand is positive and contractive on the algebra of essentially bounded measurable functions. The latter properties are encoded at the L^2-level by the Beurling-Deny conditions: the form reduces under so-called normal contractions.

A noncommutative theory requires a proper analogue of both 'symmetry', now with respect to a state or weight on a von Neumann algebra, and 'Dirichlet'. In this talk 'KMS-symmetry' will be contrasted with 'GNS-symmetry' and the noncommutative Beurling conditions will be described. If time permits then recent work, with S. Goldstein and A. Skalski, extending the theory to nonsymmetric semigroups, will also be outlined.

Sergey Neshveyev

Quantium groups as noncommutative manifolds.

For the q-deformation G_q, $0 < q < 1$, of any simply connected simple compact Lie group G we construct an equivariant spectral triple which is an isospectral deformation of that defined by the Dirac operator D on G. The construction depends on the choice of a twist, which can be thought of as a 2-cochain on the dual discrete quantum group $\hat{G}$. The key properties of our quantum Dirac operators depend not on the twist but on the associator, that is, the corresponding coboundary on $\hat{G}$. What allows us to analyze the quantum Dirac operators, is that by results of Drinfeld and Kazhdan-Lusztig we can always find a twist such that the corresponding associator is determined by the monodromy of a system of partial differential equations. (Joint work with Lars Tuset.)

James A. Mingo

Random Matrices and Second Order Freeness.

Second order freeness gives a universal rule for the calculation of limiting fluctuation moments of random matrices. We present an approach based on cumulants and planar diagrams. We will show how to use this calculate cumulants of products

Yasuyuki Kawahigashi

Classification of superconformal nets of factors.

Super Virasoro nets with central charge less than 3/2 are constructed as Fermionic extensions of certain coset nets arising from the SU(2)-nets. We study their extensions, and give a complete classification, using the work on modular invariants by Cappelli and Gannon-Walton. This is a "super" counterpart of our previous complete classification of local conformal nets with central charge less than 1. This is a joint work with Sebastiano Carpi and Roberto Longo.

Benoit Collins

A linearization theorem for non-commutative probability spaces and applications to Connes embeddability conjecture.

Abstract: We prove a non-commutative probability space analogue of a C^*-algebra linearization trick by Haagerup-Thorbjornsen. We apply it to derive new criteria for a finite von Neumann algebra to be embeddable in the ultraproduct von Neumann algebra R^ω. This is joint work with Ken Dykema.

Alexandru Nica

On infinite divisibility for free additive convolution, and a semigroup of transformations related to it

Abstract: Free additive convolution is an operation $\boxplus$ with probability measures on the real line, which reflects the addition of free selfadjoint elements in a C^*-probability space. The property of infinite divisibility with respect to $\boxplus$ was thoroughly studied in papers from the 80's and the 90's by Voiculescu, Bercovici-Voiculescu, Bercovici-Pata. It is known to parallel quite nicely the classical theory for infinite divisibility with respect to the usual convolution of probability measures.

The definition of $\boxplus$ and of infinite divisibility with respect to it extend naturally to the multi-variable framework of (non-commutative) joint distributions for k-tuples of selfadjoint elements in a C^*-probability space. In this talk I will present some recent results obtained in joint work with Serban Belinschi, which go in the multi-variable framework. We put into evidence a multi-variable counterpart for an important bijection found by Bercovici and Pata in their study of relations between infinite divisibility in free and in Boolean probability. We observe a semigroup B_t, $t \geq 0$, of transformations of the space of distributions, where the multi-variable Bercovici-Pata bijection appears at $t = 1$. We prove that the transformations B_t are homomorphisms with respect to the related operation of free multiplicative convolution, and that they have an interesting relation with the free Brownian motion.

Ken Dykema

Microstates in amalgamated free products.

We describe a method of constructig microstates for certain regular generators of amalgamated free products of von Neumann algebras, where we amalgamate over a finite dimensional or hyperfinite subalgebra. This allows us to compute the fre entropy dimension of such generators. The proof makes use of a "ubiquity of freeness" result. This is joint work with Nathanial Brown and Kenley Jung.

Marius Junge

Free probability methods in operator space theory.

Through the work of Pisier and Shlyakhtenko on the operator space version Grothendieck's inequality it has become clear that free probability methods provide an important tool in operator space theory. However, there are many more situation where free probability is required to prove Khintchine type inequalities-even for hyperfinite factors. This has applications to embedding problems motivated by operator space versions of classical p-stable random variables. We will also briefly discuss why Connes' embedding problem has to be verified for certain free products which enters as a technical tool. Joint work with Parcet.

Roland Speicher

Multiplication of free random variables and the S-transform in the case of vanishing mean

This note extends Voiculescu's S-transform based analytical machinery for free multiplicative convolution to the case where the mean of the probability measures vanishes. We show that with the right interpretation of the S-transform in the case of vanishing mean, the usual formula makes perfectly good sense. This is joint work with Raj Rao.

Andreas Thom

Sofic groups and diophantine approximation.

We prove the algebraic eigenvalue conjecture of J.Dodziuk, P.Linnell, V.Mathai, T.Schick and S.Yates for sofic groups. Moreover, we give restrictions on the spectral measure of elements in integral group rings. Finally, we define a notion of integer operators and prove a quantization of the operator norm below 2. To the knowledge of the author, there is no group known, which is not sofic.

Siegfried Echterhoff

K-fibrations and non-commutative torus bundles.

In this lecture we introduce the notion of K-fibrations and KK-fibrations which serve as non-commutative analogues of the classical Serre fibrations in topology. We show that many C*-algebra bundles which appear naturally as crossed products and group algebras are KK-fibrations. We give a version of the Leray-Serre spectral sequenz for these bundles, which provides a new invariant for RKK-equivalence of such fibrations. We then show that for a certain class of non-commutative torus bundles this invariant allows to answer the question under what conditions such torus bundles are RKK-equivalent to commutative or trivial principal torus bundles. (joint work with Ryszard Nest and Herve Oyono-Oyono)