ABSTRACTS

A unified approach to index theorems for holomorphic maps and foliations.

Marco Abate

Based on joint work with **F. Bracci** and **F. Tovena**.

In 1982, Camacho and Sad proved that if S is a compact leaf of a holomorphic foliation F on a complex surface M, then it is possible to associate to each singular point p of F in S a complex number (the index of F at p along S), depending only on the local behavior of F near p, so that the sum of the indeces is the first Chern class of the normal bundle of S into M.

This theorem, which has profound implications in the theory of holomorphic foliations of complex surfaces, has later been generalized by Lehmann, Suwa and others to triples (M, F, S) where S is a (possibly singular) subvariety of a complex manifold M tangent to a (possibily singular) holomorphic foliation F of S. In particular, Lehmann and Suwa showed how to recover the Camacho-Sad index theorem from the existence of a suitable partial connection on the normal bundle, using a localization procedure starting from a vanishing theorem essentially due to Bott.

In 2004, Abate, Bracci and Tovena showed how to get a Camacho-Sad-like index theorem when the foliation F is replaced by a holomorphic self-map f of the complex manifold M leaving the subvariety S pointwise fixed. Furthermore, they obtained such a result both in the case when S is (in a suitable sense) tangent to the map f, and when S is (in a suitable sense) transversal to the map f, in the latter case under some interesting geometrical hypotheses on the embedding of S into M.

Finally, in the last couple of years Camacho-Movasati-Sad and Camacho-Lehmann have proved particular instances of Camacho-Sad-like index theorems for subvarieties transversal to a holomorphic foliation.

The aim of this talk is to show that all these index theorems are just particular instances of a single construction, based on the existence of a universal partial connection on the normal bundle of a submanifold of a complex manifold.

On a class of nonlinear boundary value problems

Sergiu Aizicovici

Based on joint work with **N. S. Papageorgiou** and **V. Staicu**.

We study periodic problems governed by the scalar p-Laplacian and a nonsmooth potential. Using the nonsmooth critical point theory for locally Lipschitz functions, we prove two existence theorems under conditions of resonance at infinity with respect to the first two eigenvalues of the negative scalar p-Laplacian with periodic boundary conditions.

Some properties of Schur multiplier of p-groups and subgroups and commutator of a pair of groups

Vahid Alamian

In 1904, I. Schur introduced a new concept of a given group G, which was called the Schur multiplier of G. J. A. Green in 1956 found an upper bound for the order of the Schur multiplier of finite p-group, In fact, he showed that if $|G| = p^n$, then $|\mathcal{M}(G)| \leq p^{1/2n(n-1)}$. In 1965, J. Weigold showed that, if $|G/Z(G)| = p^n$, then $|G'| \leq p^{1/2n(n-1)}$. In 1976, A. Gut introduced a concept which was later on called the Schur multiplier of a pair of groups, by G. Ellis, and denoted by $\mathcal{M}(G, M)$.

Let G be a finite p-group of order p^n, then there exists a non-negative integer $t(G)$, such that $|\mathcal{M}(G)| = p^{1/2n(n-1)-t(G)}$. In 1999 G. Ellis gave the structure of p-groups, when $t(G) = 0, 1, 2$ or 3. But Ellis proved all the cases including $t(G) = 3$ with different technique. We have characterized the structure of p-groups G, for which $t(G) = 4$. If (G, M) is a pair of groups with G/M and $M/Z(M, G)$ are of orders p^m and p^n, respectively. Then there exists a non-negative integer $s(G, M)$ such that $|[M, G]| = p^{1/2n(n+2m-1)-s(G,M)}$. Also there exists a non-negative integer $t(G, M)$ such that $|\mathcal{M}(G, M)| = p^{1/2n(n+2m-1)-t(G,M)}$. We also study the structure of pair of p-groups, when $s(G, M) = 1$, and also for the case $t(G, M) = 0, 1$ and 2.

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Subdirect and Krull dimension rings, modules, and lattices

Toma Albu

A right R-module M is said to be *subdirectly irreducible* (or *cocyclic*) if it is nonzero and has a smallest nonzero submodule, or equivalently, if M has a simple essential socle. A submodule N of M is said to be a *subdirectly irreducible* (or *cococyclic*) *submodule of* M if the quotient module M/N is subdirectly irreducible. These concepts can be naturally relativized with respect to a hereditary torsion theory τ on Mod-R, or more generally, can be defined for arbitrary posets with 0 and 1.

A nice result from 1999 of Carl Faith establishes that a right module M_R is Noetherian if and only if M is QFD (i.e., *quotient* (*Goldie*) *finite dimensional*) and satisfies the ascending chain condition on subdirectly irreducible submodules. Since an arbitrary poset P satisfies

the ascending chain condition (i.e., is Noetherian) if and only if it has *dual Krull dimension* $k^0(P) \leqslant 0$, it follows that Faith's result can be reformulated in a dual Krull dimension setting for the lattice $\mathcal{L}(M)$ of all submodules of M and its subset $\mathcal{S}(M)$ of subdirectly irreducible submodules of M as follows: $k^0(\mathcal{L}(M)) \leqslant 0 \Longleftrightarrow \mathcal{L}(M)$ *is a* QFD *lattice and* $k^0(\mathcal{S}(M)) \leqslant 0$.

A natural, still open, question raised by Albu and Rizvi in 2001 asks whether the above reformulation of Faith's result holds for an arbitrary ordinal α instead of 0. The aim of the talk is to present a positive answer to this question for any finite ordinal. In fact, the result holds for any upper continuous modular lattice satisfying certain additional conditions which are automatically verified by the lattice $\mathcal{L}(M_R)$ for any module M_R. To do that, we first characterize upper continuous modular lattices L which have dual Krull dimension $k^0(L) \leqslant \alpha$, by relating that with the property of L being QFD and with other conditions involving subdirectly irreducible elements and/or meet irreducible elements. Many of our results in this respect are true for arbitrary or countable ordinals. Next, we will discuss subdirect irreducibility in module categories and relate it with the concepts of *completely irreducible, primal*, and *primary submodule* of a module.

Comments on the behavior of quasiconformal mappings in Ahlfors regular spaces

Anca Andrei

The Ahlfors $Q-$ regular Loewner spaces, $Q > 1$, are a natural setting for the theory of quasiconformal mappings since this theory recovers much of the properties in the case of Euclidean $n-$spaces, $n \geq 2$. It is very important the fact that in these spaces the three definitions of quasiconformality in Euclidean spaces of dimension at least two, can be formulated. If these spaces are proper then the three definitions are echivalent, moreover every quasiconformal mapping is a quasisymmetric mapping and conversely.

In this talk we shall deal with some boundary extension theorems for quasiconformal mappings $f : D \to D'$ where D and D' are domains in Ahlfors regular spaces .

Lim inf Lipschitz and bi-Lipschitz conditions for direct products, skew products and other mappings

Cabiria Andreian Cazacu

Based on joint work with **S. Cerchez** and **E. Rusu**.

The starting point of this work was the problem M. Cristea proposed to S. Cerchez: what become A.P. Karmazin's results [5] if lim sup is replaced by lim inf.

Let U and V be open sets in $\mathbb{R}^k$ respectively $\mathbb{R}^m$ and $D = U \times V$ in $\mathbb{R}^n$, $n = m+k$; $x, x_0 \in U$, $y, y_0 \in V$, $z = (x, y)$ and $z_0 = (x_0, y_0)$.

A.P. Karmazin [5] established the sequence of equivalences: $F = f \times g$ qc (quasiconformal) $\Leftrightarrow$ f and g compatible $\Longleftrightarrow$ f and g bi-Lip $\Longleftrightarrow$ F bi-Lip. E. Rusu [6], [7] extended the results for qr (quasiregular) mappings and for skew-products. J. Heinonen and P. Koskela [4] proved that lim inf qc $\Longleftrightarrow$ qc. M. Cristea [2] extended this result to qr mappings and completed Karmazin's sequence by $F = f \times q$ qc $\Longleftrightarrow$ f and g $L - BLD$ [3]. In [1] S. Cerchez solved the first step of M. Cristea's problem: $F = f \times g$ lim inf qc $\Longleftrightarrow$ f and g lim inf compatible.

In this paper we concentrate on lim inf Lip (Lipschitz) and then bi-Lip condition and treat for both conditions the punctual, local and global cases.

In **1** we deal with direct products: $f : U \to \mathbb{R}^k$, $g : V \to \mathbb{R}^m$, $F = f \times g : D \to \mathbb{R}^n$, $F(z) = (f(x), g(y))$; in **2** with skew products $f : U \to \mathbb{R}^k$, $g : D \to \mathbb{R}^m$, $F(z) = (f(x), g(x, y))$; and in **3** with the general case $f : D \to \mathbb{R}^k$, $g : D \to \mathbb{R}^m$, $F(z) = (f(z), g(z))$.

As an example we present some of our results for Lip condition in the punctual case:

We prove:

In **1**: F is lim inf Lip at z_0 iff at least one of f or g is lim inf Lip at x_0, respectively at y_0; the other can be arbitrary.

More exactly, F is lim inf l-Lip $\Rightarrow$ f is lim inf $l\sqrt{1+a^2}$-Lip if

$$|y_n - y_0| \leq a|x_n - x_0| \text{ for } 0 < a < \infty \tag{1}$$

and n sufficiently large. Similarly for g.

In **2**: if F is lim inf Lip at z_0, then g is lim inf Lip at z_0; under a condition similar to (1), f is also lim inf Lip at z_0. If f and g are lim inf Lip at x_0 respectively z_0, by sequences x_n and $z_n = (x_n, y_n)$ with the same x_n, then F is lim inf Lip too.

In **3**: if F is lim inf Lip at z_0, then f and g are lim inf Lip at z_0. If f and g are lim inf Lip at z_0 with the same sequence z_n, then F is also lim inf Lip at z_0.

In conclusion, unlike lim inf qc and lim inf compatible which coincide with qc respectively compatible [4], [1], lim inf Lip or lim inf bi-Lip does not imply Lip respectively bi-Lip.

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The pectrum of spherical Dirac-type operators

Nicolae Anghel

The spectral decompositions of the classical Dirac operator on spherical spinors and the Laplace-Beltrami operator on spherical forms have been worked out independently by various methods, ranging from representation theory to separation of variables. Since these operators belong to the same family (of Dirac-type operators) it ought to be possible to obtain their spectra in a unified way. The purpose of this talk is to elaborate on this point.

3D inverse problem of dynamics

Mira-Cristiana Anisiu

The spatial inverse problem of dynamics consists in finding the potentials $V = V(x, y, z)$ under whose action a particle of unit mass can describe the curves of a given family $f(x, y, z) = c_1$, $g(x, y, z) = c_2$. As it was proved by Anisiu (2005), Bozis and Kotoulas (2005), such a potential satisfies two partial differential equations. These equations do not contain the total energy, which is present in Szebehely-type ones (Váradi and Érdi, 1983), and relate merely the potential and the given family. They are useful when no information on the energy is given in advance, as it happens in some models of galactic potentials.

We present solutions of these equations and also discuss the case of the general autonomous force fields.

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Porosity in Banach spaces

Valeriu Anisiu

The small sets in Banach spaces can be introduced in several ways. In finite dimensional spaces the σ-ideal containing the Lebesgue null sets has been used in many situations, but in infinite dimensional Banach spaces (where a translation invariant probability measure does not exist) many other σ-ideals appear in concrete problems. For example, the porous sets are suitable in the characterization of the sets where a continuous convex function defined on a Banach space with a separable dual can be Fréchet non-differentiable. In this talk we present some generalized porosity notions and their relation with other existent concepts such as the Haar-null sets, Γ-null sets and Gauss-null sets.

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Homological symbols and a vanishing conjecture

Marian Anton

We define certain homological classes for a linear group over a ring of arithmetic interest and explain their role in connecting the group structure with the arithmetic of the ring. Then we state and verify a vanishing conjecture in certain particular cases showing its relevance for certain unstable versions of Quillen-Lichtenbaum conjectures.

A dual generalization of monotone mapping

M. Apetrii

Starting with a Crouzeix caracterization of convex function (f is convex if and only if $f+x^*$ is quasiconvex for every $x^* \in X^*$), if we consider the "dual perturbation" only in a subset $M \subset X^*$ we obtain a general class of functions, namely the class of convex function related to a subset M of X^*, called M- convex. For these functions we followed to find necessary and suffiecint condition which must be satisfied by set M to mentained some propertyes known in the case of convex function.
Concomitently with M-convexity we also introduced the subdifferentiability property and respectively the M-monotonicity property.The main aspects followed in this talk are: the continuity of a function on the interior of its domain, the caracterization of extrema points with the aids of subdifferential, the caracterization of M-convexity of a function with the aids of M-monotonicity property of associated subdifferential.
Finally we establish some relationships between monotonicity, quasimonotonicity respectively pseudomonotonicity and M-monotonicity.

Fredholm property of infinite dimensional discrete operators

Narcisa Apreutesei

We present topological properties of linear and semilinear infinite dimensional discrete operators of the form

$$(Lu)_j = a^j_{-m}u_{j-m} + ... + a^j_0 u_j + ... + a^j_m u_{j+m},\ j \in \mathbb{Z} \tag{1.1}$$

and

$$(Au)_j = a^j_{-m}u_{j-m} + ... + a^j_0 u_j + ... + a^j_m u_{j+m} + F(u_j),\ j \in \mathbb{Z}, \tag{1.2}$$

in the Banach space E of sequences $\{u_j\}_{j\in\mathbb{Z}}$ with the supremum norm. They generalize the operators in the left-hand side of the discrete variants of second order ordinary differential equations on the whole axis:

$$au'' + bu' + cu = f \text{ and } au'' + bu' + F(u) = f \text{ on } \mathbb{R}.$$

We discuss the normal solvability for such discrete operators, the properness, and the Fredholm property. We construct a topological degree for the semilinear operator (1.2) . The index of the associated operators is computed and solvability conditions for the nonhomogeneous problem are established. We also study the multi-dimensional case, i.e. discrete operators corresponding to the Laplace operator from the continuous case. Conditions of the normal solvability of the corresponding discrete operators are formulated in terms of limiting problems. The results on the location of the spectrum and the solvability conditions allow various applications to linear and nonlinear problems.

Syzygies and geometry of complex curves

Marian Aprodu

Syzygies of projective varieties are fine invariants that encode information about their intrinsic geometry. G. Farkas has shown recently how to use syzygies to make explicit cycle calculations on the moduli space of curves. In particular, he generalizes a previous joint work with M. Popa invalidating the slope conjecture. I intend to discuss about recent developments of the theory, including the Green and Green-Lazarsfeld conjectures which predict that two basic invariants of curves, namely the Clifford index and the gonality could be read off syzygies. The above-mentioned conjectures are known to be valid for generic curves, and for curves with small Brill-Noether loci.

Harmonic maps on Riemannian polyhedra

Monica Alice Aprodu

Harmonic maps between smooth Riemannian manifolds, introduced by Eells and Sampson in the sixties, play a major role in geometry and analysis. They are defined as critical points of a functional, called the *energy*, or the *action integral*. This notion originates from the Plateau problem on the existence and uniqueness of soup bubbles.

Some years ago, the notion of harmonicity was extended to maps between singular spaces, and applied to rigidity problems (Gromov - Schoen '92).

In this talk (partially based on some joint works with T. Bouziane) we discuss recent results in the theory of harmonic maps on Riemannian polyhedra, with emphasis on harmonic morphisms, and other special classes of mappings.

Some theoretically aspects of the seismic problems

Mihaela Arsene

Based on joint work with **Horia Ene** and **Bogdan Nicolescu**.
The case of a wave propagation generated by a boundary source into a heterogeneous medium occupying a half-space is one of the important actually seismic problems. This work presents some theoretically aspects about these kind of problems.

More general credibility models

Virginia Atanasiu

This communication gives some extensions of the original Bühlmann model. The talk is devoted to semi-linear credibility, where one examines functions of the random variables representing claim amounts, rather than the claim amounts themselves. The main purpose of semi-linear credibility theory is the estimation of $\mu_0(\theta) = E[f_0(X_{t+1})|\theta]$ (the net premium for a contract with risk parameter: θ) by a linear combination of given functions of the observable variables: $X' = (X_1, X_2, \ldots, X_t)$. So the estimators mainly considered here are linear functions of several functions $f_1, f_2, \ldots, f_n$ of the observable random variables. The approximation to $\mu_0(\theta)$ based on prescribed approximating functions $f_1, f_2, \ldots, f_n$ leads to the optimal non-homogeneous linearized estimator for the semi-linear credibility model. Also we discuss the case when taking $f_p = f$ for all p, try to find the optimal function f. It should be noted that the

approximation to $\mu_0(\theta)$ based on a unique optimal approximating function f is always better than the one furnished in the semi-linear credibility model based on prescribed approximating functions: $f_1, f_2, \ldots, f_n$. The usefulness of the latter approximation is that it is easy to apply, since it is sufficient to know estimates for the structural parameters appearing in the credibility factors. From this reason we give some unbiased estimators for the structure parameters. For this purpose we embed the contract in a collective of contracts, all providing independent information on the structure distribution. We close the talk by giving the semi-linear hierarchical model used in the applications chapter.

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On CH-quasigroups with maximum conditions

Aleona Babiy

It is proved that the maximum condition for subquasigroups in a CH-quasigroup Q is equivalent with the conditions of finite generating of different subquasugroups of Q and different subgroups of the multiplication group of Q.

The theory of commutative Moufang loops (CML's) is one of the deepest and subtlest areas of the present-day theory of quasigroups and loops. It is certainly one of the most interesting areas, chiefly because of its many connections with other topics, in particular, with CH-quasigroups [1 – 3]. Such quasigroups are investigated in this communication with the help of CML. Concretely, the various equivalent maximum conditions of CML and its multiplication groups from [4] transferred on CH-quasigroups.

Let $(Q, \cdot)$ be a non-empty set Q provided with a binary operation $(x, y) \to x \cdot y$. One says that $(Q, \cdot)$ is a *TS-quasigroup* of any equality of the form $x \cdot y = z$ remains true under all the permutations of x, y, z (equivalently $xy = yx$ and $x \cdot xy = y$). A quasigroup is said to be *medial* if $xu \cdot xy = xv \cdot uy$ identically. $(Q, \cdot)$ is a *CH-quasigroup* iff $(Q, \cdot)$ is a TS-quasigroup such that $xy \cdot xz = xx \cdot yz$ identically. The last condition is equivalent with the following condition: every subquasigroup generated by 3 elements is medial. For loop the identity $xy \cdot xz = xx \cdot yz$ characterize the CML [1 – 3].

Given four elements x_1, x_2, x_3, k of any quasigroup $(Q, \cdot)$ *the mediator of x_1, x_2, x_3 with respect to k* is the element a of Q uniquely determined by the equality $x_1x_2 \cdot kx_3 = x_1a \cdot x_2x_3$. We denote $a = [x_1, x_2, x_3]_k$. We say that *the mediators of weight i with respect to k* are the mediators of the form $[x_1, x_2, \ldots, x_{2i+1}]_k$ defined inductively by $\alpha_1 = [x_1, x_2, x_3]_k$ and $\alpha_{i+1} = [\alpha_i, x_{2i+2}, x_{2i+3}]_k$. A quasigroup Q is called medially nilpotent of class n if it satisfies the identity $[x_1, x_2, \ldots, x_{2n+1}]_y = y$, but doesn't satisfy the identity $[x_1, x_2, \ldots, x_{2n-1}]_y = y$.

The *multiplication group* $\mathfrak{D}$ of a CH-quasigroup $(Q, \cdot)$ is the group generated by all *translations* $L(x)$, where $L(x)y = xy$, $x \in Q$.

Theorem. *For an arbitrary non-medial CH-quasigroup Q with the multiplication group $\mathfrak{D}$ and its subgroup $\mathfrak{D}^0$ consisting of products of even number of translations $L(x)$, $x \in Q$, the following statements are equivalent:*

1) *Q is finitely generated;*

2) *Q satisfies the maximum condition for subquasigroups;*

3) *if Q contains a medially nilpotent subquasigroup of class n, then all its subquasigroups of this type are finitely generated;*

4) *at least one maximal medial subquasigroup of Q is finitely generated;*

5) *non-normal medial subquasigroups of Q are finitely generated;*

6) *normal subquasigroups of Q are finitely generated.*

7) *$\mathfrak{D}$ (respect. $\mathfrak{D}^0$) is finitely generated;*

8) *$\mathfrak{D}$ (respect. $\mathfrak{D}^0$) satisfies the maximum condition for subgroups;*

9) *all normal subgroups of $\mathfrak{D}$ (respect. $\mathfrak{D}^0$) are finitely generated;*

10) *all non-normal abelian subgroups of $\mathfrak{D}$ (respect. $\mathfrak{D}^0$) are finitely generated;*

11) *at least one maximal abelian subgroup of $\mathfrak{D}$ (respect. $\mathfrak{D}^0$) is finitely generated;*

12) *if $\mathfrak{D}$ (respect. $\mathfrak{D}^0$) contains a nilpotent subgroup of class n, then all its subgroups of this type of $\mathfrak{D}$ are finitely generated;*

13) *if $\mathfrak{D}$ (respect. $\mathfrak{D}^0$) contains a solvable subgroup of class s, then all its subgroups of this type of $\mathfrak{D}$ are finitely generated;*

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Additive and multiplicative Schwarz methods for inequalities with contraction operators

Lori Badea

Keywords: domain decomposition methods, fixed-point problems, nonlinear problems, two-level methods, Navier-Stokes problem.

The problem is stated in a reflexive Banach space and it generalizes the well known fixed-point problem in the Hilbert spaces. Error estimation theorems are given for three multiplicative algorithms and two additive algorithms. We show that these algorithms are in fact Schwarz methods if the subspaces are associated with a decomposition of the domain. Also, for the one- and two-level methods in the finite element spaces, we write the convergence rates as functions of the overlapping and mesh parameters. They are similar with the convergence rates of these

methods for linear problems. Besides the direct use of the five algorithms for the inequalities with contraction operators, we can use the above results to obtain the convergence rate of the Schwarz method for other types of inequalities or nonlinear equations. In this way, we prove the convergence and estimate the error of the one- and two-level Schwarz methods for some inequalities in Hilbert spaces which are not of the variational type, and also, for the Navier-Stokes problem. Finally, we give conditions of existence and uniqueness of the solution for all problems we consider. We point out that these conditions and the convergence conditions of the proposed algorithms are of the same type.

Nonparametric efficiency analysis: recent developments

Luiza Badin

Keywords: Farrell efficiency measures, Data Envelopment Analysis (DEA), Free Disposal Hull (FDH), bootstrap, conditional efficiency.

In efficiency analysis, *the scores* of economic producers are often evaluated by the radial distance from each producer to an estimated production frontier. The two major approaches to efficiency measurement proposed by the economic literature are based on econometric and mathematical programming techniques. The former is parametric, therefore dependent on the specification of the structure of technology, while the latter is nonparametric, based on envelopment techniques. The nonparametric methods are more robust to model specification errors, since they do not rely on functional specification for the production frontier. On the other hand, the nonparametric estimators are typically affected by the presence of outliers as well as subject to the so-called *curse of dimensionality*. Other important issue in productivity analysis is the sensitivity of the efficiency scores with respect to external, environmental factors that might affect the production process but cannot be controlled by the producer.

In this talk we present recent advances on nonparametric estimation of efficiency measures. Statistical inference is now available in the nonparametric approach and parametric approximations of robust nonparametric frontiers are possible as well, using various functional specifications. Moreover, recent developments in parametric and nonparametric estimation of efficiency converge towards a more flexible, combined technique, which is intended to overcome the limitations while exploiting the advantages of both approaches.

Ramanujan graphs and representation theory of p-adic Groups

Cristina Ballantine

Ramanujan graphs were introduced in the late '80s by Lubotzky, Phillips and Sarnak and independently by Margulis.

A k-regular graph G on n vertices is a Ramanujan graph if the eigenvalues λ of its adjacency matrix satisfy the following condition:

$$|\lambda| = k \quad \text{or} \quad |\lambda| \leq 2\sqrt{k-1}.$$

Asymptotically (i.e., for an infinite family of k-regular graphs with the number of vertices approaching infinity), Ramanujan graphs have the smallest possible second largest eigenvalue. Moreover, one can introduce a notion of primes on graphs and thus one can define a Zeta function and a Riemann hypothesis for graphs. Ramanujan graphs are precisely the graphs satisfying the Riemann hypothesis.

For $n = 2$ or 3, we use the Hecke algebra of $GL_n(\mathbb{Q}_p)$ with respect to $GL_n(\mathbb{Z}_p)$ and the classification of the unramified representations of $GL_2(\mathbb{Q}_p)$, resp. $U_3(\mathbb{Q}_p)$ to show that quotients of the Bruhat-Tits building corresponding to $GL_n(\mathbb{Q}_p)$, $n = 2, 3$, are Ramanujan graphs.

If time permits, we will also discuss Ramanujan bigraphs arising from quotients of the Bruhat-Tits tree of $U(3)$.

Malliavin calculus for locally smooth law and applications to jump type diffusions

Vlad Bally

We develop a finite dimensional calculus of Malliavin type for a random variable $V = (V_1, ..., V_m)$ which has a locally smooth density and we establish an integration by parts formula based on this density. But we are not able to extend the calculus in an infinite dimensional framework, analogues to the Wiener space. Nevertheless, if the bounds produced by the integration by parts formula in finite dimensions are uniform with respect to the dimension of the space, then they produce a bound in infinite dimension as well, and this bound is used in order to study the density of a random variable which depends on an infinite number of noises.

We apply this technique in order to study the density of a jump type diffusion with discontinuous coefficients - which is out of rich using already known variants of Malliavin calculus on the Poisson space.

Phenomenological to molecular modelling of hysteresis in viscoelastic polymers: elastomers to biotissue

H. T. Banks

The mathematical modelling of *viscoelasticity* (sometimes also loosely referred to as *hysteresis*) in materials using ideas from elasticity has attracted the attention of a large number of investigators over the past century. One of the most widely used empirical constitutive models for viscoelasticity in materials is the Boltzmann convolution law, one form of which is given by

$$\sigma(t) = g_e(\epsilon(t)) + C_D\dot{\epsilon}(t) + \int_{-\infty}^{t} Y(t-s)\frac{d}{ds}g_v\left(\epsilon(s), \dot{\epsilon}(s)\right) ds, \tag{1}$$

where σ is the stress, ϵ is the infinitesimal strain, Y is the convolution memory kernel, and g_e and g_v are nonlinear functions accounting for the elastic and viscoelastic responses of the material, respectively. This form of model, when incorporated into force balance laws, results in *integro-partial differential equations* which are most often phenomenological in nature as well as being computationally challenging in simulation, estimation and control design.

An alternative formulation, which employs the *internal variable* approach to overcome both conceptual and computational challenges, is consistent with the belief that hysteresis is actually a manifestation of the presence of multiple scales in a physical or biological material system that is frequently modelled (and masked) with a phenomenological representation such as an hysteresis integral as given in (1) for the macroscopic stress-strain constitutive law. Internal variable modelling results in a coupled system of PDE/ODEs and leads to an efficient computational alternative for the corresponding integro-partial differential equation models. In addition, it provides a "molecular" basis for the models.

In this presentation, we summarize the historical development of hysteresis laws and briefly outline two recent advances: (*i*) a new constitutive model that has been developed which combines the virtual stick-slip continuum "molecular-based" ideas of Johnson and Stacer with the Rouse bead chain ideas as described in Doi and Edwards; (*ii*) a two dimensional version of a model that accounts for stenosis driven shear wave propagation in biotissue.

The Bayesian approach in computer simulation of a system reliability.

Gheorghe Barbu

It is well known that any coherent system is composed of elements. The functioning of system depends of the functioning of its elements. The failure of system depends of the failure of its elements, which are influenced of random events. The failure of the elements of system has different influence on system functioning. In order to increase the system reliability it is important to know the reason of the failure of the elements, which lead to the failure of system. Based on Bayesian inference it is possible to simulate the inference of the failure of elements to the failure of system.

Towards a good constructive definition of order completeness

Marian Baroni

Partially ordered sets are examined within the framework of Bishop's style constructive mathematics which can be viewed as the constructive core of mathematics and whose theorems can be translated into many formal systems of computable mathematics.

Particular attention is paid to the notion of order completeness. Unlike the classical case, the least–upper–bound principle does not hold constructively. However, the order completeness of the real number set is expressed constructively by an equivalent condition for the existence of supremum, a condition of (upper) order locatedness which is vacuously true in the classical setting. This condition leads to a general definition of upper locatedness which, in turn, can be used for a general definition of order completeness.

Two classically equivalent, but constructively inequivalent, notions of supremum are also examined. It turns out that the supremum of a set S exists if and only if S is upper located and has a weak supremum—that is, the classical least upper bound.

Modelling reticulate evolution

Mihaela Carmen Baroni

Since Darwin's first sketch of an evolutionary tree, biologists have used trees to describe evolutionary relationships between species. However, it is now well–accepted that processes such as hybridization, lateral gene transfer, and recombination would be better described by a network that is more complex than a phylogenetic tree, with some species arising from the genetic contribution of two ancestral lineages.

For representing and analyzing reticulate evolution, one can use the mathematical model of a hybrid phylogeny, a rooted acyclic digraph satisfying certain properties which attempt to

capture biological reality. In such a digraph, vertices of indegree at least two represent reticulate events.

An important question of interest for biologists is the following: What is the smallest number of reticulation events needed to explain two conflicting gene trees? This problem can be interpreted and solved within the framework of hybrid phylogenies.

Co-Galois theory : a topological, group theoretic approach

Şerban Basarab

An abstract (topological and group theoretic) approach of the field theoretic co-Galois theory is initiated in [1], [3] and developed in the recent work [5]. Given a profinite group Γ and a quasicyclic discrete group A on which Γ acts continuously, several classes of closed subgroups of Γ and related classes of subgroups of the cocycle group $Z^1(\Gamma, A)$, called *radical, hereditarily radical, Kneser, hereditarily Kneser*, *co-Galois* and *strongly co-Galois*, are defined, investigated, characterized and classified.

A more general frame for the abstract co-Galois theory is the object of a recent project of the author [4]. In this more general setting one considers a continuous action of a profinite group Γ on a (not necessarily commutative) profinite group G together with a continuous 1-cocycle $\eta : \Gamma \longrightarrow G$ with the property that the profinite group G is topologically generated by the image $\eta(\Gamma)$. The main object of study is the natural Galois connection induced by the cocycle η relating the topological lattice of all closed subgroups of Γ and the topological lattice of all Γ-invariant closed normal subgroups of G. The theory developed in [1], [3], [5] which we may call *abstract cyclotomic co-Galois theory*, is the first stage of this more general theory, where $G := \mathrm{Hom}(Z^1(\Gamma, A), A)$ is the Pontryagin dual of the discrete torsion Abelian group $Z^1(\Gamma, A)$ with the action induced by the action of Γ on the quasicyclic group A, while the cocycle $\eta : \Gamma \longrightarrow G$ is defined by $\eta(\gamma)(\alpha) = \alpha(\gamma)$ for $\alpha \in Z^1(\Gamma, A), \gamma \in \Gamma$.

The main results of the field theoretic co-Galois theory are easily obtained via Hilbert's Theorem 90 from their abstract cyclotomic co-Galois theoretic versions. Moreover, new results in the field theoretic co-Galois theory as well as interesting connections with the model theory of Henselian fields and p-adically closed fields [7], [8], [2], [6] are consequences of specific results of the abstract cyclotomic co-Galois theory. On the other hand, the more general frame for the abstract co-Galois theory provides a suitable extension of the frame of the classical field theoretic co-Galois theory.

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CMC surfaces in Euclidean and Finsler spaces

Vladimir Bălan

Recent advances in the theory of Finsler surfaces have been provided after 2002 by Z.Shen - who has intensively studied 2-dimensional Finsler metrics and their flag curvature. Recently Z.Shen [9], and further M.Souza and K. Tenenblat [10] have investigated minimal surfaces immersed in Finsler spaces from differential geometric point of view. Still, earlier rigorous attempts using functional analysis exist in the works of G.Bellettini and M. Paolini (after 1995, e.g., [5,6,7]). In 1998, based on the notion of Hausdorff measure, Z.Shen [9] has introduced the notion of mean curvature on submanifolds of Finsler spaces. In the present work, within the ansatz of locally Minkowski Finsler spaces of Randers and Kropina type - which extend the Euclidean ones, is determined and studied the mean curvature field of immersed hypersurfaces. Further, for the 2-dimensional case, is explicitely determined the mean curvature function. The obtained results are applied to Monge charts and to surfaces of rotation of Kropina and Randers spaces, while relevant MAPLE plots exemplify the developed theory.

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On maxentropic reconstruction method and applications

Costel Bălcău

Based on joint work with **Vasile Preda**.

We use the maxentropic reconstruction method for obtaining some countable probability distributions, multiple Markov chains and nonnegative multidimensional matrices with fixed marginal sums.

The Stratification of the Orbit Space of Proper Groupoids

An inequality in the phase space of the planar three body problem

Mihail Bărbosu

This talk is concerned with the planar three body problem in Jacobi coordinates. For fixed values of the constants of energy and angular momentum we prove an inequality that leads to a qualitative study of the motion of the three bodies. The classic inequalities that give the set of admissible configurations and the regions where the motion can occur are particular cases of our result. We also consider further explorations, along with a geometrical interpretation of our inequality.

Strong convergence theorems for common fixed points of Banach operator pair

Ismat Beg

Keywords: Banach operator pair; uniformly R-subweakly commuting map; asymptotically I-nonexpansive map; strong convergence; Gâteaux differentiable norm.

We obtain results concerning strong convergence of common fixed points of asymptotically I-nonexpansive map T for which (T, I) is a Banach operator pair in a Banach space with uniformly Gâteaux differentiable norm. Several common fixed point and best approximation results for this newly defined class of maps are proved.

Polyharmonic Dirichlet problems

Heinrich Begehr

There are 2 boundary value problems for the polyharmonic equation $\Delta^n w = f$, the Dirichlet problem

$$\partial_\nu^\mu w = \gamma_\mu \text{ on } \partial D \text{ for } 0 \leq \mu \leq n-1$$

and the Neumann problem

$$\partial_\nu^\mu w = \gamma_\mu \text{ on } \partial D \text{ for } 1 \leq \mu \leq n.$$

For the case $n = 1$ they are the harmonic Dirichlet and Neumann problems. For the biharmonic case $n = 2$ besides the Dirichlet problem

$$w = \gamma_0, \partial_\nu w = \gamma_1 \text{ on } \partial D$$

and the Neumann problem

$$\partial_\nu w = \gamma_1, \partial_\nu^2 = \gamma_2 \text{ on } \partial D,$$

there are the Dirichlet-2 problem

$$w = \gamma_0, \Delta w = \gamma_2 \text{ on } \partial D,$$

the Dirichlet-Neumann problem

$$w = \gamma_0, \partial_\nu \Delta w = \gamma_3 \text{ on } \partial D,$$

the Neumann-Dirichlet Problem

$$\partial_\nu w = \gamma_1, \Delta w = \gamma_2 \text{ on } \partial D,$$

and the Neumann-2 problem

$$\partial_\nu w = \gamma_1, \partial_\nu \Delta w = \gamma_3 \text{ on } \partial D.$$

While the last 4 boundary value problems are related to the decomposition of the biharmonic equation into a system of 2 harmonic equations, the first 2 are not. Using this decomposition these problems may be solved by iterating the respective solutions to the harmonic problems. It results in biharmonic Green functions, given as convolutions of harmonic Green and Neumann functions, respectively. Of particular interest is the Green-Neumann function, a hybrid Green function which is not symmetric in its variables as it behaves as a Green type function in one variable and as a Neumann kind function in the other variable.
For the triharmonic operator the situation is more involved. Besides the first formulated 2 boundary value problems from the decomposition into a system of 3 harmonic equations one has the following possibilities for choosing boundary conditions:

DDD DDN DND NDD
NNN NND NDN DNN

Here D means Dirichlet, N Neumann condition. But one can also decompose the triharmonic equation into a biharmonic and a harmonic equation leading to some 8 other boundary value problems.
One can also include Robin boundary conditions. This will cause more suitable boundary value problems. For the general inhomogeneous poplyharmonic equation only 3 boundary value problems will be considered in the particular case of the unit disc in the complex plane: the Dirichlet, the Dirichlet-n, and the Neumann-n problems. Their solutions will be given in explicit forms. Some results are also available for the upper half plane.

Star structures and noncommutative Riemannian geometry

Edwin Beggs

Based on joint work with **S. Majid**.

I will talk about star structures in noncommutative geometry from an abstract point of view, and give several examples. It turns out that this has an application to the theory of connections in noncommutative geometry. The idea of a Riemannian structure as a Hilbert C* module forces us to consider star structures on the forms on an algebra with differential structure. The resulting idea of connections preserving the metric has a surprising amount in common with the classical theory, though problems remain.

Manifolds of quasi-constant sectional curvature

Cornelia-Livia Bejan

A manifold of quasi-constant sectional curvature is defined as a Riemannian manifold endowed with a unit vector field U, such that the curvature of any plane depends only on its angle with U and the point of manifold, [Vranceanu, Chen & Yano, Boju & Popescu, Ganchev & Mihova]. Here we deal with the complex correspondent of this notion, that is Kahler manifolds of holomorphic quasi-constant curvature. We characterize them, give some properties of their curvature, construct some examples and study their characteristic classes. At the end, these manifolds are seen as a subclass of generalized Einstein manifolds. Some other subclasses, given by locally decomposable manifolds, are also studied.

From Riemann to Lebesgue by a.s. averaging

Alexandra Bellow

Basic examples, probabilistic considerations (a non-conventional martingale), the role of the primes; some famous open problems.

Congestions in priority queueing systems

Olga Benderschi

There is a wide range of real service systems in which not all demands have the same priority. Priority queueing systems are mathematical models for some of such phenomena.

Consider priority queueing systems $M_r|G_r|1$. For such systems there are analytical results for distribution of the busy periods, queue length, virtual waiting times, etc. Traffic coefficient, which is defined in the classical theory of queueing systems as a ratio of expected service time and expected interarrival time, makes an important part of these results. This coefficient is an important measure of the system performance and it is responsible for the workload of the system. Namely, one can show that if the traffic coefficient is greater than one, then the number of waiting requests will be growing illimitable and the system will be congested by buffered requests—the service process will be blocked then.

Numerical modelling was used to study preemptive priority queueing systems which are specified by concrete values of parameters of the incoming flows and distribution functions of service times. Obtained numerical results are in agreement with the analytical results. This work was done under support of the SCOPES grant IB7320-110720

Some applications of Mawhin continuation theorem

Cristian Bereanu

In this talk we will survey some results obtained using Mawhin continuation theorem. In particular, we will show how this theorem can be used to prove existence of positive solutions of some competition or prey-predator systems and to prove existence of solutions of some problems at resonance.

Some perturbation results for quotient morphisms

Dana Bereanu

In this talk we present some perturbation results for quotient morphisms with applications to linear relations. The main tool in our proofs is the perturbation theory for complexes of Banach spaces.

On uniform exponential instability of evolution operators

Larisa Biris

Based on joint work with **M. Megan**.
In this talk we present necessary and sufficient conditions for uniform exponential instability of evolution operators in Banach spaces. Variants for uniform exponential instability of some well-known results due to Datko, Neerven and Zabczyk are given. As consequences, some results proved some previous papers are obtained.

Saint–Venant's principle for Cosserat elastic shells

Mircea Bîrsan

This talk is concerned with the linear deformation of thin elastic shells, modeled as Cosserat surfaces. We investigate the equilibrium of cylindrical Cosserat surfaces with arbitrary open or closed cross–sections, loaded by contact forces and couples distributed over its end edges. In this context, we prove the Saint–Venant's principle which asserts that, if two sets of loadings are statically equivalent at each end of the cylinder, then the difference of the two solutions is negligible, except possibly near the ends. We obtain estimates for the strain energy $U(z)$ contained in that portion of the cylindrical shell which lies beyond a distance z from the end edges. We show that the strain energy $U(z)$ associated to the difference of the two solutions has an exponential decay as a function of z.

Special directions in real and complex contact geometry

David E. Blair

If on a contact metric manifold M^{2n+1}, the sectional curvature of plane sections containing the characteristic (Reeb) vector field ξ are negative, then there exist "special directions" in the contact subbundle along which ξ falls "forward" and "backward". If in addition M^3 is compact and ξ generates an Anosov flow whose stable and unstable subbundles agree with the special directions, then M^3 is a compact quotient of $\tilde{SL}(2,\mathbb{R})$. We first review these ideas which date from the mid 1990s and then turn to recent joint work with B. Korkmaz in the case of complex contact metric manifolds. We will discuss holomorphic and real special directions on complex contact metric manifolds and in particular on the Lie group $SL(2,\mathbb{C})$.

Adaptive refinement in peridynamics for multiscale material models

Florin Bobaru

We present a new method, peridynamics, which can simulate fracture, fragmentation, and damage in all its complexity. We introduce three types of convergence for this non-local meshfree method and present new refinement strategies that can lead to the development of multiscale models in a very direct way.

The peridynamic theory [1-4] is a new non-local method that differs from other non-local methods because it avoids using spatial derivatives in the formulation of the equations of motion. Peridynamics offers an important advantage, compared to other methods, in modeling of dynamic fracture. In peridynamics, the same equations apply everywhere regardless of discontinuities. Cracks initiate, grow spontaneously and unguided. Transition between fracture modes happens without enforced ad-hoc criteria. In peridynamics, cracks are part of the solution, not part of the problem.

For multiscale modeling the peridynamic method appears to be well suited since a single consistent model (the same type of equations and type of discretization) can be used across scales, reducing complexities and modeling difficulties currently faced by other numerical models at transition zones between different scales.

In transitioning between scales, grid refinement and adaptivity has to be employed. Due to its non-local character, refinement in peridynamics is different than in classical methods. We develop a method for adaptive refinement in peridynamics. This is a first step in constructing multiscale algorithms for peridynamics. We discuss the three different types of numerical convergence that can be introduced for a computational approximation of the peridynamic equations. We also show that the particular shape and smoothness of the micromodulus has an influence on the rate of convergence of the adaptive schemes. Moreover, we show that the use of "exotic" micromoduli, while valid from the point of view of material stability, can lead to ill-conditioning of the stiffness matrix for static problems. Even in these cases, however, a correct solution is possible using dynamic relaxation instead of forming the stiffness matrix. We implement refinement schemes for both static and dynamic analyses in 1D, and show that the adaptive peridynamic solution achieves the theoretical rates of convergence. Current efforts are directed at extending these ideas crack propagation in 2D and 3D.

Acknowledgements. Partial funding for adaptive methods in peridynamics has been provided by The Boeing Company.

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Paramedial topological grupoids

Natalia Bobeică

Based on joint work with **L. Chiriac**.

A topological groupoid $(G, \cdot)$ is called:

- *medial* if $xy \cdot zt = xz \cdot yt$ for all $x, y, z, t \in G$;
- *paramedial* if $xy \cdot zt = ty \cdot zx$ for all $x, y, z, t \in G$.

Theorem 1. *If a paramedial topological groupoid $(G, \cdot)$ satisfying the following conditions:*

1. contains a bijective idempotent e;

2. the maps $x \to xe$ and $x \to ex$ are homeomorphisms of G onto itself;

3. there exists on G a binary operation $(\circ)$ such that $(ex) \circ (ye) = yx$,

then $(G, \circ)$ is a commutative semigroup having e as identity. Furthermore, the maps $x \to xe$ and $x \to ex$ are antihomomorphisms of $(G, \circ)$.

Theorem 2. *Let $(G, \circ)$ be a paramedial topological groupoid. If there exist two topological automorphisms $f, g : (G, \circ) \to (G, \circ)$ such that*

1) $x \cdot y = f(x) \circ g(y)$;

2) $gg = ff$,

then $(G, \cdot)$ is paramedial, too.

Theorem 3. *Let $(G, \circ)$ be a medial topological groupoid. If there exist two topological antiautomorphisms $f, g : (G, \circ) \to (G, \circ)$ such that*

1) $x \cdot y = f(x) \circ g(y)$;

2) $fg = gf$,

then $(G, \cdot)$ is paramedial, too.

Primary solutions to general Beltrami equations

Bogdan Bojarski

In the lecture some new aspects of the geometry of general Beltrami equations and related problems of the theory of Bers-Vekua generalized analytic functions will be discussed.

Dependence of the optimal capital stock and consumption evolution on the consumers growth rate in the framework of Ramsey model on finite horizon

Nadia Bonchiş

Based on joint work with **Şt. Balint**.

In this talk the Ramsey optimal growth of the capital stock and consumption on finite horizon is analyzed when the growth rate of consumers is strictly positive.

The main purpose is to establish the dependence of the optimal capital stock and consumption evolution on the growth rate of consumers.

The analysis reveals: for any initial value $k_0 \geq 0$ there exists a unique optimal evolution path of length $N+1$ for the capital stock; if k^o is strictly positive then all the elements of the optimal capital stock evolution path are strictly positives except the last one which is zero; the optimal capital stock evolution of length $N+1$ starting from $k^o \geq 0$ satisfies the Euler equation; the value function V_N is strictly increasing, strictly concave and continuous on $\mathbb{R}_+$.

The family of functions $\{V_{N-T}\}_{T=0...N-1}$ satisfies the Bellman equation and it is the unique solution of this equation which is both continuous and satisfies the transversality condition. The Mangasarian Lemma is also satisfied.

For N tending to infinity the optimal evolution path of length N of the capital stock tends to those on the infinite time horizon.

For any $k^o > 0$ the value function in k^o decreases when the consumers growth rate increases.

Some geometric aspects of discrete metric spaces and metric graphs

Anca-Iuliana Bonciocat

We introduce and analyze lower (Ricci) curvature bounds for discrete metric measure spaces (M, d, m) and for metric graphs. The approach is based on convexity properties of the relative entropy $Ent(.|m)$ regarded as a function on the L_2-Wasserstein space of probability measures over the metric space (M, d). Our lower curvature bounds are stable under an appropriate notion of $\mathbb{D}$-convergence of metric measure spaces introduced by K.-T. Sturm. Furthermore we study discretizations of metric measure spaces, in particular discretizations of Riemannian manifolds. We obtain explicitly the curvature of the homogeneous planar graphs. The curvature bound yields a Talagrand type inequality, which entails the concentration of the reference measure.

Furthermore, we study a curvature-dimension condition for discrete metric measure spaces and metric graphs. This condition has more geometric consequences, like for instance a generalization of the Bonnet-Myers Theorem and a Brunn-Minkowski type inequality.

Prime numbers and irreducible polynomials

Nicolae Ciprian Bonciocat

There are several irreducibility criteria in the literature that rely on the existence of a suitable prime divisor of the value that a given polynomial takes at a specified integral argument. One such criterion is given by the famous result of A. Cohn:

Theorem A. *If a prime p is expressed in the decimal system as $p = \sum_{i=0}^{n} a_i 10^i, \quad 0 \leq a_i \leq 9$, then the polynomial $\sum_{i=0}^{n} a_i X^i$ is irreducible in $\mathbb{Z}[X]$.*

Brillhart, Filaseta and Odlyzko extended this result to an arbitrary base b:

Theorem B. *If a prime p is expressed in the number system with base $b \geq 2$ as $p = \sum_{i=0}^{n} a_i b^i, \quad 0 \leq a_i \leq b-1$, then the polynomial $\sum_{i=0}^{n} a_i X^i$ is irreducible in $\mathbb{Z}[X]$.*

M. Ram Murty established an analogue of Theorem B for polynomials with coefficients in $\mathbb{F}_q[t]$, with $\mathbb{F}_q$ a finite field. Filaseta obtained another generalization of Theorem B by replacing the prime p by a composite number wp with $w < b$:

Theorem C. *Let p be a prime number, w and b positive integers, $b \geq 2$, $w < b$, and suppose that wp is expressed in the number system with base b as $wp = \sum_{i=0}^{n} a_i b^i$, $\quad 0 \leq a_i \leq b-1$. Then the polynomial $\sum_{i=0}^{n} a_i X^i$ is irreducible over the rationals.*

Cohn's irreducibility criterion was also generalized by permitting the coefficients of f to be different from digits:

Theorem D. *Let $f(X) = \sum_{i=0}^{n} a_i X^i$ be such that $f(10)$ is a prime. If the a_i's satisfy $0 \leq a_i \leq a_n 10^{30}$ for each $i = 0, 1, \ldots, n-1$, then $f(X)$ is irreducible.*

Inspired by these particularly elegant results, we proved several irreducibility criteria for polynomials that take a prime value and have a coefficient of sufficiently large modulus:

Theorem E. *If we write a prime number as a sum of integers $a_0, \ldots, a_n$, with $a_0 a_n \neq 0$ and $|a_0| > \sum_{i=1}^{n} |a_i| 2^i$, then the polynomial $\sum_{i=0}^{n} a_i X^i$ is irreducible over $\mathbb{Q}$.*

Theorem F. *If all the coefficients of a polynomial f are ± 1 and $f(m)$ is a prime number for an integer m with $|m| \geq 3$, then f is irreducible over $\mathbb{Q}$.*

One way to extend the results of this type is to replace the condition that f takes a prime value by a more general one, by asking the resultant of f and another polynomial g to be a prime number. Together with some extra conditions on the location of their roots, this will ensure simultaneously the irreducibility of f and g:

Theorem G. *Two polynomials $f, g \in \mathbb{Z}[X]$ are irreducible over $\mathbb{Q}$ if $|Res(f, g)|$ is a prime number and $|\theta - \xi| > 1$ for every root θ of f and every root ξ of g.*

In particular, we obtain:

Theorem H. *If we write a prime number as $(a_0 - a_2 + a_4 - \ldots)^2 + (a_1 - a_3 + a_5 - \ldots)^2$ with $a_i \in \mathbb{Z}$ and $|a_0| > \sum_{i \neq 0} |a_i| 2^i$, then the polynomial $\sum_i a_i X^i$ is irreducible over $\mathbb{Q}$.*

Once we have found a pair f, g of polynomials satisfying the hypotheses of Theorem G, we may use the fact that the roots of a complex polynomial are continuous functions of its coefficients to obtain irreducibility conditions for linear combinations of the form $n_1 f + n_2 g$, with $n_1, n_2 \in \mathbb{Z}$. We also establish similar irreducibility conditions for multivariate polynomials over an arbitrary field, such as:

Theorem I. Let K be a field, $f(X, Y) = \sum_{i=0}^{n} a_i Y^i$, $g(X, Y) = \sum_{i=0}^{m} b_i Y^i \in K[X, Y]$, with $a_0, \ldots, a_n, b_0, \ldots, b_m \in K[X]$, $a_0 a_n b_0 b_m \neq 0$, and assume that $Res_Y(f, g)$ as a polynomial in $K[X]$ is irreducible over K. If

$$\deg a_0 > \max\{\deg a_1, \ldots, \deg a_n\} \quad \textit{and} \quad \deg b_m \geq \max\{\deg b_0, \ldots, \deg b_{m-1}\},$$

then both f and g are irreducible over $K(X)$.

Semireflective product of two reflective subcategories

Dumitru Botnaru

Based on joint work with **Olga Cerbu**.

Let C_2V be the category of locally convex (topological vector) Hausdorff spaces, S the subcategory of spaces with weak topology, and Γ_0 the full subcategory of complete spaces.

Definition 1 (see [2]). The reflective subcategory R is called c-reflective, if $S \subset R$ and the reflective functor $r : C_2V \longrightarrow R$ is exact to the left.

Theorem 1 (see [2]). Let R be a reflective subcategory of the category C_2V. The next affirmations are equivalent:

1. R is a c-reflective subcategory.

2. There is a coreflective subcategory K of the category C_2V so that the reflective functors $r : C_2V \longrightarrow R$ and the coreflective functors $k : C_2V \longrightarrow K$ have the properties:

a) $kr \sim k$;

b) $rk \sim r$.

In this case (K, R) is called a conjugated pair of subcategories.

Definition 2. Let R_1 and R_2 be two reflective subcategories of the category C_2V. A space X of category C_2V is called (R_1, R_2)-semireflective, if its R_1-replique belongs to the subcategory R_2. The subcategory of all (R_1, R_2)-semireflective spaces is called the semireflective product of all R_1 and R_2 subcategories and is denoted

$$L = R_1 \times_{sr} R_2.$$

Theorem 2. The subcategory of semireflective product of two reflective subcategories is multiplicative.

Theorem 3. Let (K, R) be a conjugated pair of subcategories of the category C_2V, Γ a reflective subcategory and $\Gamma_0 \subset \Gamma$, $L = R \times_{sr} \Gamma$. Then:

1. L is a reflective subcategory of the category C_2V,

2. The semireflective product of the subcategories R and Γ is equal to the right product of the subcategories K and $R \cap \Gamma$:

$$R \times_{sr} \Gamma = K \times_d (R \cap \Gamma).$$

3. $R \cap \Gamma \subset L \subset \Gamma$.

4. Let (E, t) be a (R, Γ) -semireflective space, $(E, r(t))$ its R - replique and $(E, k(t))$ its K - coreplique. For every locally convex topology u such that

$$r(t) \leq u \leq k(t),$$

the space (E, u) is (R, Γ) - semireflective.

5. $(K \cap L, R \cap L)$ is a conjugated pair of subcategories of the category L.

Examples:

1. (S, M) is a conjugated pair of subcategories, where M is the subcategory of spaces with Mackey topology. Then

$$S \times_{sr} \Gamma = M \times_d \prod,$$

where $\prod$ is the subcategory $\prod = S \cap \Gamma_0$.

2. Let $c\Gamma_0$ be a subcategory of quasicomplete spaces and sR the subcategory of semireflective spaces [4], then $S \times_{sr} (c\Gamma_0 = M \times_d (S \cap c\Gamma_0)) = sR$.

3. The subcategory Sch of Schwarz spaces is c-reflective and $Sch \times_{sr} \Gamma_0$ is the subcategory of inductive semireflective spaces [3].

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Closedness type regularity conditions in convex optimization

Radu Ioan Boţ

One of the leading parts in the optimization theory from both theoretical and practical points of view is played by the duality theory. A challenging and difficult undertaking in this field is to give regularity conditions which guarantee strong duality, the situation when the optimal objective values of the two problems are equal and the dual problem has optimal solutions. Several regularity conditions were given for convex optimization problems, where along with the classical interior some generalized interior notions have been used. Unfortunately, for a number of optimization problems, not just in theory but also in practice, they may be not applicable, for instance, when the generalized interior of the set involved in the regularity condition is empty.
In this presentation we describe a general approach for formulating and characterizing so-called closedness type regularity conditions for a general convex optimization problem. These should be stated in the space of the dual variables and should lead to corresponding new regularity conditions for different classes of convex optimization problems, weaker than the generalized interior-point conditions given so far in the literature (cf. [1], [2]).
As an application we give a weak sufficient condition for the maximal monotonicity of the operator $S + A^*TA$, where $S : X \rightrightarrows X^*$, $T : Y \rightrightarrows Y^*$ are two maximal monotone operators, $A : X \to Y$ is a linear continuous mapping and X, Y are reflexive Banach spaces (cf. [3], [4]). In this way we show once more the usefulness of convex analysis in treating the problem of maximality of monotone operators.

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The effect of the buoyancy and Marangoni forces on the dopant distribution in the case of a single crystal rod grown from the melt by the edge-defined film-fed growth (EFG) method

Liliana Brăescu

The dependence of the dopant distribution on the combined effect of the buoyancy and Marangoni forces, for two cylindrical bars grown from the melt by EFG method with central

capillary channel (CCC) shaper and melt replenishment, is analyzed. The partial differential equations used for describing the growth process are those of the incompressible Navier-Stokes equations in the Boussinesq approximation and of the convection-conduction and conservative convection-diffusion equations. For numerical solutions, the finite-element numerical technique using COMSOL Multiphysics 3.3 software for a 2D axis-symmetric model is utilized. The Marangoni effect (i.e., surface tension driven flows due to the temperature gradient along the liquid free surface) is taken into account, and for its implementation the weak form of the boundary application mode is employed.

The computations are carried out in the stationary case, for different rates $d\gamma/dT$ of the surface tension. Numerical results are given for two materials ($Nd:YVO_4$ and $Nd:LiNbO_3$) which lead outward ($d\gamma/dT<0$) and inward ($d\gamma/dT>0$) flows along the liquid surface, respectively.

Computations show that if absolute value of the $d\gamma/dT$ increases then:

- the velocity field in the capillary channel decreases;
- the velocity field in the meniscus increases as effect of the Marangoni forces;
- the axial concentration C_{ax} increases;
- the maximum value of the dopant concentration C_{max} is moved from lateral surface of the crystal to the axis, due to increases of the effect of the Marangoni forces;
- the radial segregation which describe non-homogeneity of the crystal, given by the absolute value of the difference between the concentrations on the lateral surface and on the axis of the crystal ($C_{lat}-C_{ax}$) at the level of the melt/solid interface, is smaller.

Certain integral operators on the classes $\mathcal{M}(\beta_i)$ and $\mathcal{N}(\beta_i)$

Daniel Breaz

In this talk we prove some properties for two general integral operators on the classes $\mathcal{M}(\beta_i)$ and $\mathcal{N}(\beta_i)$.

Let $\mathcal{M}(\beta)$ be the subclass of $\mathcal{A}$, consisting of the functions $f(z)$, which satisfy the inequality $\operatorname{Re}\left\{\frac{zf'(z)}{f(z)}\right\}<\beta, z\in\mathbf{U}$ for some $\beta>1$, and let $\mathcal{N}(\beta)$ be the subclass of $\mathcal{A}$, consisting of functions $f(z)$, which satisfy the inequality: $\operatorname{Re}\left\{\frac{zf''(z)}{f'(z)}+1\right\}<\beta, z\in\mathbf{U}$.

These classes are studied by Uralegaddi, Ganigi, Saragangi, Owa and Srivastava.

Consider the integral operator F_n introduced by D. Breaz and N. Breaz, having the form $F_n(z)=\int_0^z\left(\frac{f_1(t)}{t}\right)^{\alpha_1}\cdot\ldots\cdot\left(\frac{f_n(t)}{t}\right)^{\alpha_n}dt$, where $f_i(z)\in\mathcal{A}$ and $\alpha_i>0$, for all $i\in\{1,...,n\}$ and the integral operator denoted by $F_{\alpha_1,...,\alpha_n}$, that was introduced by D. Breaz, S. Owa and N. Breaz,having the form,$F_{\alpha_1,...,\alpha_n}=\int_0^z[f_1'(t)]^{\alpha_1}\cdot\ldots\cdot[f_n'(t)]^{\alpha_n}dt$, where $f_i(z)\in\mathcal{A}$ and $\alpha_i>0$, for all $i\in\{1,...,n\}$.

Theorem 1. *Let $\alpha_i>0$, for $i\in\{1,...,n\}$, and β_i the real numbers with the property $\beta_i>1$ and $f_i\in\mathcal{M}(\beta_i)$, for $i\in\{1,...,n\}$. If $\sum_{i=1}^n\alpha_i(\beta_i-1)>0$, then $F_n\in\mathcal{N}\left(1+\sum_{i=1}^n\alpha_i(\beta_i-1)\right)$.*

Theorem 2. *Let $\alpha_i>0$, for $i\in\{1,...,n\}$, $\beta>1$ real number and $f_i\in\mathcal{M}(\beta)$, for $i\in\{1,...,n\}$. If $\sum_{i=1}^n\alpha_i(\beta-1)>0$, then $F_n\in\mathcal{N}(\gamma)$,* where $\gamma=1+(\beta-1)\sum_{i=1}^n\alpha_i$.

Theorem 3. *Let $f_i \in \mathcal{N}(\beta_i)$,$\beta_i > 1$ be real numbers and $\alpha_i > 0$, for all $i \in \{1, ..., n\}$. If $\sum_{i=1}^{n} \alpha_i (\beta_i - 1) > 0$, the integral operator $F_{\alpha_1,...,\alpha_n}$, is in the class $\mathcal{N}\left(1 + \sum_{i=1}^{n} \alpha_i (\beta_i - 1)\right)$.*

Theorem 4. *Let $f_i \in \mathcal{N}(\beta)$, $\beta > 1$ be real number and $\alpha_i > 0$, for all $i \in \{1, ..., n\}$. If $\sum_{i=1}^{n} \alpha_i (\beta - 1) > 0$, the integral operator $F_{\alpha_1,...,\alpha_n}$ is in the class $\mathcal{N}\left(1 + \sum_{i=1}^{n} \alpha_i (\beta - 1)\right)$.*

Spline models in fitting and smoothing problems

Nicoleta Breaz

The regression models constitute one of the most important fields in statistics, importance won by their wide applicability in many practical problems. Our talk is focused on the applications of spline functions, in regression models. We can state that from a statistical point of view, the spline functions are a link between the parametric regression and the nonparametric one, being estimating tools in both situations.

We begin with regression models based on the least squares fitting with a polynomial spline function. These models are justified by an advantage on the polynomial fitting, from the flexibility to data point of view. Thus, the estimator comes closer to data as much as the assumed parametric forme allows. The knots and the degree of the spline function can be selected by a cross validation procedure. In order to prove the efficiency of a least squares spline model in spite of a polynomial model, we construct a test function for a Matlab environment implemented algorithm and use the data simulated from this function.

On the other side, we deal with regression models based on a smoothing technique of noisy data, the resulted model being the smoothing spline model in a nonparametric framework. Here the class of function in which one is looking for the estimator, do not assume a certain parametric form but just some smoothness properties of the function. These spline models are claimed by the assumption that the data are suspected of containing errors. Thus, one will use an estimator which even if it has great flexibility will however smooth too perturbated data, assuming some smoothness. The trade-off between the smoothness and flexibility is controlled by a smoothing parameter that can be selected by a cross validaton procedure. In a numerical exeperiment, we pointing out, on the simulated noisy data from a proposed test function, the advantage of the CV selected smoothing parameter based estimator on randomly selected smoothing parameter based estimators.

Finally, we make a comparative study between the least squares fitting spline estimator and the smoothing spline estimator.

From tffr groups to mixed groups

Simion Breaz

In the introduction of [Arnold, D.M.: Finite Rank Torsion Free Abelian Groups and Rings, Lecture Notes in Mathematics, 931, Springer-Verlag, (1982)] the author say "a secondary goal of these notes is to survey ... with an eye toward eventual applications to mixed groups of finite torsion-free rank ...". Following this point of view, we have the natural problem: "Find classes $\mathcal{C}$ of mixed groups (of finite torsion free rank) such that properties of tffr groups can be extended to these classes".

During the time some results concerning self-small groups and quotient divisible groups suggest us that good candidates for $\mathcal{C}$ are $\mathcal{S}$, the class of self-small groups of finite torsion-free rank, and $\mathcal{QD}$, the class of (mixed) quotient divisible groups.

In this talk I present some similarities and differences between results and proofs about tffr groups and groups which belongs in $\mathcal{S}$, respectively in $\mathcal{QD}$. Results about mixed groups are obtained in joint works with U. Albrecht, C. Vinsonhaler and W. Wickless.

Constructive reverse mathematics

Douglas S. Bridges

AMS Classification: 03F60 Constructive and recursive analysis; 03F55 Intuitionistic mathematics.
Keywords: constructive, reverse mathematics, uniform continuity, Specker's theorem

Reverse mathematics is a program in mathematical logic that seeks to determine which axioms are required to prove theorems of mathematics. Constructive reverse mathematics is a relatively new programme of reverse mathematics carried out in Bishop-style constructive math (BISH)—that is, using intuitionistic logic and, where necessary, constructive ZF set theory; see [6]. There are two primary foci of constructive reverse mathematics:

- first, investigating which constructive principles are necessary to prove a given constructive theorem;
- secondly, investigating what non-constructive principles are necessary additions to BISH in order to prove a given non-constructive theorem.

I will present recent work on constructive reverse mathematics, carried out with Josef Berger, Hannes Diener, and Marian Baroni. The main theme of the talk is the connection between the antithesis of Specker's theorem, various continuity properties, versions of the fan theorem, and Ishihara's principle **BD**-$\mathbb{N}$.

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Variational methods in Dirichlet Spaces

Gheorghe Bucur

We give some continuity properties of the Stampackia Projection operator in the frame of Dirichlet Spaces and then we study the continuity of multiple contractions in Dirichlet functional spaces.

On the Helly and Bray theorem

Ileana Bucur

The following assertion of Helly and Bray is well known: If $g_n : [a, b] \to \mathbb{R}, n \in \mathbb{N}$ are functions with uniformity bounded variation on $[a, b]$ and the sequence $(g_n)_n$ is pointwise contingent to a function $g : [a.b] \to \mathbb{R}$ then we have

$$\lim_{n\to\infty} \int_a^b f dg_n = \int_a^b f dg,$$

for any continuous function $f : [a, b] \to \mathbb{R}$

We give a necessary and sufficient condition for the above equality.

Relations among modular points on elliptic curves

Alexandru Buium

Given a correspondence between a modular curve and an elliptic curve A we study the group of relations among the CM points of A. In particular we prove that the intersection of any finite rank subgroup of A with the set of CM points of A is finite. We also prove a local version of this global result with an effective bound, and valid for certain infinite rank subgroups. We deduce the local result from a "reciprocity" theorem for CL (canonical lift) points on A. Furthermore we prove similar global and local results for intersections between subgroups of A and (non CM) isogeny classes in A. Finally we prove Shimura curve analogues and, in some cases, higher dimensional versions of these results. The local results are proved via an arithmetic analogue of the theory of ordinary differential equations.

Subordination and superordination-preserving convex integral operators

Teodor Bulboaca

Let $H(\mathrm{U})$ be the space of analytic functions in the unit disk U. For the integral operator $A^h_{\alpha,\beta,\gamma} : \mathcal{K} \to H(\mathrm{U})$, with $\mathcal{K} \subset H(\mathrm{U})$, defined by

$$A^h_{\alpha,\beta,\gamma}[f](z) = \left[\frac{\beta+\gamma}{z^\gamma}\int_0^z f^\alpha(t)h(t)t^{\delta-1}\,\mathrm{d}\,t\right]^{1/\beta},$$

where $\alpha, \beta, \gamma, \delta \in \mathbb{C}$ and $h \in H(\mathrm{U})$, we will determine sufficient conditions on g_1, g_2, α, β and γ such that

$$zh(z)\left[\frac{g_1(z)}{z}\right]^\alpha \prec zh(z)\left[\frac{f(z)}{z}\right]^\alpha \prec zh(z)\left[\frac{g_2(z)}{z}\right]^\alpha$$

implies

$$z\left[\frac{A^h_{\alpha,\beta,\gamma}[g_1](z)}{z}\right]^\beta \prec z\left[\frac{A^h_{\alpha,\beta,\gamma}[f](z)}{z}\right]^\beta \prec z\left[\frac{A^h_{\alpha,\beta,\gamma}[g_2](z)}{z}\right]^\beta.$$

The symbol "$\prec$" stands for subordination, and we will call such a kind of result a *sandwich-type theorem*.

In addition, $z\left[\frac{A^h_{\alpha,\beta,\gamma}[g_1](z)}{z}\right]^\beta$ will be the *largest* function and $z\left[\frac{A^h_{\alpha,\beta,\gamma}[g_2](z)}{z}\right]^\beta$ the *smallest* function so that the left-hand side, respectively the right-hand side of the above implication hold, for all f functions satisfying the differential subordination, respectively the differential superordination of the assumption.

We will give some particular cases of the main result obtained for appropriate choices of the h, that also generalize classic results of the theory of differential subordination and superordination.

The concept of differential superordination was introduced by S. S. Miller and P. T. Mocanu in 2003 like a dual problem of differential subordination.

Self-similar dilatation structures and automata

Marius Buliga

A dilatation structure is a concept in-between a group and a differential structure. We show that on the boundary of the dyadic tree, any self-similar dilatation structure induces a web of interacting automata. If time is available, applications to linearization of self-similar groups will be given.

Quantum doubles for a certain class of rank two pointed Hopf algebras

Sebastian Burciu

A certain class of rank two pointed Hopf algebras is considered. The simple modules of their Drinfel'd doubles is described using Radford's method for graded Hopf algebras with commutative and cocommutative bottom term. The socle of the tensor product of two such simple modules is computed. The formula for the socle is similar to the one obtained by H.X. Chen for doubles of Taft algebras.

On some (topological) invariants for closed manifolds with large fundamental groups

Dan Burghelea

Based on a joint work with **S.Haller**.
For a finitely presentaed group Γ the set of its rank n complex representations $Rep(\Gamma; n)$, can be regarded as an affine complex algebraic variety. It is possible (and rewarding) to use this variety as the locus for topological invariants of closed manifolds with fundamental group Γ. One such invariant is a rational function on $Rep(\Gamma; n)$. It can be defined using topology (Milnor Turaev torsion) or Riemannnian geometry (complex Ray Singer torsion) or smooth dynamics (dynamical torsion).All very different definitions lead to the same invariant which seems to be as fundamental for the topology of a closed manifold as the characteristic polynomial for a linear transformation of a f.d vector space. This invariant generalizes the Alexander polynomial of a knot, Lefschetz zeta function of a diffeomerphism and the Ruelle dynamical zeta function.

A vortex model of a helicopter rotor

Valentin Butoescu

Keywords: unsteady flows, vortex methods, helicopter rotor.

A vortex model of a helicopter rotor is presented. Each blade of the rotor has three degrees of freedom: flapping, lagging and feathering. The motions after each degree of freedom are also known for all blades. The blade is modelled as a thin vortex surface. The wakes are free fluid surfaces. A system of five equations are obtained: the first one is the integral equation of the lifting surface (rotor), the next three describe the wakes motion, and the last one relates the vortex strength on the wakes and the variation of vorticity on the rotor. A numerical solution of this system is presented. To avoid the singularities that can occur due to the complexity of vortex system, a Rankine model of the vortex core was adopted. A Mathcad worksheet containing the method has been written. The numerical results obtained with the worksheet are good.

The method of calculation the motion of the free vortex system of the wakes, the development of the vortex cores in time and a new method to approximate the aerodynamic influence of remoted wake regions.

On perfect compactifications of T_0-spaces

Laurenţiu Calmutchi

All spaces are considered to be T_0-spaces.

A set $A \subseteq X$ is r-connected in X if there are not two open subsets U, V of X such that $U \bigcap A \neq \emptyset$, $V \bigcap A \neq \emptyset$, $U \bigcap V = \emptyset$ and $A \subseteq U \bigcup V$.

A continuous mapping $f : X \to Y$ is r-monotone if the set $f^{-1}(y)$ is r-connected in X for any $y \in Y$. Every compact continuous mapping $f : X \to Y$ is a composition of an r-monotone mapping $\varphi : X \to Z$ and a zero-dimentional mapping $\psi : Z \to Y$.

A generalized extension or a g-extension of a space X is a pair (Y, f), where $f : X \to Y$ is a continuous mapping and the set $f(X)$ is dense in Y. If f is an embedding, then Y is an extension of X and it is assumed that $X = f(X)$ and $f(x) = x$ for any $x \in X$.

Let (Y, f) be a g-extension of a space X. If U is an open subset of $f(X)$, then $< U >= Y \setminus cl_Y(X \setminus U)$. Let $Fr_Y A = cl_Y A \bigcap cl_Y(Y \setminus A)$ for any $A \subseteq Y$. The g-extension (Y, f) is perfect if $cl_Y Fr_{f(X)} U = Fr_Y < U >$ for any open subset U of $f(X)$.

Let (Y, f) and (Z, g) be two g-extension of a space X. The g-extension (Y, f) is projective larger that (Z, g), and we denote this by $(Z, g) \leq (Y, f)$, or $(Y, f) \geq (Z, g)$, if there exists a continuous mapping $\varphi : Y \to Z$ such that $g = \varphi \circ f$. If φ is a homeomorphic mapping onto Z and $g = \varphi \circ f$, then we say that the g-extensions (Y, f) and (Z, g) are equal and we write $(Y, f) = (Z, g)$.

We say that a g-extension of a space X is condensed if the set $f^{-1}(y)$ is compact provided the set $\{y\}$ is not closed in Y. Every extension is condensed.

For every space X there exists the Wallman compactification $(\omega X, \omega_X)$, where $\omega_X : X \to \omega X$ is an embedding and every point $\xi \in \omega X \setminus \omega_X(X)$ is associated with a maximal ultrafilter of closed subsets of X for which $\bigcap \xi = \emptyset$. We considered that $X \subseteq \omega X$.

A g-extension (Y, f) of a space X is called an $\omega\alpha$-g-compactification if (Y, f) is a condensed extension and there exists a continuous closed mapping $\varphi : \omega X \to Y$ such that $f = \varphi | X$.

Theorem 1. *Let (Y, f) be an $\omega\alpha$-g-compactification of a space X, the mapping $f : X \to Y$ is r-monotone and $\varphi : \omega X \to Y$ be a closed mapping for which $f = \varphi | X$. The following assertion are equivalent:*

1. *The g-extension (Y, f) is perfect.*
2. *The mapping φ is r-monotone.*

If (Y, f) is a perfect completely regular compactification of a space X, then the natural mapping $\psi : \beta X \to Y$ of the Stone Čech compactification βX onto Y is monotone. Theorem 1 for completely regular spaces it was proved by E. G. Skljarenco and for T_1-spaces is due to P. Osmatescu.

Theorem 2. *Let (Y, f) be a $\omega\alpha$-g-compactification of a space X. Then there exists a minimal perfect $\omega\alpha$-g-compactification (Z, g) of X such that $(Y, f) \leq (Z, g)$.*

If X is a semi-compact T_0-space, then the Freudenthal-Morita compactification γX of X is the minimal perfect $\omega\alpha$-compactification of X. In particular, if X is a locally compact T_0-space, then the one-point Alexandroff compactification aX of X is perfect.

The halting problem and experimental mathematics

Cristian S. Calude

Most mathematical problems can - directly or indirectly - be reduced to finitely refutable problems which can be solved by checking the halting status of appropriately constructed small-size programs.

Using Chaitin's register machine language, one needs to decide the non-halting status of a program of 3,484 bits to prove Goldbach's conjecture; to solve the Riemann hypothesis one has to decide the non-halting status of a 7,780-bit program.

However, the halting problem is undecidable. This is the major obstacle in automatically solving finitely refutable problems.

Fortunately, we show that most programs which do not stop quickly, roughly in time less than 4^{n+3}, where n is the size in bits of the program, never halt. The reason is that programs that run a long time before halting, stop at an algorithmically non-random time. As a consequence, the undecidability of the halting problem can be located in a set of measure zero. This result suggests a possible probabilistic approach.

Challenge: Find efficient probabilistic methods for solving the halting problem for small-size programs and use them to probabilistically solve finitely refutable problems.

On the stochastic modelling of interacting populations

Vicenzo Capasso

Particular attention is being paid these days to the mathematical modelling of the social behaviour of individuals in a biological population, for different reasons; on one hand there is an intrinsic interest in population dynamics of herds, on the other hand agent based models are being used in complex optimization problems (ACO's, i.e. Ant Colony Optimization). Further decentralized/parallel computing is exploiting the capabilities of discretization of nonlinear reaction-diffusion systems by means of stochastic interacting particle systems.

Among other interesting features, these systems lead to self organization phenomena, which exhibit interesting spatial patterns.

As a working example, an interacting particle system modelling the social behaviour of ants is proposed here, based on a system of stochastic differential equations, driven by social aggregating/repelling "forces". Specific reference to observed species in nature will be made.

Current interest concerns how properties on the macroscopic level depend on interactions at the microscopic level. Among the scopes of the seminar, a relevant one is to show how to bridge different scales at which biological processes evolve; in particular suitable "laws of large numbers" are shown to imply convergence of the evolution equations for empirical spatial distributions of interacting individuals to nonlinear reaction-diffusion equations for a so called mean field, as the total number of individuals becomes sufficiently large.

In order to support a rigorous derivation of the asymptotic nonlinear integrodifferential equation, problems of existence of a weak/entropic solution will be analyzed.

Further the existence of a nontrivial invariant probability measure is analyzed for the stochastic system of interacting particles.

A numerical method of $2D$ flow around a flapping airfoil

Adrian Carabineanu

Based on joint work with **A. Dumitrache**.

It is considered a rigid, two-dimensional airfoil undergoing prescribed, periodic, flapping motions at zero angle of attack in an incompressible, viscous flow with constant horizontal (chord-wise) free stream velocity. The flapping consists of heaving (vertical), lagging (horizontal), and pitching (angular) motions and the airfoil is in cruise conditions: level flight with constant average velocity.

For computational simplicity, the problem is formulated in a non-inertial reference frame fixed to the airfoil such that the origin coincides with the pitch-axis, the x-axis is fixed parallel to the chord, and the y-axis is perpendicular to both the x-axis and the span.

A numerical study for two-dimensional flow around an airfoil undergoing prescribed oscillatory motion in a viscous flow is developed. The model is used to examine the flow characteristics and power coefficients of a symmetric airfoil heaving sinusoidally over a range of frequencies and amplitudes. Both periodic and aperiodic solutions are found. Additionally, some flows are asymmetric in that the up-stroke is not a mirror image of the down-stroke. For a given Strouhal number, the maximum efficiency occurs at an intermediate heaving amplitude. Above the optimum frequency, the efficiency decreases similarly to inviscid theory.

The computational model is used as the basis for developing a reduced-order model for active control of flapping wing. Using POD, sets of orthogonal basis functions are generated for simulating flows at the various heaving and pitching parameters. With POD, most of the energy in the flow is concentrated in just a few basis functions. The suitability of this approach for controlling a flapping wing over a broad range of parameters is analyzed.

Large deviations estimates of the crossing probability for small time and applications to Monte Carlo Methods in finance

Lucia Caramellino

The talk deals with the asymptotic behavior of the bridge of a process conditioned to stay in n fixed points at n fixed past instants and in particular, functional large deviation results are stated for small time. Several examples are considered, e.g. diffusion processes and fractional Brownian motion. The asymptotic behavior of the exit probability is then studied to be used for the numerical computation, via Monte Carlo, of the time at which the process hits some boundary. As an application, numerical methods for Finance can be set up, e.g. for the pricing of barrier options or also bonds and other contingent claims subject to default risk as well as interest rate risk.

Theoretical aspects regarding the fluid dynamics in the catalyst layer of a Pem fuel cell

Elena Carcadea

Based on joint work with **Horia Ene**, **Bogdan Nicolescu**, and **Ioan Ştefănescu**.

Fuel cells generate electricity from a simple electrochemical reaction in which oxygen and hydrogen combine to form water. The performance of a proton exchange membrane (PEM) fuel

cell is limited by electrochemical kinetics, proton conductance through the polymeric material, and mass transport limitations. The focus of this talk is to develop a three dimensional steady state single phase mathematical model that describes the fluid dynamics in the catalyst layer of a PEM fuel cell.

One of the main sources of energy loss in a PEM fuel cell is the slow kinetic of the oxygen reduction reaction (ORR) at the catalyst particles in the cathode catalyst layer. The description of the fuel cell cathode, especially the modeling of the mass and charge transfer limitations within the active cathode layer, is therefore of particular importance. The model use a CFD software (Fluent) as a basic tool in order to incorporate all the phenomena that takes place inside a fuel cell and simulate them.

The fuel cell operation is governed by the conservation of mass, momentum, species and charge equations. The fluid flow, reactant species transport and the electrical potential in the catalyst layer fuel cell can be expressed by following conservative equations:

$$\nabla \cdot (\varepsilon \rho \overrightarrow{u}) = S_m \tag{2}$$

$$\nabla \cdot (\varepsilon \rho \overrightarrow{u}\,\overrightarrow{u}) = -\varepsilon \nabla p + \nabla \cdot (\varepsilon \mu \cdot \nabla \overrightarrow{u}) + S_u \tag{3}$$

$$\nabla \cdot (\varepsilon \overrightarrow{u} Y_i) = \nabla (D_i^{eff} \cdot \nabla Y_i) + S_i \tag{4}$$

$$\nabla \cdot (\sigma_e^{eff} \cdot \nabla \phi_e) + S_\phi = 0 \tag{5}$$

Where $\overrightarrow{u}$, Y_i and ϕ_s denote the velocity vector of the fluid flow, the mass fraction of the i^{th} species and the electric potential, respectively, within the fuel cell. The source term from the conservative equations are given by the formulas:

Equation / Source term
Mass conservation equation $S_m = \begin{cases} -M_{H_2O} \cdot \alpha - M_{H_2} \cdot \frac{j_{anode}}{2F} & \text{at the anode} \\ M_{H_2O} \cdot \frac{j_{chatode}}{2F} + M_{H_2O} \cdot \alpha - M_{O_2} \frac{j_{cathode}}{4F} & \text{at the cathode} \end{cases}$
Species conservation equation - for H_2 $S_{H_2} = \frac{M_{H_2}}{2F} \cdot j_{anode}$
Species conservation equation - for H_2O $S_{H_2O} = \begin{cases} -M_{H_2O} \cdot \alpha & \text{in the anode catalyst layer} \\ M_{H_2O} \cdot \alpha + \frac{M_{H_2O}}{2F} \cdot j_{chatode} & \text{in the cathode catalyst layer} \end{cases}$
Species conservation equation - for O_2 $S_{O_2} = -\frac{M_{O_2}}{4F} \cdot j_{chatode}$
Potential conservation equation $S_\phi = -j$

A three dimensional model of the PEM fuel cell was developed. Important transport phenomena's was dealt with in detail, and a particularly feature of this 3D model is the full resolution of the catalyst layer as a homogeneous porous media. This comprehensive model allows us to couple the potential distribution with the concentration distribution of reactants in detail, and allow us to solve the three dimensional activity in the catalyst layer.

M-hypoelliptic pseudo-differential operators on $L^p(R^p)$

Viorel Cătană

Following Wong's point of view we construct the minimal and maximal extension in $L^p(\mathbb{R}^n)$, $1 < p < \infty$ for M-hypoelliptic pseudo-differential operators which have been introduced and studied by Garello and Morando. We give some facts about the domain of minimal and maximal extensions of M-hypoelliptic pseudo-differential operators. For M-hypoelliptic pseudo-differential operators with constant coefficients, the spectum and essential spectrum are computed.

Definition. For any $\ell, m \in \mathbb{R}, \ell \leq m$, we shall write $\sigma \in HM^{m,\ell}_{\rho,\Lambda}$, if $\sigma \in C^\infty(\mathbb{R}^n \times \mathbb{R}^n)$ and the following conditions are satisfied:

1. There exist positive constants C_1, C_2 and R such that

$$C_1 \Lambda(\xi)^\ell \leq |\sigma(x,\xi)| \leq C_2 \Lambda(\xi)^m, x \in \mathbb{R}^n, |\xi| \geq R. \tag{1}$$

2. There exist a positive constant R and for all multi-indices $\alpha, \beta \in \mathbb{Z}^n_+, \gamma \in \{0,1\}^n$ we can find a positive constant $C_{\alpha,\beta,\gamma}$ depending on α, β and γ only, such that

$$\left|\xi^\gamma \partial_\xi^{\alpha+\gamma} \partial_x^\beta (x,\xi)\right| \leq C_{\alpha,\beta,\gamma} |\sigma(x,\xi)| \Lambda(\xi)^{-\rho|\alpha|}, x \in \mathbb{R}^n, |\xi| \geq R. \tag{2}$$

A symbol α in $HM^{m,\ell}_{\rho,\Lambda}$ is said to be M-hypoelliptic of order (m,ℓ).

Let $\sigma \in HM^{m,\ell}_{\rho,\Lambda}$. Then we define the pseudo-differential operator T_σ associated to the symbol σ by

$$(T_\sigma \varphi)(x) = (2\pi)^{-n/2} \int_{\mathbb{R}^n} e^{ix\xi} \sigma(x,\xi) \widehat{\varphi}(\xi) d\xi, x \in \mathbb{R}$$

for all φ in the Schwartz space S.

We say that T_σ is M-elliptic if $\sigma \in EM^m_{\rho,\Lambda} = HM^{m,m}_{\rho,\Lambda}$.

Our results are the following :

Theorem 1. Let $\sigma \in HM^{m,\ell}_{\rho,\Lambda}, 0 < \ell \leq m$. Then, there exist positive constants C_1 and C_2 such that

$$C_1 \|u\|_{\ell,p,\Lambda} \leq \|T_\sigma u\|_{0,p,\Lambda} + \|u\|_{0,p,\Lambda} \leq C_2 \|u\|_{m,p,\Lambda}, u \in H^{m,p}_\Lambda. \tag{3}$$

Theorem 2. Let $\sigma \in HM^{m,\ell}_{\rho\Lambda}, m \geq \ell > 0$ be M-hypoelliptic. Then

$$H^{m,p}_\Lambda \subseteq D(T_{\sigma,0}) \subseteq D(T_{\sigma,1}) \subseteq H^{\ell,p}_\Lambda. \tag{4}$$

Theorem 3 Let $\sigma \in HM^{m,\ell}_{\rho,\Lambda}, m \geq \ell > 0$, be M hypoelliptic. Suppose that σ is independent of x in $\mathbb{R}^n$. If λ is a complex number such that

$$\sigma(\xi) \neq \lambda, \xi \in \mathbb{R}^n,$$

then $\lambda \in \rho(T_{\sigma,0})$.

Theorem 4. Let $\sigma \in HM^{m,\ell}_{\rho,\Lambda}, m \geq \ell > 0$ be such that σ is independent of x in $\mathbb{R}^n$ and let λ be a complex number. If σ is not bounded away from λ for $\xi \in \mathbb{R}$, then λ is in the essential spectrum $\sum_e(T_{\sigma,0})$ of $T_{\sigma,0}$.

Theorem 5. Let $\sigma \in HM^{m,\ell}_{\rho,\Lambda}, m \geq \ell > 0$ be M-hypoelliptic and such that σ is independent of x in $\mathbb{R}^n$. Then

$$\sum\nolimits_e (T_{\sigma,0}) = \sum (T_{\sigma,0}) = \{\sigma(\xi) : \xi \in \mathbb{R}^n\}.$$

Remark. If $m = \ell$ we find from Thorem 1-5 the similar results obtained by Wong in [2].

Example. The heat operator

The symbol of the heat operator $\sigma(D) = \dfrac{\partial}{\partial x_1} - \dfrac{\partial^2}{\partial x_2^2} - \cdots - \dfrac{\partial^2}{\partial x_{n+1}^2}, n \geq 1, n \in \mathbb{N}$ is given by

$$\sigma(\xi_1, \xi_2, \ldots, \xi_{n+1}) = i\xi_1 + \xi_2^2 + \cdots + \xi_{n+1}^2, \xi_1, \xi_2, \ldots, \xi_{n+1} \in \mathbb{R}.$$

Then it is easy to see that σ is an M-hypoelliptic symbol in $HM_{1/4,\Lambda}^{1/2,1/2n}$ where

$$\Lambda(\xi_1, \xi_2, \ldots, \xi_{n+1}) = 1 + \xi_1^2 + \left(\xi_2^2 + \cdots + \xi_{n+1}^2\right)^2, \xi_1, \xi_2, \ldots, \xi_{n+1} \in \mathbb{R}.$$

By Theorem 2, it follows that

$$H_\Lambda^{1/2,p} \subseteq \mathcal{D}(T_{\sigma,0}) = \mathcal{D}(T_{\sigma,1}) \subseteq H_\Lambda^{1/2n,p} \text{ for } 1 < p < \infty.$$

By Theorem 5, the spectrum and the essential spectrum of the minimal and hence maximal extension of the heat operator on $L^p(\mathbb{R}^{n+1}), 1 < p < \infty$ are both equal to the set $\left\{i\xi_1 + \xi_2^2 + \cdots + \xi_{n+1}^2 : \xi_1, \xi_2, \ldots, \xi_{n+1} \in \mathbb{R}\right\}$, which is the right half plane.

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On modelling plastic anisotropy and strength differential effects in hexagonal metals

Oana Cazacu

Hexagonal closed packed (hcp) metals and alloys are important to nearly every modern industry, ranging from biomedical applications, such as Ti implants and NiTi superelastic catheter guides; to new Sn-based lead-free solders under exploration by the microelectronics industry; to Ti (or TiAl) alloy jet engine compressor components. These advanced materials have excellent properties for their target applications (e.g. Ti implants exhibit biocompatibility, light weight, high strength, and low stiffness); however, their application is still limited because of problems associated with poor formability and consequently high manufacturing costs. Some are brittle, like Be and many TiAl-based alloys, exhibit little room temperature plasticity at all; many others are ductile, yet have poor cold-forming characteristics, such as Mg and Ti alloys. All of these hcp metals display strong anisotropy and/or asymmetries in strength and hardening behavior, which is the result of the operation and interaction between crystallographic slip and deformation twinning. Although the significant contribution to evolution of deformation textures from lattice orientations of the twinned regions has been revealed clearly in systematic experimental investigations (e.g. Salem et al, 2003; Gray, 2004), incorporation of deformation twinning as an additional mode of plastic deformation in the crystal plasticity framework has been an outstanding problem. Likewise, due to the lack of adequate macroscopic criteria for hcp materials, hcp sheet forming FEM simulations are still performed using classic anisotropic formulations for cubic metals such as Hill (1948) (see Takuda et al, 1999; Kuwabara, 2000).

In this talk, recent macroscopic level anisotropic formulations for hcp materials are presented. The anisotropy coefficients as well as the size of the elastic domain are considered to be functions of the accumulated plastic strain. The specific expressions for the evolution laws are determined based on experimental data and numerical test results performed with a self-consistent viscoplastic model together with a macroscopic scale interpolation technique. Application to the simulation of the three-dimensional deformation of a pure zirconium beam subjected to four-point bend tests along different directions is presented. Comparison between predicted and measured macroscopic strain fields and beam sections shows that the proposed model describes very well the difference in response between the tensile and compressive fibers and the shift of the neutral axis. Furthermore, an overstress approach is used to account for rate effects. The very good agreement between the simulated and experimental post-test geometries of the Taylor impact specimens in terms of major and minor side profiles and impact-interface footprints shows the ability of the model to describe the evolution of anisotropy as a function of the strain rate.

On the travelling wave motion

Mircea Dimitrie Cazacu

Considering the Gerstner potential expression for the travelling wave, one presents the determination mode of the second constant of this potential in function of the wave high; the deduction of the Bernoulli relation, adequate to the non permanent motion of the wave and its time function, in the aim to calculate the local pressure variation at the travelling wave propagation, as well as the both velocity components for the wave tide calculation, due to wave incidence on the inclined beaches. Also, transforming this velocity potential for the relative motion case of the travelling wave on a right and then an inclined bottom, with respect to a mobile trihedron, which moves with the wave propagation velocity c, respectively raises with the velocity c tg , one obtains a series of interesting relations. Thus, one shows the harmonicity property of the relative velocities potential, the availability of the Riemann-Cauchy relations in the case of a constant angle and one obtains the Bernoulli's relation, necessary for the pressure distributions calculation, as well as the tendency of wave free surface modification in his propagation on the inclined bottom.

The Mukai pairing on Hochschild homology

Andrei Căldăraru

Given a Calabi-Yau three-fold X, string theory constructs two so-called topological twists, the A-model and the B-model. A piece of the mathematical incarnation of the A-model is the singular cohomology ring of X (or its quantum deformation). The corresponding piece in the B-model is encoded by the Hochschild cohomology ring of X. Physics predicts both sets of data are Frobenius algebras, i.e., they are endowed with a non-degenerate pairing. In the A-model, this is given by the Poincare pairing on cohomology. In my talk I shall discuss the construction of the corresponding pairing on Hochschild (co)homology. I shall also discuss several important properties of this pairing, including the Cardy condition from open-closed topological string theory.

No prior knowledge of Hochschild theory or physics is required.

A geometric method for Computing Heat Kernels for Hermite-type operators

Ovidiu Călin

Differential geometry and variational techniques have useful and important applications in PDEs and quantum mechanics. We shall present a geometric approach which deals with the construction of the kernels of Hermite-type operators $\partial_t - \Delta_x + U(x)$ with linear, quadratic and quartic potential case.

In general, given a second order differential operator, one may associate a geometry with it. For instance given the usual Laplacian on R^3 given by $\Delta = \partial_x^2 + \partial_y^2 + \partial_z^2$, the associated geometry is given by the 3-dimensional Euclidean space. The fundamental solution of Δ is given by the inverse of the Euclidean distance in the associated space, i.e., $C/\sqrt{x^2+y^2+z^2}$. This idea can be carried out in general. Using Hamiltoniann formalism one may generalize it to several operators.

This talk is based on chapter 10 of the book "Geometric Mechanics on Riemannian Manifolds" by Ovidiu Calin and Der-Chen Chang, publiched by Birkhauser, 2005.

Heisenberg geometry

Ovidiu Călin

A few important discoveries in the field of Thermodynamics in the 1800's made the first step towards SubRiemannian geometry. Carnot discovered the principle of an engine in 1824 involving two isothermes and two adiabatic processes, Jule studied adiabatic processes and Clausius formulated the existence of the entropy in the second law of Thermodynamics in 1854. In 1909 Caratheodory clarified the relationship between the connectivity of two states by adiabatic processes and non-integrability of a distribution, which is given by the one-form of work. Chow proved the general global connectivity in 1934, and the same hypothesis was used by Hormander in 1974 to prove the hypoellipticity of a sum of squares of vector fields operator.

The study of the geometry of the Heisenberg group, which is the prototype of the subRiemannian geometry was started by Gaveau in 1975. Since 1990's the understanding of the geometry of this group led Beals, Gaveau and Greiner to characterize the fundamental solutions for heat sub-elliptic operators and Heisenberg sub-Laplacians. Meantime many examples have been considered. Some of them behave similarly with the Heisenberg group but others are very different. However, a unitary and general theory of these sub-Riemannian manifolds is still missing at the moment.

The talk deals with the study of subRiemannian manifolds which have the Heisenberg principle built in. This brings the hope that Heisenberg manifolds (step 2 subRiemannain manifolds) will play a role in Quantum Mechanics in the future, similar to the role played by the Riemannian manifolds in Classical Mechanics.

The subRiemannian geodesics behavior, which is very different than in the Riemannian case, plays an important role in finding heat kernels and propagators for the Shrodinger's equation, as well as in finding fundamental solutions for subelliptic operators using a very ingenious geometric method involving complex modified action.

One of the novelties of this treatment is to introduce the complex Hamiltonian mechanics techniques and apply them in describing the fundamental solutions and heat propagators. The fact that the propagator depends on each subRiemannian distance is an expected fact in Quantum Mechanics. The fact that each path has a contribution to the propagator led Feynman to introduce the path integrals. In this work we can manage without them, just working in the proper geometric framework.

The interested reader can find more details in the book "Heisenberg group and its generalizations", by O. Calin, D.C. Chen and P. Greiner, published by AMS/IP, 2007.

On a class of abstract implicit evolution variational inequalities

Anca Căpăţână

This paper deals with the mathematical analysis of a class of abstract implicit evolution variational inequalities which is a generalization of some parabolic variational inequalities of second kind and of implicit inequalities related to quasistatic unilateral contact problem with friction in linear elasticity.

The problem is formulateted as the following evolution system of coupled variational inequalities :

$$(P)\begin{cases} \text{find } u \in W^{1,2}(0,T;V) \text{ such that} \\ u(0) = u_0\,, \\ u(t) \in K(f(t)) \quad , \quad \forall t \in [0,T], \\ a(u(t), v - \dot{u}(t)) + j(f(t), u(t), v) - j(f(t), u(t), \dot{u}(t)) \\ \quad \geq b(u(t), v - \dot{u}(t))\,, \forall v \in V \text{ a.e. on } [0,T], \\ b(u(t), z - u(t)) \geq 0\,, \forall z \in K \quad , \forall t \in [0,T]. \end{cases}$$

In order to show that there exists a solution of problem (P), we give an incremental formulation obtained by using an implicit scheme for time discretization.

Foundations of the rewriting in an algebra

Virgil Emil Căzănescu

This paper includes two main ideas. Rewriting in an algebra in the first one. The second one, boolean rewriting, can be found in many papers but we were never able to find a clear comparison with the classic one.

We prefer rewriting in an algebra to term rewriting. This is our way to give a unique theory of rewriting. If the algebra is free then we get the term rewriting. If the algebra is a certaine quotient of a free algebra then we get rewriting modulo equations.

Rewriting is said to be boolean when the condition of each conditional equation is of boolean sort(in the free algebra it is a boolean term). We prove the classic rewriting is equivalent to boolean rewriting in a specific algebra, therefore, boolean rewriting is more general than the classic one.

Direct products of L-BLD homeomorphisms on direct product spaces

Serafima Cerchez

In this paper I develop the theory of L-BLD mappings for direct products of homeomorphisms on the direct product spaces $\mathbb{R}^k \times \mathbb{R}^m$. We use the distance between $z = (x, y)$ and $z_0 = (x_0, y_0)$ points in $\mathbb{R}^k \times \mathbb{R}^m$:

$$d(z, z_0) = \max(d(x, x_0), d(y, y_0)),$$

where $d(x, x_0)$ and $d(y, y_0)$ are the Euclidean distances.

The maximum principle for constrained differential inclusions with delay

Aurelian Cernea

Consider the following Mayer optimal control problem with dynamics given by a non convex differential-difference inclusion, whose trajectories are constrained to a closed set

$$\text{minimize} \quad g(x(T)) \tag{1}$$

subject to

$$\begin{aligned} &x'(t) \in F(t, x(t), x(t-\Delta)) \quad a.e.\,([t_0, T]),\\ &x(t) = c(t) \quad t \in [t_0 - \Delta, t_0),\\ &x(t_0) = x_0, \quad x(T) \in K_1,\\ &x(t) \in K \quad \forall t \in [t_0, T], \end{aligned} \tag{2}$$

where $K, K_1 \subset \mathbf{R}^n$ are given subsets, $F(.,.,.) : \mathbf{R} \times \mathbf{R}^n \times \mathbf{R}^n \to \mathcal{P}(\mathbf{R}^n)$ is a given set-valued map, $g(.) : \mathbf{R}^n \to \mathbf{R}$ is a given function, $\Delta \in (0, T - t_0)$ and $c(.) : [t_0 - \Delta, t_0) \to \mathbf{R}^n$ is a given essentially bounded function.

Necessary optimality conditions in the form of the maximum principle are obtained for problem (1)-(2) by using convex linearizations of differential inclusions and convex linearizations of constraints along optimal trajectories. Our result extends the existing results in the literature for differential inclusions without delay (e.g., [1]) and the results for differential inclusions with delay but without constraints (e.g., [2]).

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On some generalizations of comutativity in topological grupoids

Liubomir Chiriac

A topological groupoid is a Hausdorff space equipped with a jointly continuous binary operation.

A topological groupoid $(G, \cdot)$ is called:

- *medial* if $xy \cdot zt = xz \cdot yt$ for all $x, y, z, t \in G$;
- *paramedial* if $xy \cdot zt = ty \cdot zx$ for all $x, y, z, t \in G$;
- *bicommutative* if $xy \cdot zt = tz \cdot yx$ for all $x, y, z, t \in G$.

Theorem 1. *For a bicommutative topological groupoid $(G, \cdot)$ the following assertions are equivalent:*

1) $(G, \cdot)$ is a medial groupoid;

2) $(G, \cdot)$ is a paramedial groupoid.

Theorem 2. *Let $(G, \cdot)$ be a paramedial topological groupoid and e, e_1 and e_2 be elements of G for which:*

1) $ee_1 = e_1$ and $e_2 e = e_2$;

2) the maps $x \to e_1 x$ and $x \to x e_2$ are homeomorphisms of G onto itself;

3) the map $x \to xe$ is surjective.

If there exists on G a binary operation $(\circ)$ such that $(e_1 x) \circ (y e_2) = yx$ then $(G, \circ)$ is a commutative topological semigroup having $e_1 e_2$ as identity.

A topological groupoid $(G, *)$ is called *radical* if the map $s : G \to G$ defined by $s(x) = x * x$ is a homeomorphism.

Theorem 3. *If $(G, \circ)$ is a topological groupoid with unit e and $(G, \cdot)$ is commutative, idempotent topological groupoid and $(x \circ y) \cdot (z \circ t) = (ty) \circ (zx)$, then $(G, \circ)$ is a commutative radical semigroup.*

Deformation of carbon nanotubes by surface van der Waals forces

Veturia Chiroiu

Based on joint work with **Petre P. Teodorescu** and **Ligia Munteanu**.

In this talk, the bending of the zigzag single-walled nanotube of carbon (10,0) is analyzed by coupling a nonlinear continuum theory to an atomistic theory. The effect of surface van der Waals forces on the binding energies is explained by using the soliton theory. The soliton theory is able to generalize the hinge concept towards a solitonic deformation concept. This concept describes the large inelastic deformations of nanotubes, as well as the mechanism nucleation, the rippling configuration and the fracture. When the external bending moment increases, the axial

compression in a (10,0) zigzag nanotube increases too, and when the compressive stress reaches a critical value, the tube will locally buckle at a bending angle of 25.58 degree. If this angle is increasing, a solitonic deformation mechanism is starting and a portion of the nanotube becomes to rotate about a central hinge line. For large distances, the van der Waals force is attractive, but when the separation between the atoms is below the equilibrium distance, it becomes strongly repulsive. Upon complete unloading from angles below 110 degree the nanotube completely recovers. At a very large bending angle of 120 degree, atomic bonds break and the deformation of the nanotube becomes irreversible.

Criteria on the orders of the centralizers of elements in a finite group

Codruţa Chiş

Based on joint work with **M. Chiş**.

In this article, we establish four arithmetical criteria on the orders of the centralizers of elements in a finite group. These can be used to determine the finite groups with a given number of conjugacy classes.

On the autocommutator subgroup of a finite group

Mihai Chiş

Based on joint work with **C. Chiş** and **Gh. Silberberg**.

In this article, we establish that any finite abelian group is isomorphic to the autocommutator subgroup of certain a finite group. We also prove some properties of finite groups with an autocommutator subgroup of a given order.

Finding your perfect matching with eigenvalues

Sebastian Cioaba

Based on joint work with **David Gregory** (Queen's University at Kingston, Canada) and **W. Haemers** (Tilburg University, The Netherlands).

The eigenvalues of a graph G are the eigenvalues λ_i of its adjacency matrix, indexed so that $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \cdots \geq \lambda_n$. The most studied eigenvalues have been the spectral radius λ_1 (in connection with the chromatic number, the independence number and the clique number of the graph), λ_2 (in connection with the expansion properties of the graph) and λ_n (in connection with the chromatic and the independence number of the graph and its maximum cut).

Let G be a k-regular graph of even order. We find a best upper bound on the third largest adjacency eigenvalue λ_3 that is sufficient to guarantee that G has a perfect matching.

Functionally determined properties of topological spaces

Mitrofan M. Choban

All spaces are assumed to be completely regular. On the category of topological spaces consider a covariant function F with the properties:

– for every space X the object $F(X)$ is a topological linear space;

– the set $C^0(X)$ of all bounded continuous functions on X is a linear subspace of $F(X)$.

Let S be the convergent sequence, N be the discrete space of integers, βX be the Stone-Čech compactification of X, $X + Y$ be the discrete sum of the spaces X and Y.

A space X is called a wF-space if for every two distinct sequences $\{x_n, y_n : n \in N\}$ ($x_n \neq y_n$ for any $n \in N$) there exists an infinite subset $M \subset N$ such that $cl_X\{x_n : n \in M\} \cap cl_X\{y_n : n \in M\} = \emptyset$.

Every F-space is a wF-space. A wF-space is a space without non-trivial convergent sequences.

A space X is called an aF-space if there exist two infinite sequences $A = \{a_n : n \in N\}$ and $B = \{b_n : n \in N\}$ such that $A \cap B = \emptyset$ and $cl_X\{a_n : n \in M\} \setminus A = cl_X\{b_n : n \in M\} \setminus B \neq \emptyset$.

Every space is a wF-space or an aF-space.

The main results are connected with the next questions:

1. When $F(X)$ and $F(X)$ are linear homeomorphic?
2. When $F(X)$ and $F(X + S)$ are linear homeomorphic?
3. When $F(X)$ and $F(X + N)$ or $F(X)$ and $F(X + \beta N)$ are linear homeomorphic?
4. Which properties of $F(X)$ determine a given property of X?

Example 1. Let $F(X)$ be the set $C^0(X)$ in the topology of uniform convergence, Y be a closed subset of X and $P(f) = sup\{|f(y)| : y \in Y\}$. The space Y contains a dense complete metrizable subspace if and only if the set $C^0(X) \setminus \{f \in C^0(X) :$ functional P is Gateaux differentiable at $f\}$ is of the first category in $F(X)$.

Example 2. Let $F(X)$ be the space $C_p(X)$ of all continuous functions in the topology of pointwise convergence. Some answers to the Questions 1-4 are proposed. In particular, we have proved that a first countable paracompact space Y is (complete) metrizable if for some (complete) metrizable space X the spaces $C_p(X)$ and $C_p(Y)$ are linear homeomorphic. If X is an aF-space, then the spaces $C_p(X)$ and $C_p(X + S)$ are linear homeomorphic.

There exists an infinite zero-dimensional space X with the next properties: X is an aF-space; X is without non-trivial convergent sequences; if Y is a finite space with m points, then the spaces X and $X + Y$ are homeomorphic if and only if the number m is even; there exists a subspace Z of X homeomorphic to βN such that Z is a retract of X; if Y is a finite space, then the spaces $C_p(X)$, $C_p(X + Y)$, $C_p(X + \beta N)$ and $C_p(X + S)$ are linear homeomorphic.

Some open problems:

P1. Let X be a compact space and the spaces $C_p(X)$ and $C_p(X + \beta N)$ are linear homeomorphic. Is it true that X contains a subspace homeomorphic to βN?

P2. Let X be a compact space and the spaces $p(X)$ and $C_p(X+S)$ are linear homeomorphic. Is it true that X is an aF-space?

P3. Let X be a normal wF-space. Is it true that βX is a wF-space?

P4. Let X be a paracompact (metrizable) locally compact space. Is it true that βX is an F-space or a wF-space?

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Computability in interaction-based systems

Gabriel Ciobanu

We refer to interacting systems with dynamically evolving topology (e.g., the global system represented by the Web), and think in the terms of interactions between distributed and concurrent processes. In order to define the partial recursive functions, we provide a representation of the natural numbers in terms of interactions between these distributed and concurrent processes. Several operations are defined: successor, addition, predecessor, subtraction, multiplication. A form of recursion is given by process replication. The representation is related to Church numerals in λ-calculus, and to the fact that any computable operator (and its operands) can be represented under Church encoding. We also represent the interactive Boolean values true and false together with corresponding conditional processes parametric upon their locations.

New problems appear: interference between several interacting processes, the ephemeral representations and how it is possible to make them persistent. All these aspects build up a particular computability which is not yet explored.

Homogenization of Integral Energies by the unfolding method

Doina Ciorănescu

We consider the periodic homogenization of nonlinear integral energies of the type

$$u \in W^{1,p}(\Omega;\mathbb{R}^m) \mapsto \int_\Omega f\Big(\frac{x}{\varepsilon_h}, \nabla u\Big)\, dx, \tag{1}$$

where Ω is a bounded open subset of $\mathbb{R}^n$ with a Lipschitz boundary, and f is a Carathéodory energy density satisfying

$$\begin{cases} f \,:\, (x,z) \in \mathbb{R}^n \times \mathbb{R}^{nm} \mapsto f(x,z) \in [0,+\infty[, \\ f(\,\cdot\,,z) \text{ Lebesgue measurable, } Y\text{-periodic for every } z \in \mathbb{R}^{nm}, \\ f(x,\,\cdot\,) \text{ continuous a.e. } x \in \mathbb{R}^n. \end{cases}$$

with Y the reference cell $]0,1[^n$. Assume that $f(x,\cdot)$ *is quasiconvex for a.e.* $x \in \mathbb{R}^n$, and furthermore that, for $p \in [1,+\infty[$, $M > 0$, and an Y-periodic $a \in L^1(Y)$, it satisfies the following growth conditions:

$$\begin{cases} f(x,z) \le a(x) + M|z|^p \quad \text{for a.e. } x \in \mathbb{R}^n, \text{ and every } z \in \mathbb{R}^{nm}, \\ |z|^p \le f(x,z) \quad \text{for a.e. } x \in \mathbb{R}^n, \text{ and every } z \in \mathbb{R}^{nm}. \end{cases}$$

It is then known that as $\varepsilon_h \to 0$, one has

$$\inf\left\{\liminf_{h\to+\infty}\int_\Omega f(\frac{x}{\varepsilon_h},\bigtriangledown u)dx : \{u_h\}\subset W^{1,p}(\Omega;\mathbb{R}^m),\ u_h\to u \text{ in } L^p(\Omega;\mathbb{R}^m)\right\}$$

$$=\inf\left\{\limsup_{h\to+\infty}\int_\Omega f(\frac{x}{\varepsilon_h},\bigtriangledown u)dx : \{u_h\}\subset W^{1,p}(\Omega;\mathbb{R}^m),\ u_h\to u \text{ in } L^p(\Omega;\mathbb{R}^m)\right\}$$

$$=\int_\Omega f_{hom}(\nabla u)dx,$$

where the homogenized energy density f_{hom} is defined by

$$f_{hom}\ :\ z\in\mathbb{R}^{nm}\mapsto\lim_{t\to+\infty}\frac{1}{t^n}\inf\left\{\int_{tY}f(y,z+\nabla v)dy : v\in W_0^{1,p}(tY;\mathbb{R}^m)\right\}.$$

This convergence result was established in [1] and [2] by sophisticated Γ-convergence arguments. Since then, many attempts at simplifying the proof (for example by using two-scale convergence), seem not to have borne fruit. In [4], a joint paper with Alain Damlamian and Riccardo De Arcangelis, we apply the tool of periodic unfolding from [3]. This gives a direct proof of the convergence result (under slightly weaker assumptions than in [1] and [2]), making use of simple weak convergence arguments in L^p-type spaces. This paper is part of a series of ongoing works concerning the applications of the periodic unfolding method to homogenization. The second one [5], treated the same problem but in the case of convex densities. The third one [6], that will be presented here, deals with constrained integral type energies.

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Gröbner bases and Pell equations

Mihai Cipu

It is a classical fact that a Pell equation $x^2-dy^2=1$, with d positive and non-square integer, has infinitely many solutions in positive integers. It is also well-known that a generalised Pell equation $ax^2-by^2=1$, with a and b positive integers and ab non-square, has either none or infinitely many positive solutions. In 1909 A. Thue has proved that a system of such Diophantine

equations has finitely many solutions. The proof is ineffective in the sense that it gives no indication on the number of solutions. The first explicite upper bound for this number has been obtained by H.P. Schlickewei in 1990. After successive improvements, due to D.W. Masser and J. Rickert (1996), M.A. Bennett (1998), P. Yuan (2002, 2004), the definite result on the problem of estimating the number of solutions to systems of generalised Pell equations just appeared. Some of the results we shall present are obtained jointly with M.A. Bennett (Vancouver), M. Mignotte (Strasbourg) and R. Okazaki (Kyoto).

A quantitative estimation of an asymptotic representation for linear positive operators

Cristina Cismaşiu

In this talk we will give a quantitative estimation, in terms of the least concave majorant of the modulus of continuity, using an asimptotic representation with the central moments upon order $2k$, for linear positive operators of probabilistic type.

Spherical unitary representations for reductive groups

Dan Ciubotaru

A classical problem in representation theory, motivated by abstract harmonic analysis and number theory, is the study of unitary representations of *reductive* algebraic groups (for example the general linear, symplectic, or orthogonal groups) defined over real and p-adic fields.

A *unitary representation* of a group G is a continuous homomorphism π from G to the group of unitary operators on a complex Hilbert space. One defines the *irreducible* unitary representations to be those without proper closed invariant subspaces. Of particular interest is the identification of the *spherical* irreducible unitary representations of G, that is, those which have nontrivial fixed vectors underthe action of a maximal compact subgroup K. One of the main motivations for their study comes from the theory of automorphic forms.

I will report on recent work on the classification of the spherical unitary dual (joint with D. Barbasch).

Solving nonlinear systems of equations and nonlinear systems of differential equations by the Monte Carlo method using Queueing networks and games theory

Daniel Ciuiu

In this talk we will solve some nonlinear systems of equations and nonlinear systems of differential equations by the Monte Carlo method using queueing networks and some results from games theory.

On empirical Bayes estimators under some types of loss functions

Roxana Ciumara

Based on joint work with **Vasile Preda**.

During the last decades, several authors focused on the problem of asymptotic optimality and convergence rates for empirical Bayes estimators. Preda (1981, 1982) examined nonparametric empirical Bayes procedures for a selection problem, considering the weighted entropy. Tiwari and Zalkikar (1990) and then Liang (1993) found conditions for asymptotic optimality of empirical Bayes estimator and derive the convergence rate, using quadratic loss. Their results were extended by Preda and Ciumara in 2006 for a weighted quadratic loss function. In 1997, Huang and Liang found properties of empirical Bayes estimators of the truncation parameter of some distribution using Linex loss. Shi et al. (2005) studied convergence rate of empirical Bayes estimators of two-dimensional truncation parameter under the same Linex loss.

In this talk we consider some types of loss functions in order to obtain the empirical Bayes estimators for the scale parameter of Pareto distribution. We discuss conditions that have to be met for asymptotic optimality. Moreover, considering these conditions we find convergence rate and some properties of the regret risk. The approach in this talk leads to interesting new results or extends some known ones.

2-ρ-DERIVATIONS ON A ρ-ALGEBRA AND APPLICATIONS TO THE QUATERNIONIC ALGEBRA

Catalin Ciupala

There is a substantial literature analyzing quaternionic objects with many applications in mathematics and physics. In this context, one observes that the formulation of the physical problems in mathematical terms often requires to introduce quaternionic differential calculus. The main difficulty is obviously presented by the non-commutative nature of the quaternionic field. On the other hand the non-commutative geometry studies the noncommutative spaces (from the geometry point of view) and has many applications in theoretical physics, so we think that is very important to study the quaternionic algebra $\mathbb{H}$ using the framework of noncommutative geometry and these things we are doing in this paper.

A ρ-(commutative) algebra A is a G-graded algebra, G is a commutative group and ρ is a twisted cocycle. The basic geometric objects over a ρ-algebra A are: the differential calculus in [1] and linear connections, distributions and metrics in [2], [3], [4], [5] and others papers. Remark that for defining linear connections on the quantum hyperplane and on the matrix algebra (which are ρ-algebras) the basic role is played by the space of ρ-derivations, but the space of ρ-derivations on the quaternionic algebra is empty, so we had to introduce and study a new space, denoted the space of 2-ρ-derivations of the ρ-algebra A. We define 2-linear connections on bimodules over the ρ-algebra A; the curvature and the torsion of a 2-linear connection are also studied. In the last part we apply these notions to the case of the quaternionic algebra $\mathbb{H}$, and thus we get a complete characterization of the space of 2-ρ-derivations on $\mathbb{H}$ and finally we define linear connections on $\mathbb{H}$.

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Cătălin Ciupală, The "Andrei Şaguna" National College,

Finite elasto-plasticity with continuously distributed dislocations

Sanda Cleja-Ţigoiu

The aim of the talk is to propose a thermodynamically consistent model for material with structural inhomogeneities such as dislocations, under large deformations and during the isothermal processes. The continuously distributed dislocations are modeled in [1], [4], [6], by the non-zero Burgers vector, related to the *curl* of the plastic distorsion $\mathbf{F}^p$. In our model the plastic measure of deformation is represented by a pair $(\mathbf{F}^p, \overset{(p)}{\mathbf{\Gamma}})$, $\overset{(p)}{\mathbf{\Gamma}}$ is an affine plastic connection, with non-zero torsion. On the background of differential geometry concepts, any pair of $(\mathbf{F}, \mathbf{\Gamma})$, with $\mathbf{F}-$ the deformation gradient, $\mathbf{\Gamma}-$ a connection with zero torsion, is decomposed into second order elastic and plastic measures $(\mathbf{F}, \mathbf{\Gamma}) = (\mathbf{F}^e, \overset{(e)}{\mathbf{\Gamma}}) \circ (\mathbf{F}^p, \overset{(p)}{\mathbf{\Gamma}})$.

Macroscopic balance laws of macro-momentum and angular momentum are postulated in the framework of classical continuum mechanics, no Cosserat type kinematics being assumed in the model, but non-symmetric Cauchy stress and couple stresses are involved as driving forces, which are power conjugated to the appropriate deformation measures. The material response is elastic with respect to the plastically deformed configuration with torsion, and characterizes the current values of the driving forces in terms of the appropriate second order elastic strain measure. The plastically deformed configuration with torsion is defined by the evolution equations for plastic distortion and plastic connection, if we take or not into account the compatibility between the plastic distorsion and plastic connection (see also [2]). The microstructural material field equations are derived based on the appropriate principle of the virtual power.

To complete the formulation of the constitutive laws and evolution equations, we determine the restriction imposed by the requirement to have the temporal variation in the free energy density, $\dot{\psi}$, less than or equal to the internal power, $\mathcal{W}_{int}$, expended during the isothermal process. The umbalanced free energy is formalized similarly to [3], [5], but in the framework of finite elasto-plasticity, via the appropriate dissipation inequality $\dot{\psi} - \mathcal{W}_{int} \leq 0$, (see also [4]). The constitutive representation for the free energy density is assumed to be written in a frame invariant form, with respect to plastically deformed configuration. The consequences inffered in the model are analyzed for different forms of the free energy. The role of the plastic spin and of the plastic torsion are also discussed and compared with the results presented in the references.

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A variational analysis of a contact interaction problem in viscoelasticity

Marius Cocou

This talk deals with the mathematical analysis of a class of dynamic contact problems for two (visco)elastic bodies when the contact interface involves adhesion and friction.

First, continuum classical formulations of the problems that describe the surface interactions and also the bulk behaviour of both bodies are presented.

Second, variational formulations are written as a coupling between evolution implicit variational inequalities and a differential equation describing the evolution of the intensity of adhesion. The existence and approximation of variational solutions are studied, based on penalty methods, incremental approaches, several estimates and compactness results.

Finally, some numerical examples are presented.

Hyperfunctions of "n" variables on a finite based algebra

Stelian Cojan

This talk is dedicated to the occasion of 75-birthday of my Professor **P. T. Mocanu** and defines two types of hyperfunctions, establishes their properties for $\mathbb{C}^{n+1}$ and for $\mathbb{C}^{n+1}$. We define tangent cones and we notice their properties. The monogen micro-analytic and the corresponding singular spectrum for $\mathbb{C}^{n+1}$ are defined by using the open and convex cones. These results are applied in the case of the $\delta(x^{\omega})$.

On general theory of skew polynomial rings

Elena Cojuhari

Our purpose is to study a monoid algebra over a non-commutative ring. First, we introduce axiomatically a family of mappings $\sigma = (\sigma_{x,y})$ defined on a ring A and subscripted with elements of a multiplicative monoid G. The assigned properties allow to call these mappings derivations of the ring A. Next, we construct a monoid algebra $A < G >$ by means of the family σ. Namely, such an algebra is our main object of study. In particular, we prove the universality property of it. We study in detail the particular case of a monoid G generated by two elements. This case is important especially for the theory of skew-polynomial, in one variable. The obtained results concerning this special case extend and generalize some related results of T. H. M. Smits. In

this respect we cite T. H. M. Smits, [Skew polynomial rings, *Indag. Math. 30* (1968), 209-224] and also [*P. M. Cohn*, Free rings and their relation, *Acad. Press, London, New-York*, 1971].

Spectral analysis of some perturbed block Toeplitz operators

Petru Cojuhari

Our purpose is to review the spectral theory (with applications) if operators obtained by perturbations of block Toeplitz operators. The main attention is paid on the study of the structure of the spectra of corresponding operators. In particular, the point, absolute continuous and singular continuous parts of the spectrum are investigated, and the problem of the absence of the singular spectrum is considered. Estimate formulae for the discrete spectrum are also given. The main results are obtained by combining the theory Toeplitz and Laurent type operators and the methods of perturbation theory.

On the Wold decomposition of some periodical stochastic processes

Alexandra Colojoară

The Wold decomposition theorem is one of the most poweful tools used to establish the structure of the stationary stochastic processes. The trend-stationarity and the difference-stationarity consider the casses of non-stationarity processes.

This talk gives a Wold decomposition theorem for some peridical stochastic processes, and used it to fit models for time series with seasonal component, not using the usual technique of desezonality.

The variants of the fastICA algorithm for noisy data by using kurtosis and negentropy

Doru Constantin

Based on joint work with **Costel Balcau** and **Paul Radovici-Marculescu**.
Key words: Independent Component Analysis (ICA), Blind Source Separation (BSS), Entropy

In this article we evaluate the estimation of the ICA model when additive noise is present. We introduced a bias-free version of the FastICA algorithm in the case of kurtosis and negentropy functions with testing your performance.

Adiabatically switched-on electrical bias in continuous systems, and the Landauer-Büttiker formula

Horia Cornean

Based on joint work with **P. Duclos**, **G. Nenciu**, and **R. Purice**.

Consider a three dimensional system which looks like a cross-connected pipe system, i.e. a small sample coupled to a finite number of leads. We investigate the current through this system, in the linear response regime, when we adiabatically turn on an electrical bias between leads. The main technical tool is the use of the finite volume regularization, which allows us to define the current coming out of a lead as the time derivative of its charge. We finally prove that in virtually all physically interesting situations, the conductivity tensor is given by a Landauer-Büttiker type formula.

Axiomatic characterization of the (co)homology theories on spectra

Cristian Costinescu

On the category of spectra S we propose a new definition for the (co)homology theories and, by using it, we assign a (co)homology theory on S to a spectrum. Now, this bijective correspondence is based on a definition "compatible" with the classical one of the (co)homology theory on the category of CW-spaces.

Identities of nilpotent A-Loops

Alexandru Covalschi

Algebra $L = (L, \cdot, \backslash, /)$ of type $(2, 2, 2)$, where identities

$$(x \cdot y)/y = y\backslash(y \cdot x) = y \cdot (y\backslash x) = (x/y) \cdot y = x, x/x = y\backslash y$$

hold true is called a *loop.* The *multiplication group* $\mathfrak{M}(L)$ of the loop L is the group generated by all the *translations* $L(x), R(y)$, where $xy = xR(y) = yL(x)$. The subgroup $I(L)$ of the group $\mathfrak{M}(L)$, generated by all the *inner mappings* $L(x, y) = L(y)L(x)L(xy)^{-1}$, $R(x, y) = R(x)R(y)R(xy)^{-1}$, $T(x) = R(x)L(x)^{-1}$ of L is called the *inner mapping group* of the CLM L. A main importance of inner mapping group for loop theory is the following statement: a subloop H of the loop L is normal in L if $HI(L) = H$.

A loop L is called an *A-loop* is all its inner mappings are automorphisms [1].

In [2] it is proved that the identities of nilpotent Moufang loops have a finite basis. In this talk we extend this result to cover also the nilpotent A-loops.

Lemma 1. *A nilpotent and finitely generated A-loop satisfies the maximum condition for subloops.*

Corollary 2. *A nilpotent and finitely generated A-loop is finitely represented.*

Lemma 3. *A nilpotent and finitely generated A-loop is residually finite.*

Theorem 4. *The identities of a nilpotent A-loop have a finite basis.*

Corrolary 5. *The quasiequational and equational theories of any variety of nilpotent A-loops are resoluble.*

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On factorization properties for ρ-contractions

Aurelian Craciunescu

Based on joint work with **B. Chevreau**.

There are very well known the obtained successes of application of the dual algebra techniques in getting some results concerning the existence of invariant subspaces and the structure (in predual factorization, reflexivity...) for certain classes of contractions. Obviously, these results may be transferred to operators that are similar to the contractions of these classes. Such a class of operators is also the $\mathcal{C}_\rho$ class or the class of ρ-contractions, introduced by B. Sz.-Nagy and C. Foiaş, which demonstrates also the similarity of these with the contractions.

In this talk we are introducing an analogy of the $\mathbf{A}$ class for ρ-contractions and we are studying directly, without the use of the similarity with a contraction, a subclass of this one analogue to $\mathbf{A}_{\aleph_0}$ class from the contraction case. The direct method may offer an advantage by obtaining somme better constant-values in the factorization processes. We sustain the idea that the techniques we have used here can also be useful in the research of other classes of operators.

On the dynamics and geometry of an autonomous underwater vehicle with a fixed control authority

Mircea Craioveanu

Based on joint work with **Mircea Puta**

In the last years there was a great deal of interest in the study of the autonomous underwater vehicles in connection with their increasingly role in underwater missions. The goal of our talk is to discuss some dynamical and geometrical properties of an autonomous underwater vehicles with a fixed control authority.

Local homeomorphisms having local ACL^n inverses

Mihai Cristea

We study a class of local homeomorphisms which is more richer than the class of local homeomorphisms which are in addition maps of finite distortion and satisfying condition $(\mathcal{A})$, or than the class of Q-mappings. Our class of local homeomorphisms satisfy condition (N) and has local ACL^n inverses, and we prove Poleckii's modular inequality in this class and we find conditions in order to calculate the weight modulus of the paths going through a point. Using this basic instruments, we give extensions in this class to the known Zoric's theorem, we calculate the radius of injectivity and we give some eliminability results, similar to the classical one from the theory of quasiregular mappings, or even more generally. The present theory will be posed later in the

more general setting of the open, discrete mappings, but some results concerning boundary extension, equicontinuity, modulus of continuity are given in the special case of homeomorphisms satisfying condition (N) and having ACL^n inverses.

Existence results for a class of nonlinear equations involving a duality mapping

Jenică Crînganu

Based on a joint work with **G. Dincă**.

In this talk we study the existence of multiple solutions to abstract equation

$$J_p u = N_f u,$$

where J_p is the duality mapping on a real reflexive and smooth Banach space, corresponding to the gauge function $\varphi(t) = t^{p-1}, 1 < p < +\infty$. We assume that X is compactly imbedded in a Lebesgue space $L^q(\Omega), p \leq q < p^*$, and continuously imbedded in $L^{p^*}(\Omega)$, p^* being the Sobolev conjugate exponent, $N_f : L^q(\Omega) \to L^{q'}(\Omega)$, $\frac{1}{q} + \frac{1}{q'} = 1$, being the Nemytskii operator generated by a function $f \in \mathcal{C}(\bar{\Omega} \times \mathbb{R}, \mathbb{R})$ which satisfies some appropriate conditions.

Real-time multiprocessor scheduling : feasibility issues

Liliana Cucu

The use of computers to control safety-critical real-time functions has increased rapidly over the past few years. As a consequence, real-time systems — compute@@r systems where the correctness of each computation depends on both the logical results of the computation and the time at which these results are produced — have become the focus of much study. Since the concept of "time" is of such importance in real-time application systems, and since these systems typically involve the sharing of one or more resources among various contending processes, the concept of scheduling is integral to real-time system design and analysis. Scheduling theory as it pertains to a finite set of requests for resources is a well-researched topic. However, requests in real-time environment are often of a recurring nature. Such systems are typically modelled as finite collections of simple, highly repetitive tasks, each of which generates jobs in a very predictable manner. These jobs have upper bounds upon their worst-case execution requirements, and associated deadlines.

We are interested in global scheduling of periodic task systems upon identical multiprocessor platforms [1]. More precisely, we present the main steps that one should follow when he/she searches for feasibility intervals in the multiprocessor case. Contrary to the uniprocessor case, if a feasible schedule (obtained in the multiprocessor case) repeats from some time instant, then it is not obvious that a feasibility interval exists. This property remains true only for predictable scheduling algorithms [2]. Otherwise, we deal with scheduling "anomalies" [3].

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Bundles, operator algebras and K-theory

Marius Dădârlat

The bundles that occur naturally in functional analysis are not necessarily locally trivial. Dixmier and Douady have shown that a continuous bundle with fiber an infinite dimensional separable Hilbert space H over a compact contractible metric space does not need to be trivial, even though the unitary group of H is contractible. Similar phenomena appear in the theory of operator algebras. The operator algebras with Hausdorff primitive spectrum are known to be isomorphic to algebras of sections in certain continuous bundles which are typically not locally trivial. We plan to give a gentle introduction illustrated by examples to the K-theory methods used in the study of these bundles.

Some applications of interval analysis in functional analysis

Nicolae Dăneţ

Let $\mathbb{IR} = \{[\underline{a}, \overline{a}] \mid \underline{a}, \overline{a} \in \mathbb{R},\ \underline{a} \leq \overline{a}\}$ be the set of all finite real closed intervals endowed with the addition $[\underline{x}, \overline{x}] + [\underline{y}, \overline{y}] = [\underline{x} + \underline{y}, \overline{x} + \overline{y}]$ and the multiplication $[\underline{x}, \overline{x}][\underline{y}, \overline{y}] = [\min\{\underline{x}\,\underline{y}, \underline{x}\,\overline{y}, \overline{x}\,\underline{y}, \overline{x}\,\overline{y}\}, \max\{\underline{x}\,\underline{y}, \underline{x}\,\overline{y}, \overline{x}\,\underline{y}, \overline{x}\,\overline{y}\}]$. Under each of this operation $\mathbb{IR}$ is an abelian semigroup. The distributivity rule is not valid in general, but a weaker law, called "subdistributivity", is true $[\underline{x}, \overline{x}]([\underline{y}, \overline{y}] + [\underline{z}, \overline{z}]) \subseteq [\underline{x}, \overline{x}][\underline{y}, \overline{y}] + [\underline{x}, \overline{x}][\underline{z}, \overline{z}]$. The equality holds in the special case when the multiplication is made with real numbers. On $\mathbb{IR}$ we consider the order relation defined by $[\underline{x}, \overline{x}] \leq [\underline{y}, \overline{y}] \Leftrightarrow \underline{x} \leq \underline{y},\ \overline{x} \leq \overline{y}$.

Let $\Omega \subseteq \mathbb{R}^n$ be an open set. Define $\mathbb{A}(\Omega) = \{f : \Omega \longrightarrow \mathbb{IR},\ f \text{ locally bounded}\}$ and $\mathcal{A}(\Omega) = \{f : \Omega \longrightarrow \mathbb{R},\ f \text{ locally bounded}\}$. If $f \in \mathbb{A}(\Omega)$, the value $f(x)$, $x \in \Omega$, is an interval denoted by $[\underline{f}(x), \overline{f}(x)]$. For this reason a function $f \in \mathbb{A}(\Omega)$ will be written in the form $f = [\underline{f}, \overline{f}]$, where $\underline{f}, \overline{f} \in \mathcal{A}(\Omega)$ and $\underline{f} \leq \overline{f}$. The order relation defined on $\mathbb{IR}$ induces an order on $\mathbb{A}(D)$ in a pointwise way: $[\underline{f}, \overline{f}] \leq [\underline{g}, \overline{g}] \Leftrightarrow \underline{f} \leq \underline{g}$ and $\overline{f} \leq \overline{g}$.

For $f \in \mathbb{A}(X)$ and $x \in \Omega$ we define the lower Baire operator $I : \mathbb{A}(\Omega) \longrightarrow \mathcal{A}(\Omega)$, $I(f)(x) = \sup\limits_{V \in \mathcal{V}_x} \inf\{z \in f(y) \mid y \in V\}$ and the upper Baire operator $S : \mathbb{A}(\Omega) \longrightarrow \mathcal{A}(\Omega)$, $S(f)(x) = \inf\limits_{V \in \mathcal{V}_x} \sup\{z \in f(y) \mid y \in V\}$. The operator $F : \mathbb{A}(X) \longrightarrow \mathbb{A}(X)$, $F(f)(x) = [I(f)(x), S(f)(x)]$ is called graph completion.

A function $f \in \mathbb{A}(D)$ is called S-continuous if $F(f) = f$. The function $f = [\underline{f}, \overline{f}] \in \mathbb{A}(D)$ is S-continuous if and only if $\underline{f}$ is lower semicontinuous and $\overline{f}$ is upper semicontinuous. A function $f \in \mathbb{A}(D)$ is called Hausdorff continuous, or H-continuous, if f is S-continuous and for any S-continuous function g the inclusion $g(x) \subseteq f(x)$, $x \in \Omega$, implies $g(x) = f(x)$, $x \in \Omega$. $\mathbb{H}(\Omega)$ ($\mathbb{H}_b(\Omega)$) denotes the set of all H-continuous (bounded) interval functions defined on Ω.

A function $f \in \mathbb{H}(\Omega)$ has point values and is continuous on a dense subset of Ω. Obviously, $C(\Omega) \subset \mathbb{H}(\Omega)$.

In this talk we present the algebraic and order structure of the set $\mathbb{H}(\Omega)$ and the norm structure of $\mathbb{H}_b(\Omega)$.

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Fixed-point theorems for some collectively families of multimaps

Rodica-Mihaela Danet

Based on joint work with **Ioan-Mircea Popovici**.

In this talk we give some collectively fixed-point theorems for a family of multimaps defined on product spaces and with or without compactness assumptions on the range sets. The starting point of our new results is some papers of Q.H.Ansari and J.C.Yao (1999), L-J. Lin, Z-T. Yu, Q.H.Ansari and L-P.Lai (2003) and a seminal paper of S. Park(2003) In the proof of our main result that is very technical but standard we use the partition of unity subordinated to a finite subcovering of a compact product space and we apply the Tychonoff's fixed-point theorem. Our results have many applications, solving some celebrated problems, like as coincidence problem, intersection problem for sets with convex sections, quasiequilibrium problem, variational inequalities problem, minimax problem and general equilibrium problems in game theory and mathematical economics.

Groups and automorphisms

Marian Deaconescu

This talk will concentrate on several problems I have considered during the last seven years or so. These are in general old, well-known problems; the number in the parantheses indicates how old they are (in years).

1) What kind of constraints would force a finite group admitting a fixed-point-free automorphism to be abelian? (cca 100)

2) Do all finite nonabelian p-groups have non-inner automorphisms of order p? (cca 50)

3) Is it possible to find a *structure theorem* , that holds for all groups? The general view is that the answer is "no". (cca 100)

4) A Mersenne prime is a prime of the type $2^p - 1$, where p is a prime. Is it possible to characterize somehow the primes p in question? (cca 300)

5) G is a group, $A \leqslant Aut(G)$ and F is the fixed point subgroup of A in G. Is it any link between F and A? (cca 100)

6) If G is a finite p-group of unknown order, is it possible to say something about the structure of G just by looking at the number of its irreducible complex characters? (cca 100)

The solutions to all these problems rely on arguments involving the action of various automorphism groups. Problems 1, 3, 5, 6 were solved in collaboration with G.L. Walls and problem 2 was tackled in collaboration Gh. Silberberg.

Moment integrals for $GL(m) \times GL(n)$: toward convexity breaking

Adrian Diaconu

This is part of a project with **P. Garrett** and **D. Goldfeld**.

A family of essentially elementary kernels, construable as Poincaré series, is described, which, integrated against $|f|^2$ for a cuspform f on $GL(n)$, yield averages of weighted critical-line integral moments of L–functions $L(*, f \otimes F)$ for cusp forms F on $GL(m)$ with $1 \leq m < n$.

The formulation of the $GL(2)$ kernel over number fields was stimulated by two joint papers with Goldfeld (Gauss-Dirichlet conference and Bretton Woods), an older paper of A. Good (Selberg conference), remarks about spectral theory in some older papers of Sarnak, and a general conviction that many sensible structures can be profitably reformulated as having to do with spectral theory (representation theory) on adele groups. The $GL(2)$ case of this was joint work with Garrett in spring 2006.

The eventual simplicity of the $GL(2)$ scenario suggested that some extension to larger groups was plausible. The ambient literature on Euler factorizations of global integrals, addressing Rankin-Selberg, Rankin-Shimura, and Langlands-Shahidi types, as well as Jacquet-Piatetski-Shapiro-ShalikaÕs work on $GL(n)$, and Cogdell-Piatetski-ShapiroÕs work on converse theorems, and many others, provided a reassuring technical context.

More broadly, it is natural to speculate that global integrals, with explicable kernels, producing integrals of Euler products are conceivably useful.

Indeed, in hindsight, many features of the extension to $GL(n)$ are obvious, though non-trivial complications are apparent. In ongoing work, for various choices of defining data, we obtain asymptotics for moment integrals compatible with the convexity bound. Any better-power-of-T error term will break t–aspect convexity for a large class of $GL(m) \times GL(n)$ automorphic L–functions.

On the approximation of equations' solutions through methods that use divided differences

Adrian Diaconu

The results in this talk concern the approximation methods of the equation's solutions that use the divided differences or some of their generalizations. Therefore, the use of the Fréchet differential is avoided.

The main place in this talk is given to the chord method. This method for the approximation of the root of a certain equation $f(x) = 0$, where appears the function $f : I \to \mathbb{R}$ with $I \subseteq \mathbb{R}$ being a real interval, uses two numbers $x_0, x_1 \in I$ arbitrarily chosen and builds an approximation of this root through the sequence $(x_n)_{n \in \mathbb{N}}$ with the help of the recurrence relation $x_{n+1} = x_n - f(x_n) / [x_{n-1}, x_n; f]$.

We are aiming for the extension of this method to the case of a nonlinear mapping $f : D \to Y$, where$D \subseteq X$, X and Y are linear normed spaces and we are concerned with the approximation of the solution of the equation $f(x) = \theta_Y$, where θ_Y is the null element of the space Y.

The similarity with the real case but also with the Newton-Kantorovich method determines the construction of the sequence $(x_n)_{n\in\mathbb{N}} \subseteq D$ of the chord method. This sequence will be generated, starting from the initial elements $x_0, x_1 \in D$, through the recurrence relation $x_{n+1} = x_n - U_n^{-1} f(x_n)$,where for any $n \in \mathbb{N}$ the mapping $U_n : X \to Y$ is a linear, continuous and invertible mapping, that will be an extension of the divided difference from the real case. As one will see, beginning with the fourth section of the paper, it is sufficient to suppose that for any $n \in \mathbb{N}$ we have the equality $U_n(x_n - x_{n-1}) = f(x_n) -$
$-f(x_{n-1})$ .The similarity with the real case is obvious. Of this reasen the name of the mapping $U_n : X \to Y$ will be **generalized abstract divided difference of the mapping** $f : D \to Y$ **on the nodes** x_{n-1}, x_n. We will use for $U_n : X \to Y$ the more perspicuous notation $\Gamma_{f;x_{n-1},x_n}$.

Before beginning the study of the convergence of the sequence $(x_n)_{n\in\mathbb{N}} \subseteq$
$\subseteq D$ we cannot overlook the important problems regarding the existence of a mapping $U_n = \Gamma_{f;x_{n-1},x_n} : X \to Y$, for a certain or for any number $n \in \mathbb{N}$, with the afrorementioned qualities. This problem is solved in the second section of the paper through an effective construction in relation with the problem of the interpolation of a function between linear normed spaces. We will build the abstract interpolation polynomial in the form of Lagrange as well as the inverse abstract interpolation polynomial.

In order to build this abstract polynomial we use the following result.

If X is a real linear normed space with the norm $\|\cdot\|_X : X \to \mathbb{R}$ and with θ_X its null element, for any $a \in X \setminus \{\theta_X\}$ there exists a linear and continuous functional $u \in X^*$ so that $\|u\| = 1$ and $u(a) = \|a\|_X$. In the previous assertion we have noted through X^* the set of linear and continuous functionals defined on X, a set that is a linear normed space with the norm $\|\cdot\| : X^* \to \mathbb{R}$, $\|u\| = \{|u(x)| / x \in X,\ \|x\|_X = 1\}$.

We consider $p \in \mathbb{N}$ and $x_0, x_1, ..., x_p \in D$ called interpolation nodes, so that for any $i, j = \overline{0,p}$ with $i \neq j$ we have $x_i - x_j \in X \setminus \{\theta_X\}$, therefore it exists $U_{i,j} \in X^*$, with $\|U_{i,j}\| = 1$, $U_{i,j}(x_i - x_j) = \|x_i - x_j\|_X$. Under these circumstances

$$\mathbf{L}(f; x_0, x_1, ..., x_p) : X \to Y,$$

$$\mathbf{L}(f; x_0, x_1, ..., x_p)(x) = \sum_{i=0}^{p} l_i(x) f(x_i)$$

where $l_i : X \to \mathbb{R}$, $l_i(x) = \prod\limits_{j=\overline{0,p};\ j\neq i} U_{ij}(x - x_j) / \|x_i - x_j\|_X$ for any $i = \overline{0,p}$, is caled abstract interpolation polynomial of the function $f : E \to Y$ on the nodes $x_0, x_1, ..., x_p \in E$.Using this inverse abstract interpolation polynomial we can obtain expressions of the approximation of the solution for the equation $f(x) = \theta_Y$.

The abstract interpolation polynomial facilitates the definition of the abstract divided difference with the p order on the nodes $x_0, x_1, ..., x_p \in D$ of the function $f : D \to Y$ that is the coefficient of the abstract monomial with the highest degree from the expression of this abstract polynomial, being denoted through $[x_0, x_1, ..., x_p; f] : X^p \to Y$ as a $n-$ linear and continuous mapping defined through $[x_0, x_1, ..., x_p; f](h_1, ..., h_p) = \sum\limits_{i=1}^{p} \omega_i(h_1, ..., h_p) f(x_i)$ where $\omega_i(h_1, ..., h_p) = \left[\prod\limits_{j=1}^{i} U_{i,j-1}(h_j) \cdot \prod\limits_{j=i+1}^{p} U_{i,j}(h_j)\right] \Big/ \prod\limits_{j=\overline{0,p};\ j\neq i} \|x_i - x_j\|_X$. In the case $n = 1$, we will have the abstract divided difference of the first order on 2 nodes $[x_0, x_1; f] : X \to Y$ and it is possible to choose $\Gamma_{f;x_0,x_1} = [x_0, x_1; f]$ for $x_0, x_1 \in X$ with $x_0 \neq x_1$. This is $[x_0, x_1; f] : X \to Y$,

$[x_0, x_1; f]\, h =$
$= [U_{0,1}(h)\, f(x_1) + U_{0,1}(h)\, f(x_0)] / \|x_1 - x_0\|_X$.

In the third section we study the case in which the spaces X and Y are spaces with a scalar product. In this case the functionals that go in the expression of the abstract interpolation polynomial have a unique, exact determination.

On account of the facts and of the results presented, the method of the chord has a meaning and therefore we have a consistent study of its convergence. The theorem that will be established, through its hypotheses asks for the existence of a generalized abstract divided difference $\Gamma_{f;x,y}$ on every $x, y \in D$, $x \neq y$. We suppose: **a)** the existence of a constant $L > 0$ such that for any $x, y, z \in D$ with $x \neq y$ and $y \neq z$ we have $\|\Gamma_{f;x,y} - \Gamma_{f;y,z}\| \leq L \|x - z\|_X$; **b)** there exists the linear and continuous mapping $\Gamma^{-1}_{f;x_0,x_1} : Y \to X$; **c)** restrictions for the choice of the initial elements $x_0, x_1 \in D$.

The conclusions of the theorem are the inclusion of the whole approximations' sequence $(x_n)_{n\in\mathbb{N}}$ in a certain ball with the center in x_0 and together with this ball in the set D; the existence for any $n \in \mathbb{N}$ of the mapping $\Gamma^{-1}_{f;x_{n-1},x_n} : Y \to X$ as inverse of the mapping $\Gamma_{f;x_{n-1},x_n} : X \to Y$; the existence in the set D of a solution of the considered equation; and finally an estimation of the error of approximation of this solution through a term x_n. The convergence order of the method is $\alpha = \left(1 + \sqrt{5}\right)/2$.

Saari's conjecture of the N-Body problem

Florin Diacu

Saari's conjecture, proposed by Donald Saari in 1970, claims that solutions of the Newtonian N-body problem with constant moment of inertia are relative equilibria. In other words, if the mutual distances satisfy a certain relationship, the configuration of the particle system rotates like a rigid body. We prove this conjecture in the collinear case for a large class of potentials [1] and discuss a generalization for large sets of initial conditions [2].

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De Rham cohomology of hypersurface complements

Alexandru Dimca

Let V be a hypersurface in the complex projective space $\mathbb{P}^n$. In a sequence of papers, see [1], [2], [3], [4], I have studied the cohomology groups $H^*(U, \mathbb{C})$ of the affine open set $U = \mathbb{P}^n \setminus V$ using the rational differential forms. The case V smooth is classical and due to Griffiths, see [5].

Interesting relations involving the mixed Hodge filtration, the polar filtration and the Brieskorn modules have emerged.

In this talk, first I will survey some of these results and then discuss in a similar spirit the cohomology groups $H^*(U, \mathcal{L})$ of U with coefficients in a rank one local system $\mathcal{L}$.

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A Summability criterion for stochastic integration.

Nicolae Dinculeanu

Let X be an adapted process. The process X is said to be p-summable if the Stochastic measure I_x, with values in L^p, defined on the ring generated by the predictable rectangles, can be extended to sigma additive measure on the predictable sigma-algebra. If X is p-summable, then the Stochastic Integral $H \cdot X$ can be defined for certain predictable processes H. The following summability criterion is proved: If X is of class (LD) and if the measure I_x is bounded in L^p, then X is p-summable. If X is a martingale, then it is automatically of class (LD). If X is a square-integrable martingale, then the measure I_x is bounded in L^2; hence, a square-integrable martingale is 2-summable.

The constructive solution of n-dimensional operator equations' system and its application in classical Maxwell theory

Irina Dmitrieva

In given abstract we propose the explicit mathematical method that is supposed to be quite simple from the engineering and applied aspects and that allows to diagonalize an n-dimensional system of the arbitrary differential operator equations over the finite-dimensional space $(x, y, z, t, u_1, \ldots, u_m)$, where $u_1, \ldots, u_m$ are the additional real parameters that describe the characteristics of media (temperature, for example, and others). The generalization of this algorithm for the operator block matrices is shown too. All desired scalar equations have an only one component of the unknown n-dimensional vector-function $\vec{F}(x, y, z, t, u_1, \ldots, u_m)$ and are obtained by the application of the corresponding differential operators to the original system's equations. The only requirement of the operator matrix elements in the original system is their commutativity in pairs. The proposed diagonalization procedure is invariant regarding initial and boundary conditions, which become necessary only when the obtained scalar equations have to be solved, i.e. when the diagonalization algorithm is finished completely.

Saying shortly, the considered method doesn't depend neither on the operator matrix structure nor on the boundary statement of the original problem, and represents the operator analogue of the algebraic systems' solution.

The proposed procedure is demonstrated in the case of "symmetrical" differential Maxwell equations' system [1]

$$\begin{cases} rot\vec{H} = (\sigma \pm \varepsilon_a)\vec{E} + \varepsilon_a \partial_0 \vec{E} + \vec{j}^{CT} \\ -rot\vec{E} = (r \pm \lambda\mu_a \vec{H}) + \varepsilon_a \partial_0 \vec{H} + \vec{e}^{CT}, \end{cases} \tag{1}$$

that appears to be the generalization of results [2]. In (1) $\vec{E} = \vec{E}(x, y, z, t, u_1, \ldots, u_m)$ and $\vec{H} = \vec{H}(x, y, z, t, u_1, \ldots . u_m)$ are the unknown vector-functions of the electric and magnetic fields' tension; the given vector-functions $\vec{j}^{CT} = \vec{j}^{CT}(x, y, z, t, u_1, \ldots, u_m)$ and $\vec{e}^{CT} = \vec{e}^{CT}(x, y, z, t, u_1, \ldots, u_m)$ describe the appropriate outside current sources and outside tensions; the positive constants σ, μ_a, ε_a are the specific conductivity, absolute permeance and dielectric permeability correspondingly; the differential operator $\partial_0 = \frac{\partial}{\partial t}$; the positive real constant λ is the parameter of signal that intrudes in media, and r at the current stage of investigations is the theoretical positive real constant that is supposed to exist.

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Two theorems for Ψ-modulus

Dan Dogaru

An important tool in studying quasiconformal and quasiregular mappings in the classical p-modulus. In the last years were also considered other kind of modulus, like discrete modulus or weighted modulus. Recently, H. Tuominen, was introduced a new modulus, called Ψ-modulus and used in the theory of Orlicz-Sobolev spaces.

We give two theorems for Ψ-modulus, concerning separated paths families from $\mathbb{R}^n$ and families of rectifiable paths, having an analogue enounce as the classical theorems which hold in the classical case

Fuzzy integer transportation problem

Gheorghe Dogaru

An algorithm for solving the transportation problem with fuzzy supply and demand values and the integrality condition imposed on the solution is presented. This algorithm is exact and computationally effective, although the problem is formulated in the general way, i.e., its fuzzy supply and demand values can differ from each other and be fuzzy numbers of any time.

On normal and non-normal holomorphic functions on complex Banach manifolds

Peter Dovbush

Throughout, X denotes a complex Banach manifold modelled on a complex Banach space of positive, possibly infinite, dimension; K_X denotes the Kobayashi distance on X; and $\mathcal{O}(X)$ denotes the set of all holomorphic functions on X.

Let $\mathbb{C}$ denote the complex plane. A family $\mathcal{G}$ of holomorphic functions in the open unit disc $\Delta := \{z \in \mathbb{C} : |z| < 1\}$ is said to be **normal** on Δ if each sequence $\{g_j\} \subset \mathcal{G}$ has a subsequence which converges uniformly (with respect to the Euclidean metric) on compacta in Δ or diverges uniformly to ∞ on compacta in Δ.

A function f in $\mathcal{O}(X)$ is a **normal function** if the family

$$\mathcal{F} = \{f \circ \varphi \,|\, \varphi : \Delta \to X \text{ is holomorphic}\}$$

is a normal family on Δ.

Theorem 1 *A function f in $\mathcal{O}(X)$ is not a normal function on X iff there exist sequences $\{x_m\}$ and $\{y_m\}$ of points in X, and a positive constant M such that $K_X(x_m, y_m) < M$ for all m, $\lim_{m\to\infty} f(x_m) = \infty$, and $\lim_{m\to\infty} f(y_m) = a \in \mathbb{C}$.*

Theorem 2 *If f in $\mathcal{O}(X)$ is a normal function on X and if h is a non-normal holomorphic function on X such that each sequence $\{x_m\}$ of points in X contains a subsequence $\{x_{m_k}\}$ on which at most one of f or h is unbounded, then $f + h$ is a non-normal function on X.*

Theorem 3 *Let $\{x_n\}$ and $\{y_n\}$ be two sequences of points in X, and let M be a positive constant such that $K_X(x_m, y_m) < M$ for all m. If f in $\mathcal{O}(X)$ is a normal function on X which omits $L \in \overline{\mathbb{C}}$ in X but $\lim_{m\to\infty} f(x_m) = L$, then $\lim_{m\to\infty} f(y_m) = L$.*

Let D be a bounded convex (in the real sense) domain in a complex Banach space. Fix $\xi \in \partial D$, and fix $a \in D$. Let $l_\xi(a) := \{y \in D : y = \xi + t(a - \xi), 0 < t \leq 1\}$ and let

$$\begin{aligned}\mathcal{K}_\alpha(\xi, a) := &\text{ set of points } x \in D \text{ for which there exists} \\ &y \in l_\xi(a) \text{ satisfying } K_D(x, y) < \alpha.\end{aligned}$$

A function f is said to have a *$\mathcal{K}$-limit* L at $\xi \in \partial D$ if for every $\alpha > 0$, $f(x) \to L$ as $x \to \xi$ within $\mathcal{K}_\alpha(\xi, a)$.

Theorem 4 *Let D be a bounded convex domain in a complex Banach space. Let $\{x_n\}$ be a sequence of points in $l_\xi(a)$ which tends to a boundary point $\xi \in \partial D$ such that there exists a positive constant M with $K_D(x_m, x_{m+1}) < M$ for all m. If f in $\mathcal{O}(D)$ is a normal function on D which omits $L \in \overline{\mathbb{C}}$ in D but $\lim_{m\to\infty} f(x_m) = L$, then f has $\mathcal{K}$-limit L at ξ.*

On the coalitional rationality of Semivalues of cooperative transferable utility n-person games

Irinel Dragan

The quasi-cores were induced by L.S. Shapley and M. Shubik (1966), in connection with market games. A comprehensive survey on the Core, by Y. kannay (1992), were some quasi-cores are discussed can be found in the Handbook of Game theory, vol.I, Chapter 12. In the following, we shall use more general quasi-cores and give, as a byproduct, necessary and sufficient conditions for balancedness for a class of symmetric quais-cores. Sufficient conditions for balancedness are obtained by proving necessary and sufficient conditions for the appurtenance of the Shapley value to a type of symmetric quasi-cores. It is well known that the problem of appurtenance of the Shapley value to the Core of a transferable utility game (TU-game) was solved by E. Innara and J.M. Usategui (1993). Different results on the same problem were obtained by J. Marin-Solano and C. Rafels (1996), as well as by Y. Takahashi (1998). In a previous joint paper, I. Dragan and J.E. Martinez-Legaz (2001), used the so called Average per capita formula for the Shapley value, due to I.Dragan (1992), in order to get another form of the necessary and sufficient conditions for the appurtenance of the Shapley value to the Core. On the other hand, an Average per capita formula for the Semivalues, introduced by P. Dubey, A. Neyman and R.J. Weber (1981), has been proved in that previous joint paper. These facts and the above mentioned results on quasi-cores will be used in the present paper.

The aim of the talk is that of introducing a new concept of coalitional rationality for values of TU-games, called w-coalitional rationality, such that this becomes the usual coalitional rationality in case of effiency, and that of giving necessary and sufficient conditions for this type of coalitional rationality for a Semivalue. in the first section, we introduce a concept of quasi-core and symetric quasi-core and provide necessary and sufficient conditions for the nonemptiness of this type of quasi-core. Further, in the second section, necessary and sufficient conditions for the appurtenance of the Shapley value to the symmetric quasi-core are obtained. The fact that any Semivalue is the Shapley value of a game easily obtained from the given game, as shown in a paper by I. Dragan (2005), will allow us to define a new concept of coalitional rationality for a value, the w-coalitional rationality, and reduce the problem of finding necessary and sufficient conditions of coalitional rationality for a Semivalue to the problem of the appurtenance of the Shapley value to a symmetric quasi-core. To be able to use this concept, in the third section we prove an Average per capita formula for the Eficient normalization of a Semivalue, introduced by E. Calvo and J.C. Santos (1997). The last result of the paper, concerning the application of the new type of coalitional rationality to a Semivalue, in order to get necessary and sufficient conditions of coalitional rationality in this case, is given the last section. We sketched the proofs of the previous results needed, to make the paper self-contained, and gave a short reference list in which the papers used above have been shown.

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Spectral methods in hydrodynamic stability

Ioana Dragomirescu

Based on joint work with **Adelina Georgescu**. Nowadays the industry and ecology need more and more results from hydrodynamics of complicated fluids occurring in complex circumstances. For a few practical examples from these fields we present some spectral methods used by us to solve the eigenvalue problems governing the stability of fluid flows against normal mode perturbations. The difficulty of solving these problems is due to the high order of the differential equations defining the eigenvalue problems, to unusual boundary conditions, to the bifurcation of the eigenvalue set,and to the presence of physical parameters, others than the one representing the eigenvalue.

The Stratification of the Orbit Space of Proper Groupoids

Oana Drăgulete

Given a proper Lie groupoid, we show how to construct a Whitney stratification of its orbit space. In the case of an action Lie groupoid, this stratification coincides with the well known stratification by orbit types. We explicit this construction for the case of the Lie gropoid integrating a Lie algebra action

Dynamic stall modelling of airfoils and comparison with experiments

Horia Dumitrescu

The talk is aimed at describing a fundamental phenomenon: the effect of rotation on the inboard blade boundary layer of a wind turbine. The three?dimensional incompressible steady momentum integral boundary layer equations coupled with the entrainment equation are used to analyze the inboard stall- delay due to rotation.

Turbulence is modeled by using empirical correlations for the entrainment coefficient and the streamwise skin-friction coefficient. A power law type of velocity profile is assumed for the

streamwise turbulent velocity profiles and for cross ? flow velocity profile the Mager parabolic model is used. The equations are written in cylindrical coordinates and are solved iteratively using a Runge-Kutta scheme.Special attention has been devoted to those term in the differential equations that change the boundary ?layer structure from that of two-dimensional steady flow. They are used for identifying the physical mechanism associated with 3-D and rotational effects and for establishing a semiempirical correction law for the lift coefficient based on 2-D airfoil data. It is found that the results using the model are in good agreement with the experimental data.

Inequalities for means of chords and related isoperimetric problems

Pavel Exner

The talk is motivated by isoperimetric problems arising in quantum mechanics as well as in classical electrostatics. Specifically, we consider the Schrödinger operator in $L^2(\mathbb{R}^2)$ with an attractive interaction supported on a closed curve Γ, formally given by $-\Delta - \alpha\delta(x-\Gamma)$ and ask which curve of a given length maximizes the ground state energy. The second problem concerns a loop-shaped thread Γ in $\mathbb{R}^3$, homogeneously charged but not conducting; we ask about the (renormalized) potential-energy minimizer. Both of them reduce to a purely geometric problem about inequalities for mean values of chords of Γ. We prove an isoperimetric theorem for p-means of chords of curves when $p \leq 2$, which gives an answer to the above questions, and find the critical $p > 2$ for which it ceases to be valid. A discrete analogue of the problem is also considered.

IStochastic models in risk evaluation for financial portofolio options

Pavel Farcaş

In this paper we study the measurement of the risk for financial portofolio option using VaR. For solving we apply the Black - Scholes method and the numerical solution given by Monte Carlo method. In the VaR evaluation we have the stochastic model for the volatility of a portofolio.

Impact-induced adiabatic wave structure in phase transforming materials

Cristian Făciu

Based on joint work with **Alain Molinari**.

In this talk we investigate dynamic aspects of solid-solid phase transitions when thermal effects are taken into account. In a recent paper (see Făciu and Molinari, 2006) we have proposed longitudinal impact experiments of thin bars as an effective mean for understanding the kinetics of stress-induced phase transformations in shape memory alloys (SMA). This problem has been investigated in a one-dimensional and isothermal setting and has provided important insight into the wave structure. Since the influence of thermal effects on the rate of phase transformation,

size and shape of pseudo-elastic hysteresis, at higher strain rates is very important we extend this analysis to take into account their influence on the wave propagation.

We set out the balance laws, the dissipation inequality, and the corresponding jump conditions governing the dynamic response of a one-dimensional bar in the adiabatic case. Based on experimental facts, we consider an explicit one-dimensional non-monotone thermo-elastic model able to describe some aspects of the thermomechanical response of a shape memory alloy. Some aspects related to the non-unicity of weak solutions for the thermo-elastic system are reminded. The thermo-elastic model is embedded in a Maxwellian rate-type constitutive equation which removes the deficiencies of the classical thermo-elastic approach.

Impact problems that gives rise to both adiabatic shock waves and propagating phase boundaries are analyzed. Thus, we formulate the problem of a semi-infinite bar in the austenite phase impacted at one end and discuss some general features of its solution. The numerical predictions of the rate-type model for the longitudinal impact of two phase transforming bars are analyzed. One focuses on the results which can be measured in laboratory experiments like the time of separation of the bars after impact, the profile of the particle velocity at the rear end of the target and the stress-history at the contact point.

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Hermite-Hadamard inequality. New results

Aurelia Florea

Based on joint work with **C. P. Niculescu**.

The classical inequality of Hermite-Hadamard [6] gives us an important estimate of the mean value of a convex function over a compact interval. A far reaching generalization in the framework of locally convex spaces is offered by Choquet's theory [6], [7], based on the concept of majorization. A.M. Fink [3], C. P. Niculescu [4], [6], J. de la Cal and J. Cárcamo [1], P. Czinder and Z. Páles [2] have obtained a series of results even more general.

The aim of our talk is to discuss the present state of art in this field based on our paper [4].

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Construction of unbounded $*$–representations in terms of 'unbounded' C^*–seminorms

Maria Fragoulopoulou

Because of the pathologies that unbounded operators may show, the following natural question arises: under which method(s) one could select $*$-representations of the best possible behaviour? A method of this kind was introduced, in 2001, by S. J. Bhatt, A. Inoue and H. Ogi. In this talk, we shall present a number of joint results with S. J. Bhatt and A. Inoue related with the selection of "nice" $*$-representations on $*$-algebras, locally convex $*$-algebras and tensor product (locally convex) $*$-algebras.

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Multistage portofolio optimization

Cristinca Fulga

Portfolio management deals with the allocation of wealth among different investment opportunities, considering investor's preferences on risk. In this paper we consider a multiperiod model where the investor chooses a portfolio at the beginning of each period facing uncertainty

associated with the prices of the assets in portfolio at future dates. Models of this decision problem tend to become very large because of its dynamic structure and incorporating uncertainty. We present a multiple period portfolio model over a finite horizon with transaction costs, a risk averse utility function and the uncertainty being modeled using the scenario approach. We propose a new approach for efficiently solving real problems which are large enough to make use of as much information as portfolio managers can reasonably provide. The procedure utilizes stochastic programming combined with decomposition and approximating techniques. First, we turn to a scenario decomposition approach. Solving the resulting optimization problem relies on approximate dynamic programming techniques. In practice, the most of the risk averse investors are only interested in their final wealth and care less about the portfolio trajectory; therefore, the utility function that governs these investors is not additive. To take into consideration the portfolio optimization problem under general utility functions we propose a further decomposition method that relies on the K best paths algorithm. The double decomposition of our portfolio problem provides a method whose effectiveness is proved by the experimental results.

On the existence of $\mathcal{K}$ - admissible limits of holomorphic mappings

Natalia Gashitoi

Let D be a domain in $\mathbb{C}^n$ with $\mathcal{C}^2$ boundary, $\xi \in \partial D$ and ν_ξ be the unit outward normal. For $\alpha > 0$, the $\mathcal{K}$ - admissible approach region of vertex $\xi \in \partial D$ of aperture α is defined by $\mathcal{K}_\alpha(\xi) = \{z \in D : k_D(z, -\nu_\xi) < \alpha\}$. Here k_D is the Kobayashi distance on D and $k_D(z, -\nu_\xi)$ represents the Kobayashi distance from z to $-\nu_\xi$.

We say that a mapping $f : D \to \overline{\mathbb{C}}$ has radial limit $l \in \overline{\mathbb{C}}$ at $\xi \in \partial D$ if $\lim_{t\to 0} f(\xi - t\nu_\xi) = l$, and say that f has $\mathcal{K}$ - admissible limit $l \in \overline{\mathbb{C}}$ at $\xi \in \partial D$ (write $(\mathcal{K} - limf)(\xi) = l$) if for every $\alpha > 0$ we have $\lim_{\mathcal{K}_\alpha(\xi)\ni z\to\xi} f(z) = l$.

We established a connection between the existence of radial and $\mathcal{K}$ - admissible limits of a holomorphic mapping $f \in H(D, \overline{\mathbb{C}})$ in terms of the spherical derivative Q_f of f with respect to the Kobayashi norm. Provided that the domain D verifies a certain assumption, we showed that if $f \in H(D, \overline{\mathbb{C}})$ has radial limit at $\xi \in \partial D$, then f has $\mathcal{K}$ - admissible limit at ξ if and only if $(\mathcal{K} - limQ_f)(\xi) = 0$.

Abstract reproducing Kernel structures and general dilations result

Dumitru Gaşpar

It is known that the fundamental dilation theorem of B. Sz.-Nagy regarding the dialtion of Hilbert space operator valued mapping on a $*$ - semigroup, which is positive definite and satisfies a boudedness condition to a representation of the given $*$ - semigroup, whose values are operators in a larger Hilberst space, can be proved by using reproducing kernel Hilbert space techniques.

The present talk is based on a descriprion of some abstract positive definite kernels and the associated reproducing kernel abstract structure as: Loynes $\mathcal{Z}$ - spaces, Hilbert C^* - modules, Hilbert modules over the algebra $\mathcal{B}(\mathcal{X})$ of all bounded linar operators on the Banach space $\mathcal{X}$. Using this thechnique some general dilation theorems, which extend the above mentioned Sz.-Nagy fundamental theorem regarding operators on the just mentioned abstract structures are also given.

As applications we mention the dilatability of a (non adjointable) contraction on a Hilbert module over a locally C^* - algebra.

On gramian (co-)isometric extensions

Păstorel Gaşpar

Unitary extensions of operators on Hilbert space play a fundamental role in obtainig results on non-selfadjoint spectral theory, developed by Sz.-Nagy and C. Foiaş.
The current talk is based on results of the author on just mentioned extensions, but foroperators on so-called Loynes spaces, which contain as a particular case the Hilbert modules over locally C^* - algebras. As it is known, the topological structure of such spaces is determined for once by a gramian (inner product) taking values in a complete, ordered locally convex space.
The talk covers results on: the Wold type decomposition of gramian isometries and the structure of the gramian shift; the gramian unitary extension of a gramian shift; the gramian co-isometric extension of an adjointable gramian contraction; the isometric dilation of an adjointable gramian contraction.
As application are indicated the way these results apply in the study of abstract stocahastic processes.

A Gould type set valued integral

Alina Cristina Gavriluţ

Based on joint work with **Anca Croitoru**.

In 1965, G.G. Gould introduced a type of integral of a bounded, real valued function with respect to a finitely additive set function taking values in a Banach space, integral which is more general than the Lebesgue one. Recently (in two papers, appeared in 2002 and 2003), A. Precupanu and A. Croitoru defined and studied a Gould type integral for multimeasures with values in the family of all nonvoid, compact, convex sets of a Banach space. Taking as starting point their works, A. C. Gavriluţ has defined and studied in several papers the notion of a Gould type integral with respect to a special multivalued set function called *multisubmeasure,* which is of finite variation and takes values in the family of all nonvoid, bounded, closed subsets of a Banach space. This notion of a multisubmeasure naturally extends the classical notion of a real submeasure in the sense of Drewnowski. A. C. Gavriluţ pointed out the remarcable properties of this type of integral. Namely, the integral is homogenous with respect to the function and to the multisubmeasure, finitely additive with respect to the set, to the multisubmeasure and also with respect to non-negative functions. Also, it is absolutely continuos, hereditary with respect to the set and monotone with respect to the function, to the multisubmeasure and also with respect to the set. It also preserves the properties of the multisubmeasure, such as, for instance, the increasing convergence, the regularity etc. So, it has all the usual properties of an integral.

If, particularly, the multisubmeasure is induced by a real submeasure in the sense of Drewnowski, having finite variation, in a recent paper, A. C. Gavriluţ and A. Croitoru established some more interesting properties of the integral, such as, a measure change type theorem and a mean one. We have also obtained a Radon-Nikodym type theorem, which bases on a construction of Maynard, using the notion of exhaustion.

Equilibria of linear dynamical systems: analytical and algebraic approach

Raluca Mihaela Georgescu

Based on joint work with and **A. Georgescu**.

The nature of the equilibrium points for a linear system is studied from two viewpoints: the first uses the eigenvalues of the linearized system around the equilibrium, while the second is based on the group theory, namely of the invariants. A particular case worked.

When are the products of normal operators normal?

Aurelian Gheondea

We consider the question of deciding when the products AB and BA of two normal operators A and B on the same Hilbert space $\mathcal{H}$ are normal. The Fuglede Theorem gives a sufficient condition, namely commutativity. The finite dimensional case is due to Gantmacher-Krein while the compact case is due to Wigmann. We consider the non-compact case and give answers in terms of the Spectral Multiplicity Theorem.

Spectral methods for non-standard eigenvalue problems

Călin Gheorghiu

We investigate the hydrodynamic stability of a thin liquid film flowing down an inclined plane and sheared simultaneously by a prescribed uniform stress. The stress is due to surface tension gradients and acts on the free boundary of the film (the so called *Marangoni-Plateau-Gibbs effect*). The competition between gravity and shear stress sets up a steady shear flow (the basic state) which is a linear combination of a Poiseuille and a Couette flows, the later being multiplied by a factor which contains the relative contribution of superficial and gravitational forces. The natural question is why should the features of the stability of these two fundamental flows disappear so completely when a free surface is present?

Clarification of this, leads in linear approximation, to some Orr-Sommerfeld ($(O-S)$ for short) boundary eigenvalue problems with boundary conditions depending nonlinearly on the spectral parameter. For long waves, the critical Reynolds number is determined by asymptotic analysis.

Based on Chebyshev polynomials, we develop a stable and accurate numerical approach that leads to banded matrices and eliminates the spurious eigenvalues which are omnipresent in the classical approaches [1]. The method is a Petrov-Galerkin one.

The asymptotic analysis is in fairly good agreement with well-established studies and is confirmed by our numerical computations as well as by independent ones [2], [3]. Up to our knowledge, no result concerning the completeness of the eigenfunctions or other analytic properties are available for such $(O-S)$ problems.

We also investigate numerically, the non-normality of the $(O-S)$ type operators, by computing their pseudospectra and some scalar measures of non-normality (Henrici number). It seems that these two measures of non-normality are complementary.

Numerical results concerning the left - most eigenvalue in the complex plane and neutral stability curves for a large set of mechanical and geometrical parameters are carried out.

We also try to analyze the non-normality of the $(O-S)$ operator in the context of a possible scenario of transition to turbulence [4].

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Singular elliptic systems of Gierer-Meinhardt type

Marius Ghergu

Based on joint work with **Vicenţiu Rădulescu**.

We study the existence and nonexistence of stationary solutions to a general Gierer-Meinhardt system with Dirichlet boundary conditions. This kind of systems involves the determination of an activator and an inhibitor concentration field. In a biological context, the Gierer-Meinhardt system has been used to model several phenomena arising in morphogenesis and cellular differentiation.

The main feature of our system is that both the activator and the inhibitor have different sources given by general nonlinearities.

Under some additional hypotheses and in case of pure powers in nonlinearities, regularity and uniqueness of the solution in one dimension is also presented.

The geometry of Blaschke product mappings

Dorin Ghisa

Based on joint work with **Ilie Barza**.

A Blaschke product B generates a (branched) covering Riemann surface (W, B) of the complex plane. The study of such surfaces is undertaken here in a very general context, namely when the set of cluster points of the zeros of B is a (generalized) Cantor set. Explicite forms and fundamental domains for the covering transformations are found and proper subgroups of the group of covering transformations are revealed.

On the Floquet-Favard theory

Valeriu Glăvan

The so-called "Favard theory", coined so after the seminal paper under the same title by B.M.Levitan and V.V.Zhikov (1977), has its roots in the works of J.L.Favard (1927), and is concerned with some separability conditions imposed on bounded solutions to assure the existence of almost periodic solutions of differential equations with almost-periodic coefficients.

From the other hand, the Floquet theory deals with linear differential systems with periodic coefficients and states reducibility of such systems to ones with constant coefficients. As a

consequence, one obtains a Jordan type structure of the (extended) phase space, i.e. a direct sum of spectral eighen-spaces each of which, in turn, decomposes into a flag of invariant subspaces - supports of secular solutions.

A series of obstacles prevent to extend the Floquet theory to almost periodic differential systems: a) the irregular systems (V.M.Millionshchikov, 1968), and closed intervals in the "dynamical spectrum" (R.Sacker and G.Sell, 1975); b) existence of topologically nontrivial vector bundles over the torus and over the Bohr's compact (I.U.Bronshtein and V.F.Cernii, 1976; B.F.Bylov, R.E.Vinograd, V.Ja.Lin and O.B.Lokutsievski, 1977; K.Palmer, 1980); c) small denominators (I.N.Blinov, 1967) and the topological effects generated by them (R.Johnson, 1980; M.G.Lubarski 1984; M.Nerurkar, 1987; A.M.Samoilenko and V.Glavan, 1989; A.Jorba, C.Nunez, R.Obaya and J.C.Tatjer, 2005).

The first who put together these two theories to study linear non-autonomous differential equations where R.Sacker and G.Sell (1978). Using the Favard condition they stated a "three-chotomy" structure theorem for linear almost-periodic systems, i.e. a decomposition of the extended phase space into the direct sum of exponentially stable, exponentially unstable vector subbundles and the subbundle generated by bounded solutions, provided there is no "secular solution".(As they have proved, it was the Favard condition, who assured that the merely vectorial (i.e. fiber-wise additive) subset of bounded solutions is in fact a vector subbundle, i.e. its fibers depend continuously on the base point).

To catch " secular solutions", the author proposed (1991) to extend the Favard condition up to all exterior powers of the linear system to assure on this way separability of solutions not only "by norm", but also "by angle", and thus, to exclude "authomorphic", i.e. minimal non-almost-periodic, motions "at the infinity".

The author has proved (1991) that under these conditions "zero" is an isolated point in the Sacker-Sell spectrum, and the corresponding spectral subbundle admits a flag of invariant vector subbundles as supports of "secular solutions". These Favard-type conditions permit one to obtain for linear almost-periodic systems a Floquet-type structure theorem, similar to that proved for linear periodic systems, that's why this title of the talk. Moreover, under a bit stronger Favard condition, some reducibility results can be obtained as well. The talk will be concerned with these results as well as with some relaxations of the Favard conditions, still enough to obtain further structure theorems for linear almost-periodic systems, as well as for Birkhoff recurrent ones.

Distortion of moduli of the ring domains in space

Anatoly Golberg

The classical Grötzsch estimate of distortion of the circular rings $\{r \le |z| \le 1\}$ and $\{\rho \le |w| \le 1\}$ yields that by a q-quasiconformal mapping $z \mapsto w$,

$$r^q \le \rho \le r^{\frac{1}{q}}.$$

In [2], the authors have established the inequalities of the Grötzsch type, which estimate the distortion of conformal moduli of the ring domains under quasiregular mappings f in $\mathbb{R}^n$, $n \ge 2$. In particular, for the spherical rings $R(a,b) = \{x : a < |x| < b\}$ and $R(c,d) = \{y : c < |y| < d\}$,

$$\left|\log\frac{d}{c} - \log\frac{b}{a}\right| \le \frac{1}{\omega_{n-1}} \int\limits_{R(a,b)} \frac{L_f(x) - 1}{|x|^n} dx.$$

Here $L_f(x)$ denotes the inner dilatation of f at x.

In this talk, we consider more general dilatations, which include the tangential and radial ones. In the case of plane, those were introduced in [1] and [3], respectively. We apply these dilatations to establishing geometric properties of generalized quasiconformal mappings in $\mathbb{R}^n$.

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Two results in combinatorial geometry based on classical theorems in real analysis

Radu Gologan

The Vitali covering theorem and some elementary probability results are used to establish relations between distribution of disks in the plane and lines that intersect them. Generalisations in more dimensions are immediate.

Singular elliptic systems of Gierer-Meinhardt type

Eugen Grebenikov

Let the following system be given

$$\begin{cases} \dfrac{dp}{dt} = -\dfrac{\partial H}{\partial q} + \mu F(p,q,t), \\ \dfrac{dq}{dt} = \dfrac{\partial H}{\partial q} + \mu G(p,q,t), \end{cases}$$

where $p = (p_1, p_2, ..., p_n)$, $q = (q_1, ..., q_n)$.
At $\mu = 0$ this system is standard Hamiltonian

$$\frac{dp}{dt} = -\frac{\partial H}{dq}, \quad \frac{dq}{dt} = \frac{\partial H}{dq}.$$

Let $p = q = 0$ be the equilibrium point of the above system. From the KAM - theory [1,2,3] it is known, that the problem of stability in Lyapunov sense of the stationary solutions (3) of systems (2) is solved for analytical hamiltonian at n=2 (Arnold-Moser theorem) [4,5]. Various applications of this theorem can be found in publications [5,6,7]. We shall call the system (1) at $\mu \neq 0$ quasi-hamiltonian system. For it the theorem of type Arnold-Moser is unknown. We wish to define conditions, with which should satisfy functions of system (1) for stability in Lyapunov's sense of the stationary solutions (3).

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Heat transfer in axisymmetric stagnation flow on a thin cylinder

Teodor Grosan

Based on joint work with **C. Bercea** and **Ioan Pop**.

The steady axisymetric stagnation flow and heat transfer on a vertical infinite cylinder of radius a is presented in this talk. Both cases of constant wall temperature and constant wall heat flux are considered. Using similarity variables the governing partial differential equations are transformed into ordinary differential equations. The resulting set of two equations is solved numerically using Runge-Kutta method combined with a shooting technique. For the special case of the Reynolds number Re »1, we obtained an asymptotic solution which include the Hiemenz solution. Our results are compared in some particular cases with existing results in the open literature and with the asymptotic approximation, and we found a very good agreement. It is shown that the Nusselt number and the skin friction increase and the boundary layer thickness decreases with the increases of the Reynolds number. Some graphs for the velocity and temperature profiles are presented. Also, tables with values related to the skin friction and Nusselt number are given.

Performance characteristics of the queueing models in network technologies

Olesia Groza

Computer networks are widespread nowadays. In the beginning these were used predominantly for scientific purposes. Yet, Internet has come through our life amazingly rapidly and vastly—one uses the global network for searching and transferring information, for communication, for e-commerce, etc. All this needs a high level of data transmission through the network. It is necessary, therefore, to analyze extensively the aspect of the quality of service (QoS) in computer networks.

Quality of Service is a general concept which refers to the ability of a network to provide better services for the selected traffic, using different technologies, among which are Frame Relay, IP-protocol, Asynchronous Transfer Mode (ATM), SONET, Ethernet and 802.1 protocols.

The problem of providing a better service can be studied by creating and studying special mathematical models of the queueing systems involved, and, after their analysis, by establishing relationships between characteristics of these systems.

There a few methods that allow QoS control. We shall consider the method of QoS control at the level of the network nodes, assuming that every such node represents a queueing system. Thus, the main objective of studying queueing systems in the framework of network management is to establish a certain adequate level of the service quality, which depends on the corresponding queueing system performance characteristics: waiting times, service times, the number of lost requests (packets), busy period, traffic coefficient, etc. This work was done under support of the SCOPES grant IB7320-110720

Limit sets of weakly contracting relations with eventual condensation

Valeriu Guţu

The well known fractal termed as "Pitagora's tree" represents the attractor of an Iterated Function System (IFS) with condensation [1]. It consists of a constant set-valued mapping with the "hypotenuse's square" as value (the "condensation") together with two similarities which map this square onto the other two squares related to the given right triangle.

As a multi-function, or a relation, this IFS with condensation is contracting with respect to the Hausdorff-Pompeiu metrics. Weakly contracting relations have been considered in [2, 3], where existence of the attractor, as well as of some characteristics of the set-valued dynamics, such as Shadowing Property, Asymptotic Phase Property, denseness of periodic points in the attractor, have been stated.

Here we are concerned with asymptotic properties of set-valued dynamical systems generated by relations with an eventual condensation. More precisely, let (X, d) denote a complete metric space, $\mathcal{P}(X)$ the set of its subsets and let $f \subset X \times X$ be a closed relation. (In case of a compact phase space X closedeness of f is equivalent with upper semi-continuity of the corresponding multi-function.)

There are some new relations associated with f: the composition $f \circ f$, the orbit relation $\mathcal{O}f := \bigcup\limits_{n\geq 1} f^n$, the limit relations $\omega f(x) := \limsup\{f^n(x)\}$ and $\Omega f := \limsup\{f^n\}$ (here $\limsup\{C_n\} := \bigcap\limits_{n\geq 1} \overline{\bigcup\limits_{k\geq n} \{C_k\}}$).

Any contracting relation f (see, e.g. [2]) possesses an unique global attractor A and the limit relations ωf and Ωf are constant with the same value A.

We say that a compact-valued multi-function (relation) $f : X \to \mathcal{P}(X)$ is an *eventual condensation* if there exists a non-empty compact $L \subset X$ and a natural n_0 (*condensation time*)

such that for all $n \geq n_0$ the multi-function f^n is constant with value L. In contrast with a condensation, an eventual condensation need not be contracting.

In [1] the structure of the attractor for an hyperbolic IFS with condensation has been studied.

The report will be devoted to the structure of the attractor and to asymptotic properties of the dynamics generated by an weakly contracting relation with an eventual condensation. We state the limit sets and the limit relations as asymptotic characteristics of set-valued dynamics, generated by iterations of the relation $F = f \cup f_0$, where f is a weak contraction and f_0 is an eventual condensation. In contrast with an IFS with condensation the limit relations ωF and ΩF need not be constant. The relation F itself, generally, may admit many local attractors.

Theorem. *Let X be a complete metric space with the property that every bounded and closed subset is compact. Let f be a weakly contracting relation with the attractor A and let f_0 denote an eventual condensation with the condensation time n_0. Then for any $x \in X$ the ω-limit set with respect to the relation $F := f \cup f_0$ has the form $\omega F(x) = A \cup \big(\bigcap_{n\geq 1} \overline{\bigcup_{m\geq n} B_m(x)} \big)$, where the sequence of subsets $\{B_m(x)\}_{m\geq 0}$ is defined as follows: $B_0(x) = \bigcup_{n\geq 0} f^n(x)$, $B_m(x) = \bigcup_{n\geq 0} f^n\big(f_0(F^{m-1}(x))\big)$ $(m \geq 1)$.*

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Giovanni Poleni's curve, elastic curves and elastic strips

Theodor Hangan

A research work in the field of classical differential geometry revealed me that some discoveries of the 18-th and 19-th centuries are few perceived or quite forgotten by the mathematical community; to remedy, I collected these in an article proposed to be read in section 8 (history of mathematics) at the Congress in Bucarest.

The integral of the square of the curvature of a regular curve, denoted B, is considered in mechanical problems as a measure of the deformation energy of a rectilinear beam. It was introduced by Daniel Bernoulli (1744) and studied by Euler and other mathematicians till in recent days. The extremals of the functional B are called "elasticae" or elastic curves. The integration of the Euler-Lagrange (E-L) equations which define elastic curves appeals to elliptic functions except one case when the solution is a curve discovered by Giovanni Poleni in 1719 when studiyng curves associated to the tractrix; it is called, in french, "courbe des forçats". Poleni's personality deserves to be emphasized with this occasion.

In the same spirit, elastic strips are extremals of a functional S introduced by Sadowsky in 1930 in order to study the equilibrium of a thin and narrow blade. Looking for helical solutions of its E-L equations one finds geodesics on cylinders having as directrix Poleni's curve; this result motivates my interest in the subject.

Semi-Ruan one-sided modules, the extreme flatness and the Arveson-Wittstock Theorem

A. Ya. Helemskii

Arveson-Wittstock Theorem on the extension of completely bounded operators is one of most important achievements of quantum functional analysis (= operator space theory). It plays the role of a "quantum" version of the classical Hahn-Banach Theorem on the extension of bounded functionals. In this talk we shall show that this theorem or, more precisely, its non-matricial version can be obtained as a rather straightforward corollary of a certain statement, concerning tensor products of some one-sided modules. This statement, probably, is of an independent interest.

Fix an infinite-dimensional Hilbert space L and denote, for brevity, $\mathcal{B}(L)$ by $\mathcal{B}$. We call a normed contractive right $\mathcal{B}$-module Y a *semi-Ruan module*, if for arbitrary orthogonal projections $P, Q \in \mathcal{B}$ and arbitrary $x, y \in Y$, we have

$$\|x \cdot P + y \cdot Q\| \leq (\|x \cdot P\|^2 + \|y \cdot Q\|^2)^{\frac{1}{2}}.$$

Further, we call a normed contractive left $\mathcal{B}$-module X *extremely flat*, if for every isometric morphism $\varphi : Y \to Z$ of right semi-Ruan $\mathcal{B}$-modules the operator $\varphi \underset{\mathcal{B}}{\otimes} \mathbf{1}_X : Y \underset{\mathcal{B}}{\otimes} X \to Z \underset{\mathcal{B}}{\otimes} X$ is also isometric.

Theorem. *Let H be an arbitrary Hilbert space, and (H, L) the space of Schmidt operators from H into L, equipped with the left outer multiplication $a \cdot \tilde{b}$ defined as just the composition $a\tilde{b}, a \in \mathcal{B}, \tilde{b} \in (H, L)$. Then (H, L) is an extremely flat module.*

This result, being combined with some simple general facts about semi-Ruan modules and extremely flat modules, provides several theorems of Arveson-Wittstock type, including the "geniune" Arveson-Wittstock Theorem.

Random systems with complete connections, iterated function systems and autoregressive time series

Ulrich Herkenrath

Based on mathematical models which had already been introduced and treated by Onicescu and Mihoc, in 1963 M. Iosifescu created the random systems with complete connections as general framework to study two connected stochastic processes in discrete time. This framework comprises quite different applications like models for learning or control, iterated function systems or models for autoregressive time series.

An essential component of a random system with complete connections is a stochastic recursion of the forum $W_{n+1} = u(W_n, X_{n+1}), n \in \mathbb{N}$, which represents a "new" state of the system as a measurable function of the preceding state W_n and an actual random "innovation" X_{n+1}. Under suitable assumptions the sequence $(W_n, n \in \mathbb{N})$ is a Markov chain. It can also be regarded as a solution of the stochastic recursion. Often this sequence $(W_n, n \in \mathbb{N})$ is the focus of interest.

Studying of the different applications of those random systems has lead to various results depending on the viewpoint or the techniques for analysis. It turns out that the unifying treatment in the framework of random systems with complete connections offers essential advantages. Several examples for that are presented which result from a joint research project with M. Iosifescu.

Algebraic operators provided from differential geometry

Iulia Elena Hirică

Some algebras of invariant operators under the action of certain groups are introduced. The algebraic approach considered has geometrical roots and the results obtained have useful applications in the study of some problems of differentiable manifolds.

Growth and diffusion of a population in a fragmented habitat

Mimmo Iannelli

We present and discuss a mathematical model describing the dynamics of an age-structured population spreading in a one-dimensional environment.

Our model takes into account two important features of the population changes, namely its spatial diffusion and its age structure. In particular we describe a population living in a one-dimensional stratified environment composed of n layers, supposing that the age-specific fertility and the age-specific mortality depend on the layer and on a significant variable which represents a way of weighting the age distribution.

Such kind of a model may describe the growth of a population in a heterogeneous habitat, fragmented into patches, where the diffusion coefficient and the vital rates vary only from one patch to another.

We will set up the mathematical framework within the context of nonlinear partial differential equations and, under suitable assumptions, which are meaningful from a biological point of view, we will discuss some problems including optimal harvesting.

Harmonic maps and Riemann submersions from quaternionic Kaehler manifolds

Steriu Ianuş

A quatiernionic Kaehler manifold (M, Q, g) is a manifold M with a metric g and a rank-3 vector bundle Q, locally generated by a triple (Ja), $a = 1, 2, 3$ of almost complex structures, which is parallel with respect to the Levi-Civita connection (cf. D.V. Alekseevsky & S. Marchiafava, Ann.Global Anal.Geom. 16 (1998) pp 419-444; S.S. Ishihara, J. Diff. Geom. 9 (1974) pp 483-500; S. Salamon, Invent. Math. 67 (1982),pp.143-171). We prove that a Q-holomorphic map between two quatiernionic Kaehler manifolds is a harmonic map (S.Ianus, R.Mazzocco, G.Vilcu, preprint 2006). The particular case of Hyper-Kaehler manifolds was considered in J.Chen & J. Li, J. Diff. Geom. 55 (2000) pp.355-384; D.Petcu, Novi Sad J. Math. 36 (2006) pp.21-34. It is well-known that a holomorphic map between Kaehler manifolds is stable. This property is not valid in general for Q-holomorphic maps between quatiernionic Kaehler manifolds. We give necessary and sufficient conditions for the stability of such maps. Then we give some harmonic Riemanian submersions from quatiernionic Kaehler manifold (see also M.Falcitelli ,S. Ianus, A.M.Pastore-"Riemannian Submersions and Related topics",World Sci. ,2004).

This talk is concerned with the thermoelastic deformation of inhomogeneous porous cylinders. We consider the case of the continuous inhomogeneity, when the constitutive coefficients are independent of the axial coordinate. First, we present a method to reduce the three-dimensional

problem to the study of plane strain problems. Then, we apply the method to study the deformation of a circular cylinder with a prescribed inhomogeneity.

Thermal stresses in porous elastic beams

Dorin Ieşan

This talk is concerned with the thermoelastic deformation of inhomogeneous porous cylinders. We consider the case of the continuous inhomogeneity, when the constitutive coefficients are independent of the axial coordinate. First, we present a method to reduce the three-dimensional problem to the study of plane strain problems. Then, we apply the method to study the deformation of a circular cylinder with a prescribed inhomogeneity.

An iterative stochastic construction for a solution of some Navier-Stokes equations

Bogdan Iftimie

We consider the following Navier–Stokes system on the whole space with Cauchy (terminal) condition (and no divergence-free condition for the solution !):

$$\begin{cases} \frac{\partial u^i}{\partial t}(t,x) & +\frac{1}{2}\Delta u^i(t,x) + \langle \nabla u^i(t,x), u(t,x)\rangle + f_i(t,x) = 0; \\ u(T,x) & = u_T(x);\ t \in [0,T],\ x \in \mathbb{R}^n,\ i = 1,\dots,n \end{cases} \tag{6}$$

We construct a strong solution of this system via an iterative procedure, using a discretisation method by splitting the time interval $[0,T]$ into "'small" subintervals by taking a sequence of partitions of the time interval with the mesh converging to 0. On each of these subintervals we apply the Feynman-Kac formula and the time is reversed. Finally, a compacity result of Prokhorov's type allows us to substract a subsequence which converges to a solution of the system on the whole space $\mathbb{R}^n$.

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Differential games with stochastic perturbation associated with non adapted solutions

Daniela Ijacu

A differential game with stochastic perturbation is determined by a dynamic of state variable $x \in X \subseteq R^n$ defined by a system of stochastic differential equations

$$1)\begin{cases} d_t x = f\left(t,x,u_1,..,u_N\right)dt + \sum_{j=1}^{d} g_j(t,x) \otimes dw_j(t) \\ x(0) = x_0, (t,x,u_1,..,u_N) \in [0,t_f] \times X \times U_1 \times .. \times U_N \end{cases}$$

where $U_i \subseteq R^{m_i}$ are some fixed closed sets and $w\left(t\right) = \left(w_1\left(t\right)..w_d\left(t\right)\right)$ is a standard Wiener d-dimensional process over a complete filtered probability space $\{\Omega, \mathcal{F}, P, \{\mathcal{F}_t\} \uparrow \mathcal{F}\}$ and " $\otimes$ " is a special type of stochastic integral. The control function $u\left(\cdot\right) \overset{\triangle}{=} \left(u_1\left(\cdot\right),..,u_N\left(\cdot\right)\right)$, is taken in a class A of admissible controls which are defined by bounded and measurable functions

$u(\cdot):\Omega\to U=\prod_{i=1}^{N}U_i$ with piecewise continuous trajectories $u(t,\omega),t\in[0,t_f]$ over the product measurable space $\{[0,t_f]\times\Omega,\mathcal{B}\otimes\mathcal{F},dt\otimes P\}$. For each $u(\cdot)\in A$ we define the corresponding solution $x=x(t,\omega;u),(t,\omega)\in[0,t_f]\times\Omega$ as the solution for (1) and associate the following pathwise functional

2) $J_i(\omega,u)=F^i\left(x\left(t_f,\omega;u\right)\right)+\int f_0^i\left(t,x\left(t,\omega;u\right),u\left(t,\omega\right)\right)dt,i\in\{1,2,..,N\}$,for each $\omega\in\Omega$.

Denote A_i the corresponding class of admissible controls $u_i(\cdot),i\in\{1,..,N\}$ and write $A=\prod_{i=1}^{N}A_i$

The following object

3) $\Gamma_N(\omega)\overset{\triangle}{=}\{[0,t_f],X=\mathbb{R}^n,U_i,\mathcal{A}_i,f,x_0,J_i(\omega,\cdot)\}_{i=1,2,..,N},\omega\in\Omega$ is called a stochastic differential game with N-players and open loop strategies.

An admissible solution $x(t,\omega;u)$ for (1) is represented as follows

4) $x(t,\omega;u)=G(t,\omega)(y(t,\omega;u))+\eta(t,\omega)$,

$t\in[0,t_f],\omega\in\Omega$ where G is a nonsingular $(n\times n)$ matrix and $\eta\in R^n$ are defined as continuous and F_t-adapted process fulfilling the following stochastic differential equation:

5) $d_tG=\sum_{j=1}^{d}A_j(t)G\circ dw_j(t),G(0,\omega)=I_n,t\in[0,t_f]$,

$\eta(t,\omega)=\sum_{j=1}^{d}\int_0^t b_j(s)\cdot dw_j(s),t\in[0,t_f]$,

where " $\circ$ " means Fisk-Stratonovich integral and " $\cdot$ " means standard Ito integral.

The vector value function $y(t,\omega;u)\in R^n$ is defined as a differentiable and non F_t-adapted process fulfilling the following system of differential equation

$$6)\begin{cases}\dfrac{dy}{dt}=[G(t,\omega)]^{-1}f\left(t,G(t,\omega)\left(y\right)\right)+\eta\left(\left(t,\omega\right),u\left(t,\omega\right)\right)\overset{\triangle}{=}\\ \qquad\overset{\triangle}{=}\tilde{f}\left(\omega,t,y,u\left(t,\omega\right)\right),t\in[0,t_f]\\ y(0)=x_0\in\mathbb{R}^n\end{cases}$$

The main theorem in this presentation allows us to convert the stochastic control problem into a deterministic one, with corresponding changes in formulating results.

Variational derivation of the Camassa-Holm shallow water equation

Delia Ionescu-Kruse

We describe the physical hypotheses underlying the derivation of an approximate model of water waves. For unidirectional surface shallow water waves moving over an irrotational flow or over a constant vorticity flow, we derive the Camassa-Holm equation by a variational approach in the Lagrangian formalism. This equation attracted a lot of interest due to its complete integrability and the existence of non-smooth soliton solutions, the so-called peakons. In contrast to the Korteweg-de Vries equation, which is a classical integrable model for shallow water waves, the Camassa-Holm equation possesses not only solutions that are global in time but models also wave breaking.

Compressible rigid-visco-plastic fluids: mathematical and numerical challenges

Ioan R. Ionescu

In the first part a general methodology for constructing compressible rigid visco-plastic fluid-type constitutive equations is presented. Starting from classic yield conditions for plastic solids, and neglecting the elastic effects, compressible rigid visco-plastic fluid models are obtained using two methods: the visco-plastic regularization and the stress superposition method. Examples of constitutive equations obtained using both procedures for the description of the behavior of geological media when subjected to large deformations and high strain rates are presented.

In the second part the onset of the flow of a rigid visco-plastic fluid is analyzed through a limit load analysis. Even if only the 2-D unidirectional flow of a Bingham fluid will be considered, some numerical results concerning the in-plane flow of a Cam-clay fluid will be given. The blocking property, characterized by the safety factor, is connected to two optimization problems in terms of velocities and stresses. For the velocity analysis we prove that the minimization problem in $BV(\Omega)$ is equivalent to a shape optimization problem. The optimal set appears to be the part of the fluid which flows whenever the loading parameter becomes greater than the safety factor. The stress approach involves a supremal optimization problem in $W^{1,\infty}(\Omega)$. The $L^p(\Omega)$ approximation technique is used to get a sequence of minimum problems for smooth functionals. Two numerical approaches, following these two analysis, are used to compute the safety factor for some applications in landslides modeling.

In the third part the flow involving high rate deformations zones coupled with rigid ones will be considered. A mixed finite-element and finite-volume strategy is developed for numerical modeling of the flow of a compressible rigid viscoplastic fluid. This numerical method is used to model the high speed penetration of a rigid projectile into geological and cementitious targets. It accurately describes the density changes around the projectile, the stress field as well as the shape and location of the viscoplastic zone in the target.

Manifolds covered by lines

Paltin Ionescu

An embedded projective manifold $X \subset \mathbb{P}^N$ is "covered by lines" if through any point of X there passes a line contained in X. From the birational point of view such manifolds are quite special, being uniruled (hence of negative Kodaira dimension). However, their biregular geometry is rich and interesting. We shall survey known results and open problems and conjectures, based on joint work with M.C. Beltrametti and F. Russo.

A new class of residuated lattices

Afrodita Iorgulescu

We introduce the $\alpha\gamma$ algebra as a residuated lattice satisfying conditions:
$(C_\rangle)$: $(x\rangle y)\rangle(y\rangle x) = y\rangle x$ and $(C_\wedge)$: $x \wedge y = [x \odot (x\rangle y)] \vee [y \odot (y\rangle x)]$,
while an α algebra (γ algebra) is a residuated lattice satisfying condition $(C_\rangle)$ ($(C_\wedge)$ respectively).

Recall that a BL algebra is a bounded residuated lattice satisfying conditions:
(prel): $(x\rangle y) \vee (y\rangle x) = 1$ and (div): $x \wedge y = x \odot (x\rangle y)$,
while a MTL algebra (bounded divisible residuated lattice = bounded commutative Rl-monoid) is a bounded residuated lattice satisfying condition (prel) ((div) respectively).

We get: (prel) $\Longleftrightarrow$ $(C_{\rangle})$ + $(C_{\vee})$ $\Longleftrightarrow$ $(C_{\wedge})$ + (C_{ε}) and (div) $\Longleftrightarrow$ $(C_{\rangle})$+ (C_{δ}) $\Longleftrightarrow$ $(C_{\wedge})$ + (C_{π}),
where $(C_{\vee})$: $x \vee y = [(x\rangle y)\rangle y] \wedge [(y\rangle x)\rangle x]$ and the independent conditions (C_{δ}), (C_{ε}), (C_{π}) must be found (open problem).

It follows that: (1) bounded $\alpha\gamma$ algebras are a common generalization of MTL algebras and of bounded divisible residuated lattices; (2) the MTL algebras with condition (DN) (Double Negation): for all x, $(x^{-})^{-} = x$, and the bounded $\alpha\gamma$ algebras with condition (DN) are the IMTL algebras (just like the BL algebras with condition (DN) and the divisible bounded residuated lattices with condition (DN) are the MV algebras); therefore, we have obtained classes of examples of MTL algebras and of bounded $\alpha\gamma$ algebras by starting with IMTL algebras and by using the ordinal product; (3) the ordinal product of two proper bounded $\alpha\gamma$ algebras is again a proper bounded $\alpha\gamma$ algebra; (4) the ordinal product: linearly ordered MTL (BL) algebra $\bigodot$ MTL (BL) algebra is again a MTL (BL) algebra, while the ordinal product: not-linearly ordered MTL (BL) algebra $\bigodot$ MTL (BL) algebra is only a bounded $\alpha\gamma$ algebra (bounded divisible residuated lattice).

We give classes of examples of finite proper IMTL algebras, MTL algebras and bounded $\alpha\gamma$ algebras, satisfying or not satisfying condition (WNM) (Weak Nilpotent Minimum): for all x, y, $(x \odot y)^{-} \vee [(x \wedge y)\rangle(x \odot y)] = 1$. We extended all the results to the non-commutative case.

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Quasi-boundedness, fixed-points, eigenvalues for nonlinear mappings and complementarity theory

George Isac

A. Granas introduced in 1962 the notion of quasi-bounded operator as a mathematical tool for the Fixed Point Theory. The goal of our lecture is to present some generalizations of the notion of quasi-bounded operator and to give some applications to the Fixed -Point Theory, to the study of eigenvalues of nonlinear operators and to the study of nonlinear complementarity problems. The complementarity problems form a class of mathematical models used in optimization, economics and engineering.

Generally, a complementarity problem is related to the study of equilibrium, the equilibrium considered in physics or in economics. We will give also some applications to the study of complementarity problems depending of parameters. Some open subjects will be also presented.

Generalized Pohožaev identities and non-existence results for p-Laplacian and $p(x)$-Laplacian

Florin Isaia

The purpose of this talk is to present two generalizations of the well-known Pohožaev identity and some non-existence results for Dirichlet problems with p-Laplacian and $p(x)$-Laplacian which follow from them.

As an illustration, we consider the Dirichlet problem

$$\begin{cases} -\Delta_p u = f(u) & \text{in } \Omega, \\ u|_{\partial\Omega} = 0, \end{cases} \tag{7}$$

where $\Omega \subset \mathbb{R}^n$ is a bounded open set of class C^1, $1 < p < \infty$ and $f : \mathbb{R} \to \mathbb{R}$ is a locally Lebesgue integrable and Borel measurable function which satisfies certain growth conditions.

Denote by $F : \mathbb{R} \to \mathbb{R}$ the primitive of f which satisfies $F(0) = 0$. The following generalization of Pohožaev's identity holds: if F is locally Lipschitz and f satisfies certain growth conditions, then every weak solution u of problem (1) verifies the following identity

$$-\frac{p-1}{p}\int_{\partial\Omega} |\nabla u|^p (x \cdot \nu)\, d\sigma = \int_\Omega \left[\left(\frac{n-p}{p} - a\right)|\nabla u|^p + auf(u) - nF(u)\right] dx, \tag{8}$$

for every $a \in \mathbb{R}$. Here, ν is the outer normal to $\partial\Omega$. The choice $p = 2$ and $a = (n-p)/p$ gives precisely the Pohožaev identity.

The main ingredient used in the proof of identity (2) is a variational identity of Pucci-Serrin type. Precisely, we show that every weak solution u of problem (1) satisfies:

$$\begin{aligned} \frac{\partial}{\partial x_i}\left[x_i\left(\frac{1}{p}|\nabla u|^p - F(u)\right) - x_j \frac{\partial u}{\partial x_j}\frac{\partial u}{\partial x_i}|\nabla u|^{p-2} - au|\nabla u|^{p-2}\frac{\partial u}{\partial x_i}\right] \\ = \left(\frac{n-p}{p} - a\right)|\nabla u|^p + auf(u) - nF(u), \end{aligned} \tag{9}$$

for every $a \in \mathbb{R}$, the equality being understood in a weak sens and the expression between square brackets belongs to $W^{1,1}(\Omega)$ for every $i = 1, \ldots, n$. The integration by parts of identity (3) leads us to identity (2).

The proof of formula (3) requires the verification of some preliminary technical results concerning the differential calculus in Sobolev spaces. The most difficult problem (raised by the presence of the term $F \circ u$) is the following: under which conditions on a function $g : \mathbb{R} \to \mathbb{R}$, the superposition operator $N_g = g \circ u$ maps $W^{m,p}(\Omega)$ into $W^{1,q}(\Omega)$, $m \geq 1$, $1 \leq p, q < \infty$ and satisfies the chain rule

$$\frac{\partial (g \circ u)}{\partial x_i} = (g' \circ u)\frac{\partial u}{\partial x_i} \quad (\forall)\, i = 1, \ldots, n,\ (\forall)\, u \in W^{m,p}(\Omega).$$

In order to solve this problem, we generalize a result due to M. Marcus and V. J. Mizel who formulated necessary and sufficient conditions for a function $g : \mathbb{R} \to \mathbb{R}$ to generate a superposition operator $N_g : W^{1,p}(\Omega) \to W^{1,q}(\Omega)$, $1 \leq q \leq p < \infty$.

Phase transitions in combinatorial optimization problems and computational complexity: towards rigorous connections

Gabriel Istrate

One of the promises of phase transitions in combinatorial problems was to shed light on the complexity of decision algorithms. Yet, despite a flurry of rigorous results, relatively few connections have been made between phase transitions and formal concepts in Computational Complexity.

In this talk I will discuss a number of results that attempt to relate the two concepts:

1. The classification of sharp and coarse thresholds of random constraint satisfaction problems, and the relations with Schafer's Dichotomy Theorem on the computational complexity of constraint satisfaction problems.

2. I show that a discontinuity in the relative size of a variation of the backbone order parameter is correlated with exponential resolution complexity (or, equivalently, the complexity of Davis-Putnam algorithms). This brings rigorous support to results of Monasson, Zecchina et al. connecting first-order phase transitions with resolution complexity.

3. Finally, I will discuss some recent results on the cluster structure of solutions of combinatorial problems, in particular random 2-SAT and graph bisection.

Invitation to n-harmonic hyperelasticity – the art of integration of nonlinear forms and free Lagrangians

Tadeusz Iwaniec

Based on joint work with **Jani Onninen**.

We study deformations $h : X \longrightarrow Y$ between bounded domains in n-dimensional Euclidean space. The general law of hyperelasticity requires that there exists an energy integral so that the deformations of interest to us are those with smallest energy. We assume here that the integrand is conformally coerced and polyconvex. Some additional regularity conditions are also imposed. Under these conditions we establish the existence and global invertibility of the minimizers. The key tools in obtaining an extremal deformation are the FREE LAGRANGIANS. Finding suitable free Lagrangians and using them for a specific energy functional is truly a work of art. We have done it here for the total harmonic energy and a pair of annuli in the plane.

The path to extinction

Peter Jagers

All real populations die out. The interesting is how long it takes and what happens before.

The mathematical prototype for a population bound for extinction is a subcritical, general branching process, i.e. a population of independently asexually reproducing individuals, where the expected number of progeny per individual is less than one, but randomness can be considerable, both in size and numbers of litters and in bearing ages of mothers. We consider large such populations, starting from say x individuals.

First we describe the behaviour of the time T_x to extinction, as $x \to \infty$. The benchmark cases of Markov branching and Galton-Watson processes are available in literature, but the

asymptotics are remarkably robust, and extend to general processes: $T_x \sim \ln x/r$, where $a = -r$ is the Malthusian parameter (which must be negative for subcritical populations). We give a more precise description in terms of extreme value distributions. Here some care has to be exercised though, distinguishing the lattice (discrete-time) and non-lattice (properly continuous) cases.

Then we study the process "half-ways" to extinction, i.e. at times uT_x, for $0 < u < 1$, e.g $u = 1/2$. It turns out that if the population starts from x individuals at time zero and Z_t^x denotes its size at time $t \geq 0$, then as $x \to \infty$, the properly scaled population size $x^{u-1} Z^x_{uT_x}$ converges weakly to a process of the form $C^{1-u}\bar{b}^u e^{-u\eta}$. In this the constant C is $C = \lim_{t\to\infty} e^{rt} \exp[Z_t^1]$, explicitly determined by life span and reproduction distributions, $\bar{b}$ is the mean of the Yaglom conditional limit distribution, and η follows the classical Gumbel distribution in the continuous time case.

Though the Yaglom limit thus enters in describing typical values of not yet extinct subcritical branching processes, its role seems less pronounced than may have been assumed.

Ordinary p-Laplacian systems with potential boundary conditions

Petru Jebelean

Keywords: Vector p-Laplacian, a priori estimates, critical point, Palais-Smale condition.

We present existence results for ordinary p-Laplacian systems associated with a general potential boundary condition. Firstly we deal with the problem

$$-(|u'|^{p-2}u')' = f(t,u), \quad \text{in } [0,T], \tag{1}$$

$$((|u'|^{p-2}u')(0), -(|u'|^{p-2}u')(T)) \in \partial j(u(0),u(T)), \tag{2}$$

where $p \in (1,\infty)$ is fixed, $j : \mathbb{R}^N \times \mathbb{R}^N \to (-\infty,+\infty]$ is proper, convex and lower semicontinuous and $f : [0,T] \times \mathbb{R}^N \to \mathbb{R}^N$ is a Carathéodory mapping. Secondly, the nonpotential system (1) is replaced by a potential one of type

$$-(|u'|^{p-2}u')' = \nabla F(t,u), \quad \text{in } [0,T]. \tag{3}$$

Here $F : [0,T] \times \mathbb{R}^N \to \mathbb{R}$ is a Carathéodory mapping, continuously differentiable with respect to the second variable. Afterwards instead of (3) will be the differential inclusions system

$$-(|u'|^{p-2}u')' \in \overline{\partial} F(t,u), \quad \text{in } [0,T], \tag{4}$$

where, this time, F is only locally Lipschitz with respect to the second variable and $\overline{\partial} F(t,\eta)$ stands for the generalized Clarke gradient of $F(t,\cdot)$ at $\eta \in \mathbb{R}^N$. For problem (1), (2) we use a fixed point technique, while in the cases of problems (3), (2) and (4), (2) our approach is a variational one. Some examples of applications are also given.

Asymptotic behavior of solutions of elliptic equations in the neighborhood of boundary and the boundary problems with singular boundary conditions

Nicolae Jitarasu

Let $G \subset R^n$ be the bounded domain with smooth boundary $\Gamma = \cup_{k=1}^{r} \Gamma_k$, where Γ_k are smooth varieties with $dim\Gamma_k = n_k, 0 \leq k \leq n-1$. We consider the elliptic equation $Lu = f$ of order $2m$ in G.

Using the Green's function of the boundary problem (or the fundamental solution) for the elliptic equation $Lu = f$ we obtain the integral representations for the solutions for the equation $Lu = 0$ in the neighborhood of Γ_k, specially, for the singular solutions.

Under some additional conditions we obtain the asymptotic representations for the solutions in the neighborhood of varieties Γ_k.

Using the asymptotic representations for the solutions we formulate the singular boundary conditions on Γ_k, and study the boundary problem with the singular boundary conditions.

Extremal completely n-positive linear maps between C*-algebras

Maria Joiţa

W. Arveson [1] characterized the extreme completely positive linear maps from a unital C^*-algebra A to $L(H)$ in the set $P_\infty(A, H, p)$ of all completely positive linear maps ρ from A to $L(H)$ such that $\rho(1) = p$ in terms of the Stinespring representation associated with these maps [6]. Later on, using theory of Hilbert C^*-modules, W.L. Paschke [5] extended the Arveson's result for completely positive linear maps between von Neumann algebras and S.K. Tsui [7] characterized the extreme completely positive linear maps between C^*-algebras.

A completely n-positive linear map from a C^*-algebra A to another C^*-algebra B is an $n \times n$ matrix $[\rho_{ij}]_{i,j=1}^n$ of linear maps from A to B such that the map ρ from $M_n(A)$, the C^*-algebra of all $n \times n$ matrices over A, to $M_n(L(H))$ defined by $\rho\left([a_{ij}]_{i,j=1}^n\right) = [\rho_{ij}(a_{ij})]_{i,j=1}^n$is completely positive. In [3] we showed that a completely n-positive linear map $[\rho_{ij}]_{i,j=1}^n$ from A to B is of the form $[\rho_{ij}(\cdot)]_{i,j=1}^n = [V_i^* \Phi(\cdot) V_j]_{i,j=1}^n$, where Φ is a representation of A on a Hilbert C^*-module E over B and V_i, $i = \overline{1,n}$ are adjointable module morphisms from B to E (see, also [2]). This extends the KSGNS (Kasparov, Stinespring, Gel'fand, Naimark, Segal) construction for completely positive linear maps [4]. In this talk, we extend the Arveson's result to the case of completely n-positive linear maps between C^*-algebras.

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Use of cubic spline for solving conservation laws system And improving available shocks in result with use of switch automatic

Mehdi Karimi

We obtain difference methods for solving conservation laws system that dissipative with suitable stability. These methods are family of triple parameters schemes of second order accurate and in another form have single parameter with third order accurate and also conditionally stable.

In numerical experiment with discontinuation initial condition gives results with intense shock that used automatic switch shoman filter for removing this shock and we observe its preference in usability of these methods.

A pragmatic overview of automated reasoning

Cristian Kevorchian

Automated reasoning provides important techniques for many computer science domains. We assist to a paradigm shift of automated reasoning, from a stand alone service, available on a local computing environment(OTTER, SPASS,etc), to a semantic reasoning service, which integrate reasoning systems as a web service(MATHServe).

WWW can be viewed as an open, heterogeneous and distributed information repozitory, but lack structured to support rigorous logic-based reasoning. Knowledge extraction, problems representation and reasoning web services will provide a more precise web.

Other important issue is content management, which is provided by OWL(Web Ontology Language), XML, RDF, and RDF Schema (RDF-S) by providing additional vocabulary along with a formal semantics. Following table is a synthetic view on web semantic provided by Tim Berners-Lee :

	Web	Semantic Web
Purpose	Human Browsing	Agent-consuming,semantic browsing
Information representation	HTML,XML	RDF,RDFS,DAML,OWL,RuleML
Search method	Keyword-based doc. retrival	Onthology-based query answering
Search results	URL, HTML doc.	Semantically enriched content
Reasoning ability	Only query	Query and inference

The automated reasoning servers will provide Web users, automated reasoning services through the SOAP(Simple Object Access Protocol). The reasoning web service consumer ask, involvs the

problem representation as a XML (extensible markup language) compling to a markup language and sends a SOAP request message including the XML file to the automated reasoning server, and the automated reasoning server represents the solution as a XML file conforming to the markup language DTD and send it back to the client as the SOAP response message. In our paper we focused on the comparison between standalone arhitectures for automated reasoning and automated reasoning systems as service oriented arhitecture which motivate the paradigm switch.

Asymptotically starlike and asymptotically spirallike mappings in several complex variables

Gabriela Kohr

Based on joint work with **Mirela Kohr**.

Key words: Asymptotic starlikeness, asymptotic spirallikeness, Loewner differential equation, spirallike mapping, starlike mapping, univalent subordination chain.

In this talk we consider the notion of asymptotic starlikeness in the Euclidean space $\mathbb{C}^n$. We prove that the notion of parametric representation which arises in Loewner theory can be characterized in terms of asymptotic starlikeness, i.e. they are equivalent notions. In particular, starlike mappings and spirallike mappings of type $\alpha \in (-\pi/2, \pi/2)$ are asymptotically starlike. However, we give an example of a spirallike mapping with respect to a linear operator which is not asymptotically starlike. In the case of one complex variable, any function in the class S is asymptotically starlike; however in dimension $n \geq 2$ this is no longer true.

In the second part of this talk we define the notion of asymptotic spirallikeness (a natural generalization of asymptotic starlikeness) in the Euclidean space $\mathbb{C}^n$. We consider the connection between this notion and univalent subordination chains. We introduce the notions of A-asymptotic spirallikeness and A-parametric representation where $A \in L(\mathbb{C}^n, \mathbb{C}^n)$, and prove that if $k_+(A) < 2m(A)$, then a mapping $f \in S(B^n)$ is A-asymptotically spirallike if and only if f has A-parametric representation, i.e. if and only if there exists a univalent subordination chain $f(z,t)$ such that $Df(0,t) = e^{At}$, $\{e^{-At}f(\cdot,t)\}_{t\geq 0}$ is a normal family on B^n and $f = f(\cdot,0)$. We also deduce that if f is a spirallike mapping with respect to an operator A such that $A+A^* = 2I_n$, then f has parametric representation. Finally we obtain some examples of asymptotically spirallike mappings.

Boundary integral equations for a three-dimensional Brinkman flow problem

Mirela Kohr

Keywords: Brinkman model, potential theory, boundary integral equations, existence and uniqueness, Sobolev spaces, Hölder spaces.

The purpose of this talk is to prove existence and uniqueness in Sobolev or Hölder spaces for a boundary value problem which describes the flow of a viscous incompressible fluid past a porous particle embedded in a second porous medium, by using the Brinkman model and potential theory. Some particular cases, which refer to Stokes flow past a porous particle, or to Brinkman's flow past a void, are also presented together with corresponding asymptotic results for the flow velocity field and the hydrodynamic force exerted on the particle.

Use of cubic spline for solving conservation laws system And improving available shocks in result with use of switch automatic

George K. Kostopoulos

We present an original method for the design of root-computing algorithms. The method is applicable to all numerical systems and leads to the design of algorithms for the extraction of any integer roots. It is simple and can be easily programmed into a loop where each iteration produces one digit of the sought-after root extending into the fractional part of the root.

Metric properties of Denjoy's canonical continued fraction expansion

Cor Kraaikamp

Based on joint work with **Marius Iosifescu**.
It is quite well-known–see e.g. [1]–that every real number x can be written as a regular continued fraction (RCF) expansion

$$x = a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \ddots}} = [a_0; a_1, a_2, \ldots], \tag{10}$$

where $a_0 = \lfloor x \rfloor$, and the a_i, $i \in \mathbb{N}_+ := \{1, 2, \ldots\}$, are positive integers. In case x is irrational, the expansion (10) is unique and infinite; in case x is rational, (10) is finite and two possible expansions exist.

Apart from the RCF expansion (10), very many other continued fraction expansions of x exist, such as the nearest integer continued fraction expansion (NICF), the singular continued fraction expansion, Nakada's α-expansions ([5]). All these expansions, and the relations between have been studied thoroughly; see e.h. [4, 1].

One such expansion, Denjoy's *canonical continued fraction expansion*, has hardly attracted any attention. Let x be a real number with RCF expansion (10), and let $d_0 \in \mathbb{Z}$ be such, that $d_0 \leq x$. Then it was shown in [2] that x has an expansion of the form

$$x = [d_0; (0,1)^{a_0 - d_0}, (1,0)^{a_1 - 1}, 1, (1,0)^{a_2 - 1}, 1, \ldots],$$

where $(1,0)^k$ is an abbreviation for the string $1,0,1,0,\ldots,1,0$ consisting of k pairs $(0,1)$, which is empty if $k=0$. Such a continued fraction expansion is called a *canonical continued fraction* (CCF) *expansion* of x.

In [2], a map $T_d : [0,\infty) \to [0,\infty)$ is studied, which 'generates' a unique CCF expansion for every $x > 0$. This 'Denjoy-map' is given by $T_d(0) = 0$, and

$$T_d(x) = \frac{1}{x} - d(x), \qquad x > 0,$$

where

$$d(x) = \begin{cases} 1, & \text{if } x \in (0,1], \\ 0, & \text{if } x \in (1,\infty). \end{cases}$$

The digits (or partial quotients) $d_n = d_n(x)$ of $x > 0$ are now given for $n \in \mathbb{N}_+$ by

$$d_n(x) = d\left(T_d^{n-1}(x)\right) \quad \text{whenever } T_d^{n-1}(x) > 0.$$

In this talk the relation between the RCF and the CCF is studied, and it is shown how techniques used for other continued fraction expansions (such as the NICF), can be used to find the ergodic sustem "undelying" the CCF-expansion.

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Self-dual codes and 3-manifolds

Matthias Kreck

In this talk I will explain the concept of self-dual binary codes and the fascinating relation to unimodular bilinear forms. Then I will attach to a 3-manifold with involution with finitely many fixed points a self-dual binary code. The obvious questions are which binary self-dual codes occur this way and are there natural constructions of such codes. Particularly interesting binary self-dual codes are the doubly even codes. I will explain a condition involving 4-manifolds and spin structures related to this question. This is joint work with Volker Puppe.

Generalized fuzzy almost continuity

Biliana Krsteska

Based on a joint work with **Yong Chan Kim**.

Keywords: Fuzzy topology, generalized fuzzy almost regular continuous mapping, generalized fuzzy almost α-continuous mapping, generalized fuzzy almost regular continuous mapping, fuzzy normal space, fuzzy regular spaces

Some new classes of generalized fuzzy mappings in Chang's fuzzy topological space have been introduced and studied. Their properties and relationships with other early defined classes of generalized fuzzy mappings have been investigated. Several preservation properties and some characterizations concerning fuzzy separation axioms have been considered.

Difference counterpart of Nevanlinna theory

Ilpo Laine

Recently, Halburd–Korhonen [HK] and Chiang–Feng [CF1], [CF2] developed Nevanlinna theory for difference expressions. Given a meromorphic function $f(z)$ and its shift $f(z+c)$, they worked out estimates for $f(z+c)/f(z)$, both pointwise and integrated in the sense of the proximity function $m(r, f(z+c)/f(z))$, corresponding to the usual logarithmic derivative estimates. As a consequence, difference counterparts of the familiar Clunie and Mohon'ko lemmas follow. We also consider generalizations of the Clunie lemma [LY], see also [YY], obtained by using Hölder type reasoning instead of the classical argument. Considering applications of the difference Nevanlinna theory, we shortly review consequences to meromorphic solutions of complex difference equations.

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Multiobjective control of time-discrete systems and dynamic games on networks

Ludovic Dan Lemle

Keywords: topology of uniform convergence on compact subsets; C_0-semigroups; adjoint C_0-semigroups; Hille-Yosida theorem.

Let $(\mathcal{X}, \| \, . \, \|)$ be a Banach space. In general, for a C_0-semigroup $\{T(t)\}_{t\geq 0}$ on $(\mathcal{X}, \| \, . \, \|)$, its adjoint semigroup $\{T^*(t)\}_{t\geq 0}$ is no longer strongly continuous on the dual space $(\mathcal{X}^*, \| \, . \, \|^*)$. In [1] Wu and Zhang consider on $\mathcal{X}^*$ the topology of uniform convergence on compact subsets of $(\mathcal{X}, \| \, . \, \|)$ denoted by $\mathcal{C}(\mathcal{X}^*, \mathcal{X})$, for which the usual semigroups in literature becomes C_0-semigroups.
The main purpose of this talk is to prove a much easier characterization of the generator of a C_0-semigroup on $(\mathcal{X}^*, \mathcal{C}(\mathcal{X}^*, \mathcal{X}))$.

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Moment problems

Luminiţa Lemnete Ninulescu

In this note,we study two generalizations of classical moment problems.The first generalization is of a scalar complex moment problem.It is given a multisequence $\{S_\alpha = (s_{ij}(\alpha))\}$, $1 \leq i, j \leq k,$, $\alpha \in \mathbf{Z}_+^2$ of k-dimensional matrices $S_\alpha \in M(k, \mathbf{C}), k \geq 1$;when does there exist a nonnegative (k, k) matrix $\Lambda = (\lambda_{ij})_{1\leq i,j\leq k}$ of complex Borel measures on $D_1 = \{z, |z| \leq 1\}$ such that

$$(s_{ij}(\alpha_1, \alpha_2))_{ij} = (\int_{D_1} z^{\alpha_1} \overline{z}^{\alpha_2} d\lambda_{ij}(z))_{ij}$$

The second generalization is of an operator valued moment problem.This is:given a multisequence $\{S_\alpha = (s_{ij}(\alpha))\}$,$1 \leq i, j \leq k, \alpha \in \mathbf{Z}_+^2$,$\{S_\alpha \in M(k, \mathbf{B}(\mathbf{H})\}$with entries bounded hermitian operators acting on a Hilbert space $\mathbf{H}$, $S_\alpha = (\Gamma_\alpha^{ij}$,$1 \leq i, j \leq k$,$\forall \alpha \in \mathbf{Z}_+^2$,$\Gamma_{00}^{ij} = Id$,when does exist a nonnegative (k, k) matrix $\Lambda = (F^{ij})$ of positive operator valued measure on D_1 such that

$$(\Gamma_{\alpha_1,\alpha_2}^{ij})_{ij} = (\int_{D_1} z^{\alpha_1} \overline{z}_{\alpha_2}))_{ij}, 1 \leq ij \leq k$$

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A survey on dynamical and asymptotical structures in numeration

Pierre Liardet

This survey is devoted to numeration systems and related structures. Starting from a fundamental request that is to facilitate symbolic arithmetical computations or to produce good approximations of numbers in a wide sense, we first exhibit the most current systems under present investigations. Then, we develop some tools related to probability and harmonic analysis to obtain more sophisticated statistical and ergodic properties of various objects issuing from number systems. Applications on progress should be given, involving both pseudo-random number generators and well uniformly distributed sequences in unit hyper-cubes having at present the best known discrepancy estimate. Recent results on the weak convergence of processes defined in terms of arithmetical additive sequences should be also investigated.

Decay estimates for Fourier transforms of densities defined on surfaces

Otto Liess

Let $S \subset \mathbb{R}^n$ be a surface which is assumed smooth except a finite number of singular points. We are interested in decay estimates of integrals of form

$$I(x) = \int_S \exp\left[-i\langle x, \xi\rangle\right] u(\xi) d\sigma(\xi), x \in \mathbb{R}^n.$$

Here the integral is in the smooth part of the surface, $u : S \to \mathbb{R}$ is a smooth function, and $d\sigma$ is the surface element on S. We are interested in decay estimates for $|\xi| \to \infty$ and in applications to decay estimates for solutions of systems of hyperbolic equations which appear in crystal theory. The main problems in the estimates come from the singular points in the surface and from the set of points where the total curvature of the surface is vanishing. Part of the talk is addressed to a study of the geometric properties of the characterstic surface of the system of crystal acoustics in a number of situations.

Arithmetic properties of differential operators on algebraic curves

Răzvan Liţcanu

Based on joint work with **Teresa Crespo**.

This presentation is devoted to differential operators on algebraic curves C, with a full set of algebraic solutions (that is, solutions that are algebraic over the function field $K(C)$). A conjecture of Dwork states the following (weak form): Let V be the set of all operators of order n on $\mathbb{P}^1$ with coefficients in $\overline{\mathbb{Q}}(x)$, with fixed sets of singular points S and local exponents E. Let V_1 be the subset of V corresponding to equations with a full set of algebraic solutions. Then V_1 corresponds to an algebraic subset of V.

Let L be a second order liniar differential operator, defined on a complex algebraic curve C. It is a classical fact that it has a full set of algebraic solutions if and only if the monodromy group of L is finite or, moreover, if and only if the wronskian is finite and the projective monodromy is finite. The case when C is the projective line $\mathbb{P}^1$ and L has only three singular points was solved by Schwarz and Klein, who proved that such operators are hypergeometric and gave an explicit list (the "Basic Schwarz List").

In the general case, the operator L can be obtained as a pull-back of a hypergeometric operator in the "Basic Schwarz List" via a rational function [1]. If there is no apparent singularity, this rational function has only three branching points. One can then use the topological and combinatorial machinery of Belyi functions and dessins d'enfants for classifying such operators. Using this method, we completely classified the Lamé operators with algebraic solutions [2], [3].

For operators having apparent singularities, one can use a classical argument of Ritter, as well as ideas of Shiga, Tsutsui and Wolfart [4]. Such an operator can be put in a family $\{L_t \,/\, t \in [0,1]\}$ with isomonodromic properties, such that L_0 and L_1 have less apparent singularities (or none).

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Time periodic solutions to a nonlinear second-order partial differential equations

Rodica Luca Tudorache

We consider the second-order partial differential system

$$\text{(S)} \qquad \begin{cases} u_t(t,x) + v_{xx}(t,x) + \alpha(x,u) = f(t,x) \\ v_t(t,x) - u_{xx}(t,x) + \beta(x,v) = g(t,x), \end{cases}$$

$$t > 0, \quad x > 0,$$

with the boundary condition

$$\text{(BC)} \qquad \begin{pmatrix} \operatorname{col}(-u_x(t,0), u(t,0)) \\ S(w'(t)) \end{pmatrix} \in -G \begin{pmatrix} \operatorname{col}(v(t,0), v_x(t,0)) \\ w(t) \end{pmatrix} + B(t), \quad t > 0.$$

The unknown functions are u, $v : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}^n$ from (S), and $w : \mathbb{R}_+ \to \mathbb{R}^m$ from (BC). The functions α and β are of the form

$$\alpha(x,u) = \operatorname{col}(\alpha_1(x,u_1), \ldots, \alpha_n(x,u_n)),$$

$$\beta(x,v) = \operatorname{col}(\beta_1(x,v_1), \ldots, \beta_n(x,v_n)),$$

S is a m-positive diagonal matrix, G is an operator in the space $\mathbb{R}^{2n+m}$, which satisfy some assumptions and $B(t) = \operatorname{col}(b_1(t), \ldots, b_{2n+m}(t)) \in \mathbb{R}^{2n+m}$, for all $t > 0$.

The existence, uniqueness and asymptotic behaviour of the strong and weak solutions of the above problem with the initial data

$$\text{(IC)} \qquad \begin{cases} u(0,x) = u_0(x), \quad v(0,x) = v_0(x), \quad x > 0, \\ w(0) = w_0, \end{cases}$$

have been investigated in [1]. In this work we present some existence results for the time periodic solutions of the problem (S)+(BC), in two different cases $B(t) = \text{const.}$ and $B(t) \neq \text{const.}$ We use several results from the theory of monotone operators and nonlinear evolution equations of monotone type (see the monographs [2], [3]), and also a fixed point theorem due to F.E. Browder and W.V. Petryshyn (see [4]).

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On endomorphism semigroups of direct products of commutative Moufang loops

N. T. Lupashco

Based on joint work with **N. Sandu**

It is proved that the endomorphism semigroups of commutative Moufang loops, which possess a direct decomposition into its subloops, is isomorphic to a semigroup of M, M-matrices.

The commutative Moufang loops (CML) is the most studied class of non-associative loops. Such loops are characterized by identity $x^2 \cdot yz = xy \cdot xz$. Let the CML $Q, \cdot)$ decompose into a direct product of its subloops

$$Q = Q_1 \times Q_2 \times \ldots \times Q_r \times \ldots . \tag{1}$$

If $u \in Q_i, v \in Q_j, w \in Q_t$ and at least two of the three indices i, j, t are distinct, then from definition of direct product it follows that $uv \cdot w = u \cdot vw$. Moreover, we consider that the product $a = (a_1 a_2 \ldots a_n)_\alpha$ with a certain distribution of parentheses α, where $a_i \in Q_j$. If $(a_1 a_2 \ldots a_n)_\alpha$ contains at most two factors a_i, a_j $(i, j = 1, \ldots, n)$, belonging to the same component Q_k, then the value of expression $(a_1 a_2 \ldots a_n)_\alpha$ doesn't change for any other distribution of parentheses β and for any rearrangement of factors $a_1, a_2, \ldots, a_n$. We defined this property of expression a as *component associative.*

In decomposition (1) each element $a \in Q$ has a form $a = \prod_{i=1}^{t} a_i$. If φ is an endomorphism of CML Q then $\varphi a = \prod_{i=1}^{t} \varphi a_i$. Hence, to define the endomorphism φ it is sufficient to indicate how φ acts upon the components of Q_i. If $\varphi a_i = \prod a_{ki}$, then we write $\varphi a_i = \prod \varphi_{ki} a_i$. As the element $a_{ki} = \varphi_{ki} a_i$ is unambiguously defined by element a_i and mapping φ, then φ_{ki} gives an unambiguous mapping of Q_i in Q_k. It is proved that φ_{ik} is a homomorphism of CML Q_i into a CML Q_k.

Lemma. *Let the CML Q decompose into a direct product (1) and let φ be an endomorphism if CML Q. Then Q_i is component associative regarding its subloops $\varphi_{ik} Q_k$.*

Now let us introduce the following notions by analogy with [1]. Let M, N are two sets and let L be a field. We will consider all types of possible functions $A, B, \ldots$ of two variables, defined on the cartesian product $A \times B$ with its values in L. We defined such functions as *M, N-matrices* on L. By $a_{\alpha\beta}$ we denote the value of the functions for arguments α and β : $A(\alpha, \beta) = a_{\alpha\beta}$ and the function itself will be denoted as $(a_{\alpha\beta})$. If $A = (a_{\alpha\beta})$ is a matrices, then fixing the first argument into it, we get a one-variable function - row $\overline{a}_\alpha = (\overline{a}_\alpha(\beta))$ and similarly, fixing the second argument, we'll get column $\overline{a}^\beta = (\overline{a}^\beta(\alpha))$. We call matrices A as *finite-rowed* if all its rows contain only a finite number of non-null values. The matrices with finite columns are defined similarly. Let now A be M, N-matrices and B be N, K-matrices, where A is finite-rowed or B is finite-columned. Then its product $C = AB$ is a M, K-matrices, determined by formulae

$$c_{\alpha\beta} = \sum_{\gamma} a_{\alpha\gamma} b_{\gamma\beta},$$

where $\alpha \in M$, $\gamma \in N$, $\beta \in K$ and we consider that by summing up the infinite number of nulls we will get again a null. Finally, the function $\overline{a} = (\overline{a}_{\alpha\alpha})$ is the principal diagonal of M, M-matrices $(a_{\alpha\beta})$.

Theorem. *The endomorphism semigroup of CML, which possess a direct decomposition into its subloops, is isomorphic to a semigroup of M, M-matrices with the condition of component associativity, indicated in Lemma.*

Corollary. *The endomorphism semigroup of periodic CML is isomorphic to a semigroup of diagonal M, M-matrices.*

We have also proved here the relationship between the direct decompositions of CML Q and its endomorphism semigroups $E(Q)$, as well its M, N- matrices that represent these endomorphisms.

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Some remarks on an elementary inequality equivalent to Riemann hypothesis

Cezar Lupu

In this note we shall provide some remarks concerning an elementary inequality equivalent to the Riemann hypothesis. The inequality stated by Jeffrey Lagarias, says that Riemann hypothesis is equivalent to

$$\sigma(n) \leq H_n + \exp H_n \log H_n, \forall n \geq 1,$$

where $\sigma(n)$ is the sum-divisor function, i.e. $\sigma(n) = \sum_{d/n} d$ and $H_n = 1 + \frac{1}{2} + \ldots + \frac{1}{n}$. In fact, we shall make some remarks to another inequality equivalent to Riemann hypothesis found by Matsumobo Keneko namely

$$\sigma(n) < \exp H_n \log H_n, \forall n \geq 61.$$

On local maximum stable sets of the corona of a path with complete graphs

Eugen Mandrescu

Based on joint work with **Vadim E. Levit**.

S is a *local maximum stable set* of G, and we write $S \in \Psi(G)$, if S is a maximum stable set of the subgraph induced by $S \cup N(S)$, where $N(S)$ is the neighborhood of S. In [1] we have proved that $\Psi(G)$ is a greedoid for every forest G. The case of bipartite graphs was analyzed in [2]. In this presentation we demonstrate that if G is the corona of a path with complete graphs, i.e., it is obtained by joining each vertex of a chordless path P_n to all the vertices of a complete graph, then $\Psi(G)$ forms a greedoid.

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Relationship between vector field valued semigroup automata and random processes

Violeta Manevska

Key words and phrases: vector valued semigroup automata, random process, matrix of probability

The aim of the talk is to examine a vector valued semigroup automata as a discrete random process. There will be established a relationship between a transition function from one to other state of vector valued semigroup automata and a appropriate matrix probabilities for transition from one to other state. Then, using this matrix probabilities will be examined properties

of vector valued semigroup automata. Also it will be discussed the ergodicity on the set of states over the matrix probability with their properties and the probability of all states after n-transitions which forms a matrix of final probabilities.

Submanifolds of (para-)quaternionic manifolds

Stefano Marchiafava

We give a survey on results and problems concerning submanifolds which are interesting to consider in a quaternionic or para-quaternionic manifold.

Work done under the program of G.N.S.A.G.A. of INdAM; partially supported by PRIN Cofin05 (Italy)

A class of unbounded operators and Schur multipliers for them

Liviu Marcoci

Based on joint work with **Anca Marcoci.**

We introduce the space $Bw(l^2)$ of linear (unbounded) operators on l^2 which map decreasing sequences from l^2 into sequences from l^2 and we find some classes of operators belonging either to $Bw(l^2)$ or to the space of all Schur multipliers on $Bw(l^2)$. For instance we show that the space $B(l^2)$ of all bounded operators on l^2 is contained in the space of all Schur multipliers on $Bw(l^2)$.

Proving and programming

Solomon Marcus

Based on joint work with **C. Calude** and **E. Calude**.

There is a strong analogy between proving theorems in mathematics and writing programs in computer science. This paper is devoted to an analysis, from the perspective of this analogy, of proof in mathematics. We will argue that:

1. Theorems (in mathematics) correspond to algorithms and not programs (in computer science); algorithms are subject to mathematical proofs (for example for correctness).

2. The role of proof in mathematical modelling is very little: adequacy is the main issue.

3. Programs (in computer science) correspond to mathematical models. They are not subject to proofs, but to an adequacy analysis; in this type of analysis, some proofs may appear. Correctness proofs in computer science (if any) are not cost-effective.

4. Rigour in programming is superior to rigour in mathematical proofs.

5. Programming gives mathematics a new form of understanding.

6. Although the Hilbertian notion of proof has few chances to change, future proofs will be of various types, will play different roles, and their truth will be checked differently.

A BOUNDARY ELEMENT ITERATIVE ALGORITHM FOR THE TEMPERATURE RECONSTRUCTION IN PIECEWISE SMOOTH DOMAINS FROM INCOMPLETE BOUNDARY DATA

Liviu Marin

Based on joint work with **Tomas Johansson**.

Consider a plane body occupying the domain $\Omega \subset \mathbb{R}^2$ with boundary $\Gamma = \partial\Omega$ which is smooth except for a finite number of corner points, and assume that only a part of the boundary, namely $\Gamma_0 \subset \Gamma$, is accessible for measurements. We wish to determine the temperature field u inside Ω from measurements of the temperature and heat flux on Γ_0, i.e.

$$\mathcal{L}u = 0 \quad \text{in} \quad \Omega, \qquad u = \varphi \quad \text{on} \quad \Gamma_0, \qquad \mathcal{N}u = \psi \quad \text{on} \quad \Gamma_0. \tag{11}$$

Here $\mathcal{L}$ denotes a linear second-order elliptic partial differential operator, whilst $\mathcal{N}$ is the co-normal derivative. The operator $\mathcal{L}$ can for example be the stationary heat operator $\mathcal{L}u = \nabla \cdot (k\nabla u)$, where k is the thermal conductivity. A general linear second-order elliptic operator corresponds to the reconstruction of the temperature in a non-homogeneous and non-isotropic medium. There can be an energy source term present in (11), but due to the linearity of the problem it is sufficient to consider the case when this term is zero.

In this talk, the method of Johansson [1] is extended to two-dimensional corner domains. This involves extending the ideas of Hào and Lesnic [2] to a general second-order elliptic operator in weighted spaces. To do so, a mixed problem for the operator $\mathcal{L}$ with the measured temperature ψ on Γ_0 and a control temperature η on $\Gamma_1 = \Gamma \setminus \overline{\Gamma}_0$ is introduced. The functional measuring the discrepancy between the restriction of the solution of this mixed problem to Γ_0 and the measured temperature φ is minimised. It is proven that this mixed problem is well-posed in a weighted L^2-space, where the weight is the product of the functions $r_j^{\beta_j - 2}$ with r_j equal to the distance to the j-th corner point and β_j is a real number in a certain interval. Furthermore, it can be shown that the above functional is twice Fréchet differentiable and strictly convex, whilst the Cauchy problem is solvable for almost all data, provided that they belong to appropriate weighted spaces. In order to minimize the discrepancy functional in a stable way, a minimal error method is employed and the convergence of the method is proven. Numerical results are presented based on this procedure, in conjunction with a stopping rule for noisy Cauchy data, by using the boundary element method, see e.g. Brebbia *et al.* [3]. The numerical results show that the procedure gives accurate and stable numerical approximations in relatively few iterations.

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A global variational principle associated with gradient stochastic control systems

Marinela Marinescu

Stochastic control systems are considered using an extended gradient stochastic control system (GSCS) and weak feedback controls. The local variational principle and dynamic programming equations are substituted by o global variational principle avoiding final constraints and martingale representations problems involved in the corresponding adjoint system.

A complet stochastic control system is driven by a finite set of controlled vector fields $\{f_0(x,u), f_1(x,u), ..., f_m(x,u)\}$ where f_0 is the drift and $f_1, f_2, ..., f_m$ are the diffusion vector fields . As far as the admissible feedback controls are smooth mappings of the state $x \in \mathbb{R}^n$ we may and do associate the corresponding Kolmogorov equation or Hamilton-Jacobi-Bellman (HJB) equations describing the evolution of a certain value function. On the ather hand, using open loop controls, a stochastic Pontryagin principle is valid including final constraints and martingal representation problems regarding the adjoint system.

There is a significant and well appreciated background of this subject and we shall only mention the references [1, 3, 5]. The extended GSCS, the admissible class of controls (weak feedback controls) and even the functionals are defined according to a smooth coordinate mapping $x = G(p)[y], x, y \in \mathbb{R}^n$, for each $p \in B(0, 2\rho) \subset \mathbb{R}^M$, such that the controlled vector fields driving $x \in \mathbb{R}^n$ can be written as a Lie derivative of $x = G(p)[x_0], p \in B(0, 2\rho)$. It reveals an extended state variable $(x, p) \in \mathbb{R}^n \times \mathbb{R}^M$ and the construction of the smooth mappings $x = G(p)[y]$, and its inverse $y = [G(p)]^{-1}(x) := \psi(p, x)$ uses a finite type Lie algebra generated by the original controlled vector fields.

The basic idea of this analysis appeared in [4] and it was applied to some SPDE in [2] considering finite dimensional Lie algebras. Using weak feedback controls ($u \in W_{ad}$) as Lipschitz functions of $p \in B(0, 2p) \subset \mathbb{R}^M$, the associated value function can be described using stochastic Hamilton-Jacobi equations driven by smooth random vector fields with respect to $x \in \mathbb{R}^n$. Though the diffusion vector fields are depending on control $u \in W_{ad}$, we get a description af the optimality by means of a global variational principle (GVP). The main results are given in Theorem 2 which contains the construction of The control variable $u \in \mathbb{R}^d$. The algoritm contained in Section 3 relies upon GVP given in Theorem1.

Well-posedness for a degenerate diffusion equation with a nonlinear operator

Gabriela Marinoschi

We study a degenerate nonlinear variational inequality which can be reduced to a multivalued inclusion by an appropriate change of the unknown function. We establish existence, uniqueness and regularity results. An application arising in the theory of water diffusion in porous media is discussed as an example.

Splitting numerical method for the solution of one-dimensional transport equation

Olga Martin

The main problem in the nuclear reactor theory is to find the neutrons distribution, hence its density. This is the solution of an integral-differential equation named the transport equation.

In this talk, we provide an approximate solution of stationary transport equation, using a plan-parallel geometry and the variational form of the integral identity method. Numerical results are presented to prove the accuracy and computational efficiency of the proposed algorithm.

Bestvina-Brady groups and arrangements of hypersurfaces

Daniel Matei

We investigate finiteness properties of fundamental groups of complements to algebraic hypersurfaces in a projective space. We show that, among these finitely presented groups, we may find, for any $n \geq 2$, groups G which have an Eilenberg-MacLane space with finite n-skeleton, but such that the homology group $H_{n+1}(G)$ is infinitely generated. This is achieved by realizing the quasi-projective Bestvina-Brady, classified by Dimca-Papadima-Suciu, as fundamental groups of hypersurface complements. To a finite simplicial graph one associates a right-angled Artin group having a presentation with one generator for each vertex of the graph, and one commuting relation between two generators if the corresponding vertices form an edge. The Bestvina-Brady group is then the kernel of the homomorphism from the Artin group to the integers sending all generators to one.

Distortion of harmonic type mappings in the space

Miodrag Mateljević

In first part of our communication, we plan to study the distortion of harmonic functions and in particular we show that quasiconformal harmonic type mappings are the quasi-isometries with respect to quasihyperbolic geometry; and in connection with this we consider versions of the Kobe theorem for quasiregular harmonic mappings.

Second part is devoted to harmonic maps and their connections with boundary data. We study equivalent norms on Lipschitz-type spaces of harmonic type mappings and subharmonic behaviour of smooth functions. We also point some differences between the theory in the plane and the space.

Galois theory of differential equations

Heinrich Matzat

This lecture gives an introduction to the Galois theory of linear differential equations in characteristic zero and in positive characteristic. Relations between these two theories leed to new algebraicity criteria for solutions of differential equations in the classical case.

From mathematical physics to mechanical engineering

Gerard A. Maugin

In this contribution we emphasize and exemplify the reciprocal fertilization of good field theory and engineering applications. Thus the theory of configurational - or material - forces that we developed with others in the period 1990-2005 smells good of its mathematical-physical

origin. For instance, the newly entertained relationship with computational techniques based on mathematical formulations akin to general conservation laws (weak form such as the principle of virtual power, classical volume balance laws, notion of ńămaterialăż force in computations) is not so surprising in the framework of material-configurational force mechanics. In fact, both the deep physical meaning and the practical usefulness of the critical expressions (e.g., of thermodynamical driving forces) obtained within this framework stem from the intimate relationship built from the start between these expressions and a field-theoretical invariance or lack of invariance. Symmetries and their eventual breaking are the physically most profound and intellectually puzzling tenets in modern physics. That engineering applications fit into this general picture is a comforting view of the unity of science, whether purely theoretical or applied to modern engineering.

Boundary value problems for nonlinear perturbations of singular or bounded ϕ-laplacians

Jean Mawhin

We survey some existence and multiplicity results recently recently obtained, in collaboration with C. Bereanu, for nonlinear differential equations of the form

$$(\phi(u'))' = f(t, u, u') \tag{12}$$

submitted to various boundary conditions.

Motivated by the special cases $\phi(s) = \frac{s}{\sqrt{1-s^2}}$ and $\phi(s) = \frac{s}{\sqrt{1+s^2}}$ respectively related to curvature and relativistic problems, the emphasis is put in (12) on homeomorphisms $\phi :]-a, a[\to \mathbb{R}$ (singular case) and $\phi : \mathbb{R} \to]-a, a[$ (bounded case).

Cohomology jumping loci and formality properties of nilpotent groups

Daniela Anca Măcinic

We examine the cohomology jumping loci of finitely generated nilpotent groups. We compute their characteristic varieties and relate resonance with partial formality properties. For instance, we show that for finitely generated nilpotent groups that are k-formal, the resonance varieties are trivial up to degree k; in the case of fundamental groups of complements to complex hyperplane arrangements, we note that triviality of resonance in degree $k = 1$ actually detects nilpotence.

We also show that the cohomology ring of a nilpotent k-formal group is generated in degree one, up to degree $k + 1$; this criterion is sufficient for two-step nilpotent groups to be k-formal. As an application, we estimate the non-triviality of higher homotopy groups, for compact Kahler manifolds having a Heisenberg-type fundamental group.

The Ghoussoub-Preiss theorem

Irina Meghea

The Ghoussoub-Preiss theorem (*I. Ekeland, Convexity methods in hamiltonian mechanics*, Springer Verlag, 1990):

" Let X be Banach space and $\varphi : X \to \mathbb{R}$ continuous Gâteaux differentiable with $\varphi'_w : X \to X^*$ continuous from the norm topology to the $*$-weak topology. One takes y_0, y_1 distinct points from X and let be $\Gamma := \{\gamma \in C([0,1]; X) \ : \ \gamma(0) = y_0, \ \gamma(1) = y_1\}$ and $c := \inf\limits_{\gamma\in\Gamma} \sup\limits_{[0,1]}(\varphi \circ \gamma)$. If there is a closed subset F such that $F \cap \{x \in X \ : \ \varphi(x) \geq c\}$ is nonempty and separates y_0, y_1, then there is $(x_n)_{n\geq 1}$ in X with the properties

$$\lim_{n\to\infty} d(x_n, F) = 0, \ \lim_{n\to\infty} \varphi(x_n) = c, \ \lim_{n\to\infty} (1 + \|x_n\|)\varphi'_w(x_n) = 0."$$

has an erroneous proof. The author, with a difficult proof, has retrieved this theorem, but replacing the property " $\lim\limits_{n\to\infty} (1+\|x_n\|)\varphi'_w(x_n) = 0$" by " $\lim\limits_{n\to\infty} (1+\|x_n\|)^{-1}\varphi'_w(x_n) = 0$", obviously weaker.

As result are presented two statements: a variant of Mountain pass theorem and a Mountain pass point theorem.

About integrability of unbound functions in the mechanical modeling

Marcel Migdalovici

The research in this talk is focuses on the study of a possible structure of the unbounded points for the integrable real function with real variable. We analyze the behavior of the function around the unbounded points of the definition domain which permit us to define the notions of the integrability and of the defined integral for the unbounded functions on the definition domain. About the possible structure of unbounded points of the function we have investigated the possible complexity for the set of unbounded points of the real function with real variable such that these function to be integrable on the definition domain, in other words, we have investigated how much can be unbounded an integrable function. Concerning the definition of the integrability notion for the function with unbounded points, the author introduce the notion of generalized Riemann integrability and the generalized Riemann integral of unbounded functions, where the finite divisions of definition domain for Riemann integral are substituted by the numerable divisions and where the finite integral sums for Riemann integral are substituted by the simple integral series, double integral series,... ,etc, that depends by the accumulation points structure of the set of unbounded points on the definition domain of analyzed function. The set of unbounded points of any function analyzed is considered as a subset of the any division points set of the definition domain of function with unbounded points. Are described some results referred to generalized Riemann integrability and generalized Riemann integral, introduced by the author. Are discussed also the possibilities of application of this research in the mechanical modeling.

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Pure powers among Fibonacci numbers

Maurice Mignotte

Keywords: Diophantine equations, Frey curves, level-lowering, linear forms in logarithms, Thue equations, Fibonacci numbers, Lucas numbers, exponential Diophantine equations

The talk is based on a joint paper (entitled *Classical and Modular Approaches to Exponential Diophantine Equations*) with **Yann Bugeaud** and **Samir Siksek**.

We combine the classical approach to exponential Diophantine equations (linear forms in logarithms, Thue equations, etc.) with a modular approach based on some of the ideas of the proof of Fermat's Last Theorem. We give new improved bounds for linear forms in three logarithms. We also apply a combination of classical techniques with the modular approach. Using these techniques, we have shown in the first paper that the only perfect powers in the Fibonacci sequence are 0, 1, 8 and 144 and the only perfect powers in the Lucas sequence are 1 and 4.

The proof uses what may be called the "modular" approach which has since been applied to many other Diophantine equations; mostly – though not exclusively – of the form

$$ax^p + by^p = cz^p, \qquad ax^p + by^p = cz^2, \qquad ax^p + by^p = cz^3, \ \ldots \quad (p \text{ prime}).$$

The strategy of the modular approach is to associate to a putative solution of such a Diophantine equation an elliptic curve, called a Frey curve, in a way that the discriminant is a p-th power up to some small factor. Next apply Ribet's level-lowering theorem to show that the Galois representation on the p-torsion of the Frey curve arises from a newform of weight 2 and a fairly small level N say. If there are no such newforms then there are no non-trivial solutions to the original Diophantine equation (a solution is trivial if the corresponding Frey curve is singular). Occasionally, even when one has newforms of the predicted level there is still a possibility of somehow showing that it is incompatible with the original Galois representation though there does not seem to be a general strategy.

A fact that has been underexploited is that the modular approach yields a tremendous amount of local information about the solutions of the Diophantine equations. For equations as above it is perhaps difficult to exploit this information successfully since we neither know of a bound for the exponent p, nor for the variables x, y, z. This suggests that the modular approach should be applied to exponential Diophantine equations; for example, equations of the form

$$ax^p + by^p = c, \qquad ax^2 + b = cy^p, \ldots \quad (p \text{ prime}).$$

For such equations, Baker's theory of linear forms in logarithms gives bounds for both the exponent p and the variables x, y. This approach through linear forms in logarithms and Thue equations, has undergone through substantial refinements, though it still often yields bounds that can only be described as astronomical.

Metric properties and dynamics for conformal non-invertible maps

Eugen Mihăilescu

The dynamics of non-invertible maps is very different than that for diffeomorphisms. We will study several cases of fractal sets constructed from iterations of conformal non-invertible maps, for instance skew products or holomorphic maps on complex projective spaces.

In this setting I will introduce some new invariants, and equilibrium measures for dynamically interesting potentials.

We will give as well results about Hausdorff dimension for such fractal sets, extending to this case theorems by Bowen, Viana, and also outlining new phenomena.

Primality proving, theoretical and practical

Preda Mihăilescu

The problem of proving whether a given integer n is prime or not has been recently solved, in 2002, by Agrawal, Kayal and Saxena to the best of theoretical expectations. It was proved that the problem can be solved in deterministic *polynomial* time - i.e. the time required for a proof is a polynomial in the length $m = \log(n)$ of the input. Furthermore, the algorithm of Agrawal, Kayal and Saxena (AKS) works unconditionally on any input n. The proof used initially an ineffective bound, yet further improvements of Lenstra and Pomerance led to an effective proof that the time can be reduced to $O(m^{6+\varepsilon})$.

From a practical point of view, the algorithm AKS can not be used for very large imputs, due not only to the large exponent of m but also to high memory space requirements. We present also a recent algorithm, which is not deterministic but has expected run - time of $O(m^{3+\varepsilon})$ and can thus be most effective in practice.

We shall present for a large mathematical public the various aspects of theoretical and practical primality proving and give an overview of the ideas of the two algorithms above mentioned.

Generalized Gromov-Witten invariants on Grassmannians

Leonardo C. Mihalcea

The (3-pointed, genus 0) Gromov-Witten (GW) invariants on a Grassmannian count rational curves of degree d intersecting three Schubert varieties in general position. Recent results of Buch-Kresch-Tamvakis realize a bijection between the curves counted by the GW invariants and points of intersection of three Schubert varieties on a two-step flag manifold, whose number is easier to compute. In joint work with Anders Buch, we extend these results to equivariant GW invariants, where no enumerative interpretation is available. I will also indicate how to further extend this result to K-theoretic Gromov-Witten invariants.

Factorization problems for finite groups

Gigel Militaru

Based on joint work with **A.L. Agore**, **A. Chirvasitu** and **B. Ion**.

We investigate two questions about bicrossed products of finite groups which we believe have the potential of being approachable for other classes of algebraic objects. The first one is the problem of classifying groups which can be written as bicrossed products of two groups of fixed isomorphism types and the second is the problem of classifying the groups which cannot be obtained as bicrossed products of smaller groups. The groups obtained as bicrossed products of two finite cyclic groups, one being of prime order, are completely described and it is shown that A_6 is never a bicrossed product.

On the modeling of high Reynolds'number fluid flows by Multiple Scales Method

Romulus Militaru

The aim of this talk is the modeling of fluid flows for high Reynolds'number and placed outside the turbulent boundary layer, using the homogeneisation of the microstructure.

We consider the Euler system of equations:

$$\begin{cases} \dfrac{\partial u^{\varepsilon}}{\partial t} + (u^{\varepsilon} \cdot \nabla)u^{\varepsilon} + \nabla p^{\varepsilon} = 0 & \text{in } \Omega \times]0,T[\\ \nabla \cdot u^{\varepsilon} = 0 & \text{in } \Omega \times]0,T[\end{cases} \tag{13}$$

where $u^{\varepsilon} = u^{\varepsilon}{}_i(x,t),\ i = \overline{1,3}$ is the velocity field and $p^{\varepsilon} = p^{\varepsilon}(x,t)$ is the pressure We postulate as initial condition

$$u^{\varepsilon}(x,0) = \underline{u^0}(x) + \underline{\omega^0}(x/\varepsilon, x) \tag{14}$$

$\varepsilon = \frac{l}{L},\ \varepsilon << 1$, a small parameter, l being the characteristic scale of turbulence and L the characteristic scale of the flow.

We assume that u^{ε} and p^{ε} admit two-scale asymptotic expansions of the form:

$$u^{\varepsilon}(x,t) \sim u^0(x,t) + \omega(y,\tau,x,t) + \varepsilon u^1(y,\tau,x,t) + ...$$

$$p^{\varepsilon}(x,t) \sim p^0(x,t) + \lambda(y,\tau,x,t) + \varepsilon p^1(y,\tau,x,t) + ...$$

where the notations are $y = \dfrac{\theta(x,t)}{\varepsilon},\ \tau = \dfrac{t}{\varepsilon}$, $\theta(x,t)$ is the inverse Lagrangian coordinates associated to the velocity u^0 and the functions ω, u^k, p^k, $k \geq 1$ are smooth and periodic in the $y - \tau$ variables.

Using the principle of singular perturbation and the hypothesis of separation of scales we arrived to a homogenized macroscopic system containing a memory term, trace of microscopic structure. Based on a constitutive relationship one relies this term with other mean quantities, simplifying the mathematical calculus of the macrostructure.

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Solar prominence models tackled via the theory of dynamical systems

Vasile Mioc

Based on joint work with **Cristina Dumitrache**.

We tackle the 2D models of Kuperus-Raadu and Kippenhahn-Schluter, extended by Malherbes and Priest, via the geometric methods of the theory of dynamical systems. We treat the models as phase portraits, and discuss them from the standpoint of mathematics only, without regard to physical significance. Some phase portraits are very intricated and we formulate questions about them. We also suggest possible phase portraits. This paper tries to draw attention to new tools, used in celestial mechanics, as useful for the solar physics, too, and for a suitable interdisciplinary cooperation.

Considerations on a boosting algorithm

Maria Miroiu

Key words and phrases: Statistical classification, boosting algorithms.

A Boosting algorithm can significantly improve the prediction performance of any single classification algorithm. In this paper we propose a generalized boosting algorithm in the context of binary classification problems, together with its properties and experimental results.

Generalized priority systems in Queueing analysis

Gheorghe Mişcoi

By Generalized Priority Systems we shall call the mathematical models of queueing systems in which the switching between priority classes need some time to be lasted. At first glance it might seem that it is enough to attribute such time losses to the process of service of the requests of certain classes and such attribution will account for switching losses. However, such issues are much more complicated from the mathematical point of view. Formalization of switchover times inevitably leads to the appearance of a wide class of new mathematical models with a range of important peculiarities. Due to the fact that the models involving switching represent generalizations (by introduction of switchover times) of the classical priority models, one can expect that the analytical results for such generalized systems contain, as particular cases, the corresponding results for the classical systems. Such a hypothesis is confirmed. We will present and discuss some examples, in particular a recurrent system of functional equations - an analog of the well-known Kendall-Takacs (1953) functional equation and an analog of Pollaczek-Khinthcin (1957) transform equation.

This work was done under support of the Scopes grant IB7321-110720 and RFFI grant 0644CRF.

Topics in invexity theory

Ştefan Mititelu

In this talk there are given necessary and sufficient conditions for the invexity of the sets. There are defined algebraic operationsfor invex sets. Some properties concerning the stationary points of the pseudoinvex functions are given. Other properties study the local minimum points of the quasiinvex functions and the relation "quasiinvex = pseudoinvex" for real functions on open invex sets.

The regularity of green potentials in non-smooth domains

Marius Mitrea

In this talk I will survey some recent developments in the area of partial differential equations with minimal smoothness assumptions. The emphasis is on the role played by singular integral operators of Calderon-Zygmund type and atomic decompositions for the classical smoothness spaces, of Hardy-Besov-Sobolev-Triebel-Lizorkin type.

The paradigm example is that of the Poisson problem for the Laplacian with Dirichlet and Neumann boundary conditions in domains satisfying uniform interior/exterior cone conditions (Lipschitz domains). For this, I will discuss some new, sharp estimates for the associated Green potentials and, in the process, provide answers to some open problems posed by C. Kenig and E. Stein.

Maximal operators in Orlicz spaces on metric measure spaces

Marcelina Mocanu

Maximal operators, a central tool in modern harmonic analysis and in the theory of PDE's, have been studied in various settings, including spaces of homogeneous type. Maximal inequalities in Orlicz spaces are used as an important approach to the theory of higher integrability of functions (see the monograph of T. Iwaniec and G. Martin, "Geometric function theory and nonlinear analysis"). We prove a necessary and sufficient condition for the boundedness of the Hardy-Littlewood maximal operator in Orlicz spaces on Ahlfors-regular metric spaces. Applications to the study of the regularity of Orlicz-Sobolev functions on metric measure spaces are also given.

Briot-Bouquet differential superordinations and sandwich theorems

Petru T. Mocanu

Based on joint work with **Sanford S. Miller**.

Briot-Bouquet differential subordinations play a prominent role in the theory of differential subordinations. We consider the dual problem of Briot-Bouquet differential superordinations. Let β and γ be complex numbers and let Ω be any set in the complex plane $\mathbb{C}$. The function p analytic in the unit disk U is said to be a *solution* of the *Briot-Bouquet differential superordination* if

$$\Omega \subset \left\{ p(z) + \frac{zp'(z)}{\beta p(z) + \gamma} : z \in U \right\}.$$

We determine properties of functions p satisfying this differential superordination and also some generalized versions of it.

In addition, for sets Ω_1 and Ω_2 in the complex plane, we determine properties of functions p satisfying a Briot-Bouquet sandwich of the form

$$\Omega_1 \subset \left\{ p(z) + \frac{zp'(z)}{\beta p(z) + \gamma} : z \in U \right\} \subset \Omega_2.$$

Generalizations of this result are also considered.

Space regularity of stochastic heat equations driven by irregular Gaussian processes

Oana Mocioalcă

We study linear stochastic evolution equations driven by various infinite-dimensional Gaussian processes, some of which are more irregular in time than fractional Brownian motion (fBm) with any Hurst parameter H, while others are comparable to fBm with $H < \frac{1}{2}$. Sharp necessary and sufficient conditions for the existence and uniqueness of solutions are presented. Specializing to stochastic heat equations on compact manifolds, especially on the unit circle, sharp Gaussian regularity results are used to determine sufficient conditions for a given fixed function to be an almost-sure modulus of continuity for the solution in space; these sufficient conditions are also proved necessary in highly irregular cases, and are nearly necessary (logarithmic corrections are given) in other cases, including the Hölder scale.

Some remarks about random graphs

Laurenţiu Modan

Cebasev inequality for random graphs, new properties for random subgraphs and new remarks about almost all random graphs will be presented using our new Boolean representing for this kind of Mathematics objects,which were introduced in our papers since 1995.

Professor Constantin Corduneanu, a life dedicated to Mathematics

Laurenţiu Modan

We dedicate some thoughts to Professor Constantin Corduneanu, one of the known Romanian mathematicians of all times, in the eve of his anniversary, which will happen during the present Congress.

Semisupervised learning using Dirichlet spaces techniques

Emil Moldoveanu

Semisupervised learning is a special form of classification. The special part of semisupervised learning is that it also uses unclassified data for a better classification. This is possible by assuming a certain structure of data. Some ideas are from graph-based methods. We propose here a new approach for using a mathematical tool which was largely developed in the last decade: Dirichlet spaces on finite sets. Some results of Kigami on Dirichlet spaces and Xiaojin Zhu on semi-supervised learning are the base of our tentative.

The connection between the stability of the differential equation and the operators norm in the stochastic case

Radu Moleriu

This work presents the stochastic perturbation of the evolution operators witch is used to obtain the solutions of the stochastic differential equations. It is given the relation between the operators norm witch appears in stochastic differential equations and the growth bound of the evolution operators in the deterministic case.

Stochastic models in risk evaluation for financial portofolio options

Radu Moleriu

Based on joint work with **P. Fărcaş**.

In this talk we study the measurement of the risk for financial portofolio option using VaR. For solving we apply the Black - Scholes method and the numerical solution given by Monte Carlo method. In the VaR evaluation we have the stochastic model for the volatility of a portofolio.

Models of exceptional compact simple Kantor triple systems

Daniel Mondoc

The history of the subject addressed in this talk goes back to the works of H. Freudenthal (1954), I.L. Kantor (1964-1970) and M. Koecher (1967-1968) who studied the construction of Lie algebras from Jordan algebras and triple systems. The construction has been extended in several ways to give 5-graded Lie algebras $\mathcal{U} = \sum_{l=-2}^{2} U_l$, starting with some nonassociative algebras or triple systems which appear as the component U_1. The concept of (ϵ, δ)-Freudenthal-Kantor triple systems covers many of these systems showing to be an important notion in the study of Jordan and Lie (super)algebras. In particular, $(-1, 1)$-Freudenthal-Kantor triple systems which coincide with the notion of Kantor triple systems, defined and studied by I.L. Kantor (1970-1973), are used in the construction of simple Lie algebras. The classical (exceptional) compact simple Kantor triple systems are connected to classical (exceptional) real simple Lie algebras. We present here models of exceptional compact simple Kantor triple systems (D. Mondoc, 2005-2007), models connected with real simple structurable algebras and the introduction of two commuting involutions on these algebras.

Exponential stability of discrete time linear equations defined by positive operators on ordered Hilbert spaces

Toader Morozan

Based on joint woirk with **V. Drăgan**.

In this talk, the problem of exponential stability of the zero state equilibrium of a discrete-time time-varying linear equation described by a sequence of linear bounded and positive operators acting on an ordered Hilbert space is investigated.

The class of linear equations considered in this work contains as particular cases linear equations described by Lyapunov operators or symmetric Stein operators as well as non-symmetric Stein operators. Such equations occur in connection with the problem of mean square exponential stability for a class of difference stochastic equations affected by independent random perturbations and Markovian jumping as well as in connection with some iterative procedures which allow us to compute global solutions of discrete time generalized symmetric or non-symmetric Riccati equations.

The exponential stability is characterized in terms of the existence of some globally defined and bounded solutions of some suitable backward affine equations (inequations, respectively) or forward affine equations (inequations, respectively).

The problem of preservation of exponential stability under some additive perturbations of sequence of operators is investigated and an estimation of stability radius is provided.

Spectral spaces and their characteristic classes

Henri Moscovici

Noncommutative geometry has the capacity of treating as geometric spaces a variety of mathematical objects which are highly singular from most other points of view. The search for an appropriate theory of characteristic classes for such spaces, defined in terms of spectral data, has led Connes to the discovery in early 1980's of cyclic cohomology. In late 1990's, work on local formulae for these classes has led Connes and myself to develop the Hopf cyclic theory. After a brief overview of the pertinent facts, my talk will focus on the most recent advances in the direction of computing characteristic classes of spectral spaces.

Calderón commutators and the Cauchy integral on Lipschitz curves, revisited

Camil Muscalu

The plan of the talk is to revisit some of the most important operators in harmonic analysis: Calderón commutators and the Cauchy integral on Lipschitz curves. We shall speak about a new proof of their L^p boundedness properties, which is conceptually simpler and avoids the usual theory of BMO functions and Carleson measures.

The irregularity of cyclic multiple planes after Zariski

Daniel Naie

Let $f(x,y)=0$ be an affine equation of a curve $B\subset\mathbb{P}^2$ and H_∞ be the line at infinity. The projective surface $S_0\subset\mathbb{P}^3$ defined by the affine equation $z^n=f(x,y)$ is called by Zariski the *n-cyclic multiple plane* associated to B and H_∞—possibly only to B if $n=\deg B$. For a given curve B, the cyclic multiple planes play an important role in the study of the fundamental group of the complement of B. At the same time they provide interesting examples of surfaces. Zariski took up the study of S_0 in the case the curve B has only nodes and cusps and answered the following question: what is the irregularity of S_0, the dimension of the vector space of global holomorphic 1-forms on a desingularization of S_0?

Zariski's Theorem. *Let B be an irreducible curve of degree b, transverse to the line at infinity H_∞ and with only nodes and cusps as singularities. Let $S_0\subset\mathbb{P}^3$ be the n-cyclic multiple plane associated to B and H_∞, and let S be a desingularization of S_0. The surface S is irregular if and only if n and b are both divisible by 6 and the linear system of curves of degree $5b/6-3$ passing through the cusps of B is superabundant. In this case,*

$$q(S)=h^1\left(\mathbb{P}^2,\mathcal{I}_Z\left(-3+\frac{5b}{6}\right)\right),$$

where Z is the support of the set of cusps.

Later on, Artal-Bartolo, Esnault, Libgober, Vaquié et others have generalised Zariski's work. We present a generalization of Zariski's Theorem to a branch curve that has arbitrary singularities and is transverse to the line at infinity bringing to the fore the theory of cyclic coverings. The irregularity will be expressed as a sum of superabundances of linear systems defined in terms of some multiplier ideals associated to the branch curve B.

Theorem. *Let B be a plane curve of degree b and let H_∞ be a line transverse to B. Let S be a desingularization of the n-cyclic multiple plane associated to B and H_∞. If $J(B,n)$ is the subset of jumping numbers of B smaller than 1 and that live in $\frac{1}{\gcd(b,n)}\mathbb{Z}$, then*

$$q(S)=\sum_{\xi\in J(B,n)}h^1(\mathbb{P}^2,\mathcal{I}_{Z(\xi B)}(-3+\xi b)),$$

where $Z(\xi B)$ is the subscheme defined by the multiplier ideal of $\xi\cdot B$.

Explicit computations of multiplier ideals are performed and some applications are presented.

Homogeneous sets and quasiconformal mappings

Raimo Näkki

A set E is said to be *homogeneous* with respect to a family $\mathcal{F}$ of mappings if for any pair of points a and b in E there is f in $\mathcal{F}$ such that $f(E)=E$ and $f(a)=b$. We will discuss homogeneous sets that are either compact or open and connected (i.e. domains) with respect

to the family $\mathcal{F}$ consisting of either conformal or quasiconformal self-mappings of the complex plane.

Computation of approximate inertial manifolds for a prey-predator model

Cristina Nartea

Key words and phrases: inertial manifold, approximate inertial manifold.

The existence of an inertial manifold and a sequence of approximate inertial manifolds for a prey-predator model is established.

It is proven that all hypothesis of an algorithm developed by Jolly, Rosa and Temam, are verified for a prey-predator model, local, near origin and thus the algorithm can be applied.

The algorithm is implemented and used to construct graphical representations of the approximate inertial manifolds and to compute errors of approximation.

Positive morphisms of quotient Hilbert spaces

Sorin Nădăban

Let H be a Hilbert space. A linear subspace $H_0 \subset H$ is said to be a Hilbert subspace of H if H_0 is a Hilbert space and the inclusion mapping $i : H_0 \to H$ is continuous. We denote by Lat(H) the family of all Hilbert subspaces of H. By a quotient Hilbert space we mean a linear space of the form H/H_0, where H is a Hilbert space and $H_0 \in Lat(H)$. L. Waelbroeck defines the notion of morphism. This is a linear mapping $T : H/H_0 \to H/H_0$ induced by an operator $T_1 \in B(H) \ : \ T_1H_0 \subset H_0$, meaning $T(x + H_0) = T_1x + H_0$. We denote by $B(H/H_0)$ the set of all morphisms from H/H_0 to H/H_0. This set can be regarded as a quotient Banach algebra A/α, where

$A = \{T_1 \in B(H) \ : \ T_1H_0 \subset H_0\}$, $\alpha = \{T_1 \in B(H) \ : \ T_1H \subset H_0\}$.

Using the Douglas's criterion, we obtain that there exists a bounded surjective operator $V : H \to H_0$ such that $(ii^*)^{1/2} = iV$. We mention that V is the unique co-isometry with the property $(ii^*)^{1/2} = iV$, $KerV = Ker(ii^*)^{1/2} = Keri^* = \overline{H_0}^{\perp}$. If H_0 is dense in H, then $V^*V = I_H$.

Further on, we will restrain our discussion to the case in which H_0 is dense in H. In this case we have an involution on $B(H/H_0)$ which comes from an involution on Banach algebra A, namely $A \ni T_1 \to \overline{T_1} = V^*T_0^*V \in A$.

As in the classic case we can define the spectrum $\sigma(T)$, the spectral radius $r(T)$, the Ptàk's function $p(T) = [r(T^*T)]^{1/2}$, the notion of self-adjoint morphism, normal morphism, etc. We will say that T is positive if T is self-adjoint morphism and $\sigma(T) \subset [0, \infty)$.

Theorem 1. *a) If* $(\forall)S \in B(H/H_0)$ *the morphism* S^*S *is positive then the involution on* $B(H/H_0)$ *is hermitian .*

b) If the involution of A is hermitian then $(\forall)S \in B(H/H_0)$ *the morphism* S^*S *is positive.*

Theorem 2. *a)* $r(T) \leq p(T)$.

b) If T, S *are self-adjoint, then* $r(TS) \leq r(T)r(S)$ *and* $r(T + S) \leq r(T) + r(S)$.

c) $p(TS) \leq p(T)P(S)$ *and* $p(T + S) \leq p(T) + p(S)$.

d) If T, S *are positive, then* $T + S$ *is positive.*

Isomorphic character tables of p-groups

Adriana Nenciu

Two character tables of finite groups are isomorphic if there exist a bijection for the irreducible characters and a bijection for the conjugacy classes that preserve all the character values. We give necessary and sufficient conditions for two finite groups to have isomorphic character tables. In the case of finite p-groups with derived subgroup of order p we classify up to isomorphism the character tables and we give a precise formula for the number of non-isomorphic character tables of these groups.

Option evaluation formulas for some classes of Levy processes

Mircea Nica

It is known that in the martingale approach the option value is the integral of a discounted transition density times a payoff function, an evaluation done in the S'- space, where S is the maturity security price. However for the Levy processes the S-space is often very complicated and it results that is much easier to compute the option value as an integral in a Fourier-space using the Parseval identity.

Some simple option evaluation formulas are presented, making use of simple integration involving only elementary functions. This approach generalizes previous work in the area using Fourier transforms and characteristic functions. Also, the residue calculus is used to obtain evaluation formulas for any European style option simple or exotic (without path dependence) under Levy processes with a known characteristic function.

Colored bialgebras and nonlinear equations

Florin F. Nichita

Based on joint work with **Deepak Parashar** (U. Warwick / MPIM Bonn).

We present new solutions for the constant and spectral-parameter Yang-Baxter equations from algebra structures, the associated FRT bialgebras, and some new constructions of Yang-Baxter systems.

We conclude with remarks, interpretations and problems.

Rigidity of higher rank abelian cocycles with values in diffeomorphism groups

Viorel Nitica

Based on joint work with **A. Katok**.

We consider cocycles over certain hyperbolic higher rank abelian actions and show rigidity properties for cocycles with values in a Lie group or a diffeomorphism group, which are close to identity on a set of generators, and are sufficiently smooth. The actions we consider are Cartan actions on compact quotients of SL(n,R) or SL(n,C), for n greater than 3. The results rely on a technique developed recently by D. Damjanovic and A. Katok.

Some unexpected problems of Moment Problem

Octav Olteanu

Based on joint work with **Alina Olteanu**.

The present work deals with some Markov-type moment problems on $\mathbb{R}$ and $\mathbb{R}^n$ with solutions that vanish on the subspaces of odd functions, subspaces of the spaces on which these solutions are defined. Scalar and operatorial problems are solved. For the one dimensional case, when the "form" of even positive polynomials on $\mathbb{R}$ is known, our proofs are partially based on some recent results ([5]) concerning the approximation of some continuous nonnegative functions by positive polynomials, on some closed unbounded subsets of $\mathbb{R}$ respectively of $\mathbb{R}^n$. In the case of several variables, some other methods, based essentially on [6], are used.

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Dynamics and bifurcation of waves in the electroconvection of nematic liquid crystals

Iuliana Oprea

The electroconvection in nematic liquid crystals provides a rich variety of pattern formation phenomena and has become a main paradigm for the study of anisotropic dissipative structures. We present here the results of a bifurcation analysis of the weak electrolyte model (WEM) for the nematic electroconvection in a planar layer, which allows for a systematic investigation of the nonlinear mechanisms generating the patterns observed experimentally.

The mathematical model consists of the generalized Navier-Stokes equations for an anisotropic electrically conducting fluid, the conservation of charge, Poisson and Ohm's laws, the equation

for the director field giving the orientation of the molecules and an evolution equation describing the effects of ionic migration and molecular dissociation-recombination reactions on the electrical conductivity.

The linear stability analysis indicates the existence of a Hopf-type primary instability involving four oblique rolls, confirming the validity of WEM. Globally coupled complex Ginzburg Landau equations are used in the weakly nonlinear analysis of the bifurcating convective patterns beyond threshold. Numerical computations for parameters corresponding to measured values of the nematic liquid crystal I52 show different types of oblique travelling rolls as well as more complex spatiotemporal structures, including heteroclinic cycles, a period doubling cascade to chaos, zigzag spatiotemporal chaos at onset and localized solutions. Eckhaus stability boundaries and quantitative comparison with the experiments done at the Liquid Crystal Institute, Kent, US, are discussed, too.

Acknowledgement This research has been supported by the National Science Foundation, under Grant No. DMS-0407418

On the motion of artificial satellites in a resisting medium

Tiberiu Oproiu

This talk is dedicated to the 50th anniversary of the launch of **Sputnik 1**, which took place on **4 october 1957**.

The study of the artificial satellite motion in the terrestrial atmosphere constitues a basic problem of the space flight dynamics. The theory of atmospheric drag and its influence on Earth orbiting satellites have been well analysed and studied. Drag is considered as orbit perturbing force causing the satellite to deviate from the idealised Kepler orbit. This perturbation is especially significant at the altitudes of LEO (Low Earth Orbit) satellites (below 1000 km). We present some results obtained in Romania in the years elapsed from the launch of the first Earth's artificial satellite. The orbital evolutions of several satellites (e.g. Explorer 19, Polyot 1, Samos 2 etc) are analized. Historical remarks are pointed out.

A CR-embedding for Sasakian manifolds

Liviu Ornea

Based on joint work with **Misha Verbitsky**.

The aim of the talk is to describe the best possible analogue of Kodaira embedding theorem for compact Sasakian manifolds, namely a CR-embedding in a manifold diffeomorphic with a sphere. I shall argue why a model space playing the role of $\mathbb{C}P^n$ cannot exist in Sasakian geometry. The proof exploits a previously proved result concerning compact locally conformal Kähler manifolds and combines techniques and results of contact, Sasakian and Kähler geometry.

Strong differential subordination

Gheorghe Oros

Based on joint work with **G. I. Oros**

Keywords: analytic function, differential subordination, subordination, strong subordination, univalent.

The concept of differential subordination was introduced in [4] by S.S. Miller and P.T. Mocanu and the concept of strong differential subordination was introduced in [1] by J.A. Antonino and S. Romaguera. This last concept was applied in the special case of Briot-Bouquet strong differential subordination. In this paper we study the strong differential subordinations in the general case, following the general theory of differential subordinations presented in [4].

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Harmonic morphisms between Weyl spaces

Radu Pantilie

Basend on joint work **E. Loubeau**.

We show that Weyl spaces provide a natural context for harmonic morphisms. This follows from the study of the interplay between harmonic morphisms and twistorial maps. Along the way, we obtain - the necessary and sufficient conditions under which on an Einstein-Weyl space of dimension four there can be defined, locally, at least five distinct foliations of dimension two which produce harmonic morphisms, - the description of the harmonic morphisms between Einstein-Weyl spaces of dimensions four and three.

Foruier transform of the "heat kernel" on the Heisenberg group

Marian-Dumitru Pantiruc

Convolution semigroups on a locally compact group can be described using Fourier transformation in terms of continuous negative-definite functions on the dual group. I computed the Fourier transform of the "heat" kernel associated with the laplacean Δ_H on the Heisenberg group

to see if such a connection still holds in a particular case of a locally compact **non-Abelian** group.

Cohomology jumping loci, groups, and algebraic manifolds

Ştefan Papadima

I will describe several special features of characteristic and resonance varieties coming from smooth complex quasi-projective manifolds. Related properties, together with applications, will also be discussed.

On semilinear spectral theory

Dan Pascali

Based on joint work with **P. Catană**.

The purpose of the talk is to compare three nonstandard spectra for semilinear operators: the asymptotic Furi-Martelli-Vignoli spectrum, the global Feng spectrum, and the local Vath phantom. We show that these spectra coincide for certain classes of mappings and indicate their application in proving some kind of nonlinear Fredholm alternative.

Domain convergence of reflecting Brownian motion

Mihai N. Pascu

Based on joint work with **Nicolae R. Pascu**.

For smooth bounded domains $D, D_n \subset \mathbb{C}$ $(n \geq 1)$ we show that if D_n converges to D with respect to the point $x \in D$ in the sense of Caratheodory kernel convergence, then the transition densities $p_{D_n}(t, x, y)$ of the reflecting Brownian motion in D_n starting from x converge to the transition density $p_D(t, x, y)$ of the reflecting Brownian motion in D starting from x. In the particular case of an increasing sequence D_n of domains, we obtain a previous result of K. Burdzy and Z.-Q. Chen.

Some extensions of the Schwarz Lemma

Nicolae R. Pascu

In this talk we give some extensions of the Schwarz Lemma for a class of analytic functions defined in angular domains.

Stability results for multi-layer Hele-Shaw flow

Gelu Paşa

Based on joint work with **Prabir Daripa**

In this talk, we will first present a new upper bound on the growth rate of disturbances in three-layer Hele-Shaw flows where each layer has constant-viscosity fluid. This upper bound is in terms of two free parameters λ_1 and λ_2 such that $\lambda_1 + \lambda_2 \leq 1$. The significance of these parameters will be stressed and the relevance of the upper bound results for practical applications will be discussed. We will also provide a near-optimal inequality relating interfacial tensions and viscosities of extreme-layer fluids resulting from the need to tame pure Saffman-Taylor instability in two-layer Hele-Shaw flows.

Next we will extend the new upper bound result on growth rate from three-layer to arbitrary number of layers with each layer having constant viscosity fluid. Generalization of this result to variable viscosity case runs into significant difficulties except in the four-layer case where partial progress has been made. Time permitting, these results will also be presented and their relevance in the context of applications will be mentioned.

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Gauss sums and their generalizations

Samuel J. Patterson

One of the most remarkable theorems of number theory is Gauss' evaluation of the quadratic Gauss sum $\sum_{0 \leq j < c} \exp(2\pi \imath j^2/c)$. This is equal to $\sqrt{c}$, 0, $\sqrt{c}\imath$ or $\sqrt{c}(1+\imath)$ according as to whether c is congruent to 1, 2, 3 or 0 modulo 4. This theorem can be proved using elementary methods but yet it implies the law of quadratic reciprocity. Another version is the evaluation of $\sum_{j \pmod c} \left(\frac{j}{c}\right) \exp(2\pi \imath j/c)$ where c is odd and $\left(\frac{\cdot}{\cdot}\right)$ is the Jacobi-Legendre symbol. This is equal to $\sqrt{c}$ if $c \equiv 1 \pmod 4$ and $\sqrt{c}\imath$ if $c \equiv 3 \pmod 4$.

It has been an open question for a long time as to what sort of analogues exist for more general sums of similar type. In the second formulation one would be interested in the case where the Jacobi-Legendre symbol is replaced by the n^{th} power residue symbol. In the first case sums of the type $\sum_{0 \leq j < c} \exp(2\pi \imath f(j)/c)$ where f is a polynomial with integral coefficients. In both cases considerable progress has been made recently and the goal of the talk will be to describe how one now understands the distribution of the values of these sums.

On the center of compact hypergroup algebras

Liliana Pavel

Hypergroups are locally compact spaces whose bounded Radon measures form an algebra which has properties similar to the convolution measure algebra of a locally compact group. A hypergroup can be viewed as a probabilistic group in the sense that to each pair x, $y \in K$ there exists a probability measure $\delta_x * \delta_y$ on K with compact support, such that $(x, y) \mapsto supp\ \delta_x * \delta_y$ is a continuous mapping from $K \times K$ into the space of compact subsets of K. Unlike the groups, $\delta_x * \delta_y$ is not in general a point measure.

Let K be a compact hypergroup. Contrary to the group case, $Trig(K)$, the linear span of coordinate functions for the irreducible representations of K, endowed with the usual multiplication does not bear an algebra structure. However, $Trig(K)$ naturally inherits an algebra structure from $\mathcal{M}(K)$, the convolution algebra of all bounded Radon measures on K. Our purpose here is to characterize the center of the convolution algebras $\mathcal{M}(K)$, $L_p(K)$ and $Trig(K)$. As an application we obtain, for a certain class of compact hypergroups, the correspondence between the structure space of the center of $L_1(K)$ and the center of $Trig(K)$.

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Asymptotic expansion for Durrmeyer operators in complex domains

Radu Păltănea

Consider the Bernstein – Durrmeyer operators, extended on a complex domain $D \subset \mathbf{C}$, such that the real segment $[0, 1]$ is included in D

$$M_n(f, z) = (n+1) \sum_{k=0}^{n} p_{n,k}(z) \int_0^1 f(t) p_{n,k}(t)\, dt,$$

where $f : D \to \mathbf{C}$ is continue, $z \in D$ and $p_{n,k}(z) = \begin{pmatrix} n \\ k \end{pmatrix} z^k (1-z)^{n-k}$.

If f is analytic on D, then there is an ellipse $E \subset D$, with foci in the points 0 and 1 on which f admit a complete development:

$$M_n(f,z) = \sum_{k=0}^{p} \frac{1}{k!}(n+2)^{-k}[x(1-z)f^{(k)}(z)]^{(k)} + o(n^{-p}), \quad (n \to \infty),$$

uniformly with regard to $z \in E$, for all $p \in \mathbf{N}$. This result is an extension for analytic functions, of a result obtained by U. Abel and is based on the eigen structure of operators M_n.

Mathematical proof and science argumentative structure

Ilie Pârvu

We will contrast the logic of mathematical proof and the structure of empirical theories with their constructive counterparts. Going beyond the analogies noticed by Thomas Mormann and Adonni Ibbara, we will advocate the unity of the argument, based also on the transcendental Kantian argument used to legitimise mathematical constructions as essential for mathematical inferences. For us, "constructivism" is not only the philosophical idea on which constructive mathematics is founded, but also the role played in mathematical proofs by numerical experiments, computer simulations, imaginary constructs, etc.

On a property of p-summing operators

Carmen Pârvulescu

Key words: p-summing operator, quasi-compact operator.

In this talk we concluded that a p-summing operator is a quasi-compact one. We present also some consequences: the property of uniform convergence for its averages and some observations about its resolvent set.

Multialgebras, universal algebras and identities

Cosmin Pelea

This talk is mainly based on the paper *Multialgebras, universal algebras and identities, J. Aust. Math. Soc,* **81**, *2006, 121–139* which is a joint work with **Ioan Purdea**.

One of the most important constructions in multialgebra theory is the formation of factor multialgebras. This has been studied from the outset of this theory, which is not surprising because the first case of multialgebras were the hypergroups and they emerged as a result of the factorization of a group modulo an equivalence relation determined by a subgroup. Later, G. Grätzer proved that any multialgebra can be obtained by an appropriate factorization of a universal algebra modulo an equivalence relation.

It is known that by the factorization of a universal algebra modulo a congruence relation which includes a relation we obtain a universal algebra in which any two elements in the given relation determine the same class. We proved that the factorization of a universal algebra modulo an equivalence relation — which gave rise to multialgebras — can be seen as an "intermediate step" of such a factorization. This leads to the study of certain (ideal) equivalences which have the property that the factor multialgebras they determine are universal algebras. Such equivalences

appear in the literature from the very first papers on hypergroups. A series of important works about these equivalences of the hypergroupoids, semihypergroups, hypergroups, hyperrings and other particular multistructures have been published after 1990 and they converge toward the study of the smallest equivalences of this kind.

The main results mentioned here concern the smallest equivalence for which the factor multialgebra is a universal algebra for which a given identity is verified. By applying this theorem to the case of (semi)hypergroups and to the identity which expresses the commutativity of the hyperproduct, one finds the relation introduced by D. Freni in *A new characterization of the derived hypergroup via strongly regular equivalences, Comm. Algebra,* **30** *2002, 3977–3989* in order to obtain a characterization of the derived (sub)hypergroup of a hypergroup. We also established a connection between the derived subgroup of a group and the derived subhypergroup of its factor hypergroup modulo an equivalence relation determined by a subgroup.

The bivariate log-skew-normal distribution. Numerical aspects

Elena Pelican

Based on joint work with **Raluca Vernic**.

Though very used in practice for its nice properties, the normal distribution is not so adequate to model real data which are usually skewed and sometimes long-tailed (e.g. insurance claims data). This is why alternative distributions derived from the normal one were considered, like e.g. the lognormal or skew-normal. Motivated by a study on a set of bivariate real data from auto insurance (third party liability and bodily injury), in this paper we consider the bivariate log-skew-normal distribution. Derived from the bivariate skew-normal the same way as the lognormal was derived from the normal, it offers a better fit to our data. Though more difficult to handle than its lognormal correspondent, the log-skew-normal distribution should often perform better for skewed data with a long-tail. Therefore, we concentrate on the numerical aspects involved by parameters estimation, and by other computations related to practical problems.

Singularly perturbed Cauchy problem for abstract system of linear equations in Hilbert spaces

Andrei Perjan

Let $H_i, i = 1, 2$ be real Hilbert spaces endowed with scalar products $(\cdot,\cdot)_i$ and norms $|\cdot|_i$ and $H = H_1 \times H_2$ be the real Hilbert space endowed with scalar product $(\cdot,\cdot) = (\cdot,\cdot)_1 + (\cdot,\cdot)_2$ and norm $|\cdot| = |\cdot|_1 + |\cdot|_2$. Let $V_i \subset H_i, i = 1, 2$ be the real Hilbert spaces endowed with norms $||\cdot||_i$ such that the inclusions are dense. By V we denote the Hilbert space $V = V_1 \times V_2$ endowed with norm $||\cdot|| = ||\cdot||_1 + ||\cdot||_2$.

Consider the following Cauchy problem, which will be called (P_ε)

$$\begin{cases} u'(t) + B_1 u(t) + B_2 v(t) = f(t), \quad t > 0, \\ \varepsilon v'(t) + B_3 u(t) + B_4 v(t) = g(t), \quad t > 0, \\ u(0) = u_0, \quad v(0) = v_0, \end{cases}$$

where $\varepsilon > 0$ is a small parameter, $u, f : [0, \infty) \to H_1$, $v, g : [0, \infty) \to H_2$, $B_i, i = 1, 2, 3, 4$ are linear operators such that $B_1 : D(B_1) = V_1 \to H_1$, $B_2 : D(B_2) = V_2 \to H_1$, $B_3 : D(B_3) = V_2 \to H_1$, $B_4 : D(B_4) = V_2 \to H_2$.

We will investigate the behavior of solution $U(t,\varepsilon) = col(u(t,\varepsilon), v(t,\varepsilon))$ to the perturbed system (P_ε) as $\varepsilon \to 0$. We will establish the relationships between the solutions to the problem (P_ε) and the corresponding solutions to the following unperturbed system, which will be called (P_0):

$$\begin{cases} z'(t) + B_1 z(t) + B_2 w(t) = f(t), \quad t > 0, \\ B_3 z(t) + B_4 w(t) = g(t), \quad t > 0, \\ u(0) = u_0. \end{cases}$$

Under some conditions on operators $B_i, i = 1,2,3,4$ and some smoothness condition on u_0, v_0, f and g we prove the relationships

$$u(t) = z(t) + O(\sqrt{\varepsilon}), \quad in \quad C([0,T]; H_1) \quad as \quad \varepsilon \to 0,$$

$$v(t) = w(t) + Y(t/\varepsilon) + O(\varepsilon^{1/4}), \quad in \quad C([0,T]; H_2) \quad as \quad \varepsilon \to 0,$$

where $Y(\tau)$ is solution to the following problem:

$$\begin{cases} Y'(\tau) + B_4 Y(\tau) = 0, \quad \tau > 0, \\ Y(0) = v_0 - w(0),\ Y(\tau) \to 0, \quad \tau \to \infty. \end{cases}$$

The size of the critical sets of some differentiable mappings

Cornel Pintea

We shall study the size of the critical set and the set of critical values of certain differentiable mappings in terms of dimension, by means of the homotopy groups of the involved manifolds.

Homogenization of thermal flows with first-order jump interfaces

Dan Polişevschi

Keywords: Boussinesq approximation, Navier-Stokes system, two-scale convergence, first-order jump condition.

We consider a radiant solid matrix which has an ε-periodic distribution of pores through which an incompressible viscous fluid flows. It is governed by the Navier-Stokes system with the heat transfer influence described by the Boussinesq approximation. Both phases are connected, but only the fluid is reaching the boundary of the domain, with non-homogeneous data. We study the case when there is a first-order temperature jump on the fluid-solid interface; the two phases have conductivity coefficients of the same order and the Rayleigh number is of ε^{-2}-order. We obtain the homogenized system by using the two-scale convergence techniques. It is similar to the Darcy-Boussinesq system, with a filtration flow and two macroscopic temperatures, the heat equations being coupled through a term accounting for the presence of the jump interface.

Large fields and inverse Galois Theory

Florian Pop

The notion "large field" was introduced by the author in the context of Inverse Galois Theory, but it proved useful in other contexts too, like the theory of rationally connected varieties, model theory, etc. I plan to review old and new facts concerning large fields, and then give some recent applications of the new results to the structure of absolute Galois groups and a complex of conjectures going back to Bogomolov, conjectures which are also related to the so called Serre's Conjecture II concerning the triviality of some homogeneous spaces.

Free convection in a porous cavity with temperature dependent viscosity and radiation effects

Serban R. Pop

Based on joint work with **T. Grosan** and **C. Bercea**

The natural convective flow in a fluid-saturated porous medium is considered for a cavity with the bottom wall being heated or cooled. The flow is assumed to follow darcy law and the fluid has temperature dependent viscosity. The heat transfer and flow under the influence of radiation is analysed analytically, by means of asymptotic expansions for shallow cavities, and numerically, using Gauss-Seidel method for the general cases. Results are presented in terms of Nusselt number and slip velocity at the bottom and top wall of the cavity for various values of radiation parameter and forms of the viscosity function. It is seen that the radiation parameter influence directly the average Nusselt number. Other additional effects and consequences are discussed.

The semi-dynamical system associated with a generator

Eugen Popa

A semi-dynamical system is defined as a map $\Phi : (0, +\infty) \times C \to C$ with the semigroup property $\Phi(t, \Phi(s, x)) = \Phi(s+t, x)$. C will be an ordered convex cone. An infinitesimal generator may be associated, through the usual formula $Ax := \lim_{t \to 0} \dfrac{\Phi(t, x) - x}{t}$, the limit being taken in some weak sense.

Under suitable assumptions, a good subcone $C_0 \subseteq C$ may be chosen, such that $A : C_0 \to C_0$ and the semi-dynamical system is recovered via the exponential formula

$$\Phi(t, x) = \sum_{n=0}^{\infty} \frac{A^n x}{n!} t^n$$

This general fact is illustrated for the cases of translation and brownian semigroups.

A Dirichlet form on a standard H-cone

Liliana Popa

In the definition of an autodual H-cone, it is considered an H-morphism $\varphi : S \to S^*$ with some properties. It is known that the associated bilinear form denoted by

$$[s,t] := \varphi(t)(s)$$

is positive and symmetric iff for any balayage B the next equality holds

$$B^*\varphi(s) = \varphi(Bs).$$

This statement is equivalent with the equalities

$$[Rf, Rf] = [Rf, f] = [f, Rf]$$

for any $f = s - t$, where s, t are from a suitable subcone of S.

In this paper we replace the morphism φ with a new correspondence between S and S^*, which generalizes the notion of measure of representation; let $\sigma : S \to S^*$ be a morphism of lattices, relative to the specific order; that means that

$$s \prec t \Leftrightarrow \sigma(s) \prec \sigma(t).$$

The main result is that if the bilinear form

$$[s,t] := \sigma(s)(t)$$

satisfies

$$[Rf, Rf] \leq [Rf, f] + [f, Rf]$$

for any $f \in S_0 - S_0$, then

$$[f, f] \geq 0$$

for any f which is a difference of nearly continuous elements.

Notes on c-free probability with amalgamation

Mihai Popa

The notion of conditionally freeness (or, shortly, c-freeness) was developed in the '90"s by R. Speicher and M. Bozejko as an extension of freeness within the framework of *-algebras endowed with not one, but two states. As in the case of free probability, many of the properties of the construction are preserved if the states are replaced by positive functionals valued in a C*-algebra. We will present several positivity results, a version of the central limit theorem, the connection to monotonic independence with respect to an operator-valued functional and an analogue of the conditionally free R-transform will be constructed by means of multilinear function series.

Schur multipliers between Banach spaces of matrices

Nicolae Popa

We consider the matrix version of Bloch space, denoted by $\mathcal{B}(\ell_2)$- the space of all upper triangular infinite matrices A such that the $B(\ell_2)$-valued function $f_A(re^{it}) = \sum_{k=0}^{\infty} A_k r^k e^{ikt}$, where A_k means the k^{th}-diagonal of A, belongs to the Bloch space of analytic $B(\ell_2)$-valued functions.

We denote by $H^1(\ell_2)$the space of all upper triangular Schur multipliers A. (See [1] for definitions.)

We get that the space $(H^1(\ell_2), \mathcal{B}(\ell_2))$ of all Schur multipliers from $H^1(\ell_2)$ into $\mathcal{B}(\ell_2)$ coincides with $\mathcal{B}(\ell_2)$, extending Proposition 2 from [2].

Also we introduce a S_1-valued Bergman space, where S_1 is the trace class of operators, and we state a theorem about matrix version of Bloch space as the space of Schur multipliers between the vector valued Bergman space above and other space of matrices. Finally we investigate Schur multipliers between $B(\ell_2)$ and $\mathcal{B}(\ell_2)$.

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Special values of P-adic and global L-functions. A conjectural program in equivariant Iwasawa theory

Cristian D. Popescu

We will describe a conjectural program linking the values of Galois-equivariant *p-adic L-functions* at arbitrary integers k to various arithmetic and geometric invariants of the associated arithmetic schemes.

For $k \leq 0$, we will show that via Iwasawa descent these p-adic conjectures imply refinements of the classical conjectures of Coates–Sinnott and Lichtenbaum ($k < 0$) and Gross–Tate and Brumer–Stark ($k = 0$), thereby linking special values of *global L-functions* to various étale cohomology groups (and implicitly algebraic K-groups.)

For $k = 0$, we will show that via Iwasawa descent the p-adic conjectures lead to previously unpredicted arithmetic properties of special values of *global L-functions.*

Finally, if time permits, we will provide ample evidence in support of the p-adic conjectures.

Stanley conjecture in small embedding dimension

Dorin Popescu

Let $S = K[x_1, x_2, ..., x_n]$ be a polynomial ring in n variables over a field K and $I \subset S$ a monomial ideal. Let $\mathcal{F}$ be a prime filtration of S/I given by some monomial ideals, that is a filtration

$$I = F_0 \subset F_1 \subset \ldots \subset F_r = S$$

such that $F_i/F_{i-1} \cong S/P_i(-a_i)$ for some monomial prime ideals

$P_i \in Spec(S)$, $a_i \in \mathbf{N}^n$. The prime filtration $\mathcal{F}$ is a *pretty clean filtration* if $P_i \subset P_j$ and $i \leq j$ implies $P_i = P_j$.

A decomposition of S/I as a direct sum of linear K-spaces of the form $uK[Z]$, where u is a monomial in S and $Z \subseteq \{x_1, x_2, ..., x_n\}$, is called a *Stanley decomposition*. Stanley conjectured that always there exists a Stanley decomposition

$$S/I = \bigoplus_{i=1}^{r} u_i K[Z_i]$$

such that $|Z_i| \geq depth(S/I)$ for all $i, 1 \leq i \leq r$. If this holds for I we say that I is a *Stanley ideal*. Sometime Stanley decompositions of S/I arise from prime filtrations. Actually, if $\mathcal{F}$ is a prime filtration of S/I as above, then taking $u_i = \prod_{j=1}^{n} x_j^{a_{ij}}$ and $Z_i = \{x_j : x_j \notin P_i\}$ we get a Stanley decomposition of S/I. If $\mathcal{F}$ is a pretty clean filtration of S/I, then J. Herzog and myself have shown that I is a Stanley ideal. If $n = 3$ then each monomial ideal of S is Stanley by Apel.

Recently I. Anwar and myself have shown that each monomial ideal of S is Stanley when $n = 4$. If $n = 5$ then we noticed that the monomial ideals $I \subset S$ with all associated prime ideals of height 2 are Stanley (this extends a result of Herzog-Soleyman Jahan-Yassemi for $n = 5$). We should point that Stanley decompositions considered in our proofs arise all from prime filtrations.

The geometry of Lie algebroids and its applications

Liviu Popescu

The notion of Lie algebroid is a generalization of the concepts of Lie algebra and integrable distribution. A. Weinstein gives a generalized theory of Lagrangians on Lie algebroids and obtains the Euler-Lagrange equations. The same equations were later obtained by E. Martinez using the symplectic formalism and the notion of prolongation of Lie algebra over a mapping introduced by P.J. Higgins and K. Mackenzie. In this paper we study the the notions such as: connections, curvature, torsion, complex structure, etc.. The applications to optimal control are presented.

Sufficient optimality condition in orbital rendez-vous problems

Mihai Emilian Popescu

Optimizing the index of performance of the dynamic systems imposes checking the necessary and sufficient conditions of extremum for the determined control.Because of the difficulties created by satisfying the sufficient conditions, most of the existent research uses only the necessary conditions of optimality, which makes the solution we have obtained less rigorous. This talk deals with determining the sufficient conditions of minimum for the class of problems in which the necessary conditions of optimum are satisfied in the strengthened form Legendre-Clebsch. To this paper, we shall use the sweep method which analysis the conditions of existence of the conjugated points on the optimal trajectory. The study we have done evaluates the command variation on the neighboring optimal trajectory. The sufficient conditions of minimum are obtained by imposing the positivity of the second variation. The results that this method offers are applied to the problem o the orbital rendez-vous for the linear case of the equations of movement.

A coincidence result between the sets of appproximate efficient points and Choquet boundaries in separated locally convex spaces

Vasile Postolica

Key Words: Choquet Boundary, Approximate Efficiency

This talk aims to to establish the coincidence between a new type of approximate efficient points sets and Choquet boundaries in ordered Hausdorff locally convex spaces, being based on the first result established by us concerning such a property as this for Pareto type efficient points sets and the corresponding Choquet boundaries of non-empty, compact sets, with respect to appropriate convex cones of real, increasing and continuous functions. Thus, the main result represents a strong connection between two great fields of Mathematics: The Axiomatic Theory of Potential, its Applications and Vector Optimization in infinite dimensional vector spaces.

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A new method of estimation of the distribution of one- and two-dimensional scan statistics

Cristian Preda

Based on joint work with **George Haiman**.

Keywords: Discrete and continuous scan statistics, Approximation, Simulation.

In this talk we present our recent results concerning a new method of estimation of the distribution of scan statistics. The method is applied to discrete and continuous one and two-dimensional scan statistics. We compare our results with several other methods proposed in the literature.

Primality proving, theoretical and practical

Valeriu Prepeliţă

A class of multi-dimensional time-variable hybrid control systems is studied. An appropriate separable multi-dimensional partial differential-difference equation is solved and a variation-of-parameters type formula is established, which is used to obtain the input-output map of the system. The concepts of controllability and reachability are analysed and suitable controllability and reachability Gramians are constructed to characterize the controllable and the reachable time-variable systems. Some necessary and sufficient conditions of reachability are provided for time-invariant multi-dimensional hybrid control systems and a geometric characterization of the space of reachable states is given.

Riemannian metrics associated to vector fields

Gabriel Pripoae

For a given nowhere vanishing vector field ξ on a differentiable manifold M, we prove there exists a Riemannian metric g whose geodesics are its trajectories. Several new examples are provided, including variations on the two-body dynamics theme; the moduli space $Riem(M, \xi)$ of these adapted metrics is quite large and offers a flexible framework with "multiple degrees of freedom" for exotic post-Newtonian theories.

In a previous paper (C.R. Acad. Sci. Paris, Ser.I, 342 (2006),865-868), we classified the Lie groups G into seven families, according to the behaviour of the left invariant vector fields with respect to the existence of *left invariant* Riemannian metrics in $Riem(G, \xi)$. We prove now that three of these families may not exist, so the respective classification rests on exactly only four kinds of specific behaviours.

Toeplitz operators associated to commuting row contractions

Bebe Prunaru

Let H be a complex separable Hilbert space and let $B(H)$ denote the algebra of all bounded linear operators on H. A sequence $\bar{T} = \{T_n\}_{n\geq 1}$ of bounded operators on H is called a row contraction if $\sum_{n\geq 1} T_n T_n^* \leq I_H$ where I_H stands for the identity operator on H. To each row contraction $\bar{T}$ one can associate its Toeplitz space defined as

$$F(\bar{T}) = \{X \in B(H) : \sum_{n\geq 1} T_n X T_n^* = X\}.$$

We show that, if $\bar{T} = \{T_n\}_{n\geq 1}$ is a row contraction with mutually commuting components, then its Toeplitz space $F(\bar{T})$ is either null or is completely isometric to a type I von Neumann algebra. A concrete representation of this von Neumann algebra is also provided.

Are the earthquakes predictable?

George Purcaru

Earthquake predictability is among the still unresolved hard problems in the earth sciences, and our limited and incomplete insights prevent the prediction, in terms of place, magnitude and time, of earthquakes with the desired certainty. Time prediction is the most difficult. Only for few, especially large and great, earthquakes successful (scientific) predictions were made, with variable precisions. But false alarm amd failure of prediction, the other two components of the prediction concept, are by far more times occurring. We cannot today measure the stress changes, the drving force to the critical point, the earthquake, in the earth to depths down to 680 km where large earthquakes occur, too. However, the gradually gained theoretical and better observational knowledge have advanced our understanding of the scale - dependent physics of earthquake rupture process, variability of complex patterns of seismicity in space and time and fault tectonics. Thus, deterministic and probabilistic forecasting methods, more realistic experimental and numerical simulations and algorithms, retrospective prediction case studies, ensamble predictions, critical behaviour of precursors, etc. Along these, we present successful concepts and methods for for prediction, such as: seismic cycle and earthquake gap, M8 algorithm, RTL method, time probability of a future earthquake, AMR method and history reconstruction of occurrences. We conclude that large earthquakes will become more predictable with increasing our actual knowledge, and laws specified to particular tectonic-seismic regions of the earth. Finally, we present successful predictions of some large M7+ and great M8+ earthquakes: 1977 M7.5, Vrancea; 1996 M7.9, Aleutines; 1997 M7.8, Kamchatka; 2006 M7.9, Tonga, and M7.9, Kuriles; 2007 M8.1, Solomon Is., and exemples of false alarms and failures of prediction.

A new algebraic point of view for the rigid body dynamics on $SO(3)$

Mircea Puta

We present some new Lie algebraic properties of the rigid body dynamics on $SO(3)$.

Algorithmic unsolvability of a problem of expressibility in the Gödel-Löb logic provability

Mefodie Raţă

Formulas of Gödel-Löb logic of provability [1,2,3] (GL logic) are constructed in the usual way from variables p, q, r (possibly with subscripts) by means of the connectives $\&, V, \supset, \neg$ and Δ (Godel's provability).

GL logic is represented by means of the calculus given by the axioms of the classical propositional calculus and the two Δ-axioms:

$$\Delta(\Delta p \supset p) \supset \Delta p \text{ (axiom of Löb) and } \Delta(p \supset q) \supset (\Delta p \supset \Delta q),$$

and by the three rules of inference: substitution rule, modus ponens and necessitation rule.

A formula F is said to be expressible in the logic L by means of the system of formulas Σ, if F can be obtain from variables and from formulas of Σ by a finite number of applications of the rules of weak substitution and replacement by equivalents in L. By the *problem of expressibility in the logic* L we mean the algorithmic problem requiring the construction of an algorithm which,

for every formula F and every finite system of formulas Σ, enables us to determine whether F is expressible in L by means of Σ.

Theorem 1. *There is no algorithm solving the problem of expressibility in the GL logic.*

Theorem 2. *The problem of expressibility in the GL logic is algorithmicaly unsolvable.*

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Convexity in symplectic geometry

Tudor Raţiu

After a quick review of the known convexity results for the momentum map, the presentation will focus on the problem of convexity for arbitrary proper Lie group actions that do not necessarily admit momentum maps. The cylinder valued momentum map will be presented, an object that exists for any proper action, and its convexity properties from the metric point of view will be presented.

Quantum dynamics in the von Neumann algebras associated with some discrete groups

Florin Rădulescu

The Berezin quantization method is used to constuct a Chapmann-Kolmogorov type of dynamics. This is used to construct some cohomological invariants.

Notes on free probability

Florin Rădulescu

An introductory lecture on free probability.

Mathematical modelling of coupled chemical mechanical interactions for shale

Simona Roatesi

This talk is motivated by the practical problem of stability of a borehole drilled in a chemically active rock and it deals with the coupling of mechanical-chemical exchanges in a porous media. The constitutive model which is used in this talk is an extension of the poroelastic constitutive equations to account for physical-chemical interactions. So, a Biot type theory of chemo-poroelasticity is developed. This new innovative theoretical solution for the borehole stability deals with the change in stress, pore pressure and salt concentration occurring in a chemically active shale around a borehole filled when submitted to a difference in salt concentration between drilling and formation fluids. This theory can be degenerated into two limiting cases, one corresponding to the absence of any chemical interaction and the other one which can be characterized as a perfect ion exclusion membrane theory. The problem is analyzed within the framework of the Biot's theory of poroelasticity extended to physical-chemical interactions. The well is drilled in a clay mass subjected to an isotropic stress 0. A pore pressure p0 and a molar fraction x0 initially characterize the interstitial fluid. The problem is analyzed assuming plane strain and an instantaneous drilling of the well. The boundary conditions and the geometry determine an axisymmetric problem. The equations are then integrated using Laplace transforms to solve the irrotational problem of an infinitely long borehole in an axisymmetric 1D far stress field. A validation of the solution is performed by comparing it with the poroelastic solution by canceling the chemical effects and also, by the results obtained with another numerical code.

Hopf bifurcation for coupled identical dynamical systems. Applications

Carmen Rocşoreanu

Based on joint work with **Mihaela Sterpu**.

Keywords: coupled dynamical systems, Hopf bifurcation, Lyapunov coefficients, advertising model.

Two identical 2D dynamical systems depending on a parameter were coupled linearly and non-symmetrically. The Hopf bifurcation in the resulting 4D system is studied. Formula for the computation of the first and second Lyapunov coefficients are obtained in two cases of Hopf bifurcation. They use only 2D vectors.

These formula are applied to the study of the Hopf bifurcation around one equilibrium point of some 4D systems obtained by coupling two 2D systems modelling economic phenomena. Even if the Hopf bifurcation in the single system is always supercritical, for the coupled system the Hopf bifurcation can be supercritical, subcritical or degenerated. The bifurcations are illustrated by numerical computations using the software winpp.

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On Baer and Quasi-Baer properties for modules

Cosmin Roman

The notions of Baer and quasi-Baer rings have their roots in functional analysis. For example, von Neumann algebras possess a plethora of structures - algebraic, geometric and topological. For an algebraist, a boon is the rich supply of idempotents which these algebras have. In order to obtain an insight into the theory of von Neumann algebras, several authors started to axiomatize this theory, including S.W.P. Steen, I.M. Gel'fand, M.A. Naĭmark, C.E. Rickart and von Neumann. Algebraically, in any von Neumann algebra (i.e. W^*-algebra) the right annihilator of any subset is generated as a right ideal by a projection (i.e. a self-adjoint idempotent with respect to the involution *). Kaplanksy, in 1955, defined the larger class of Baer *-rings as rings with involution in which the right annihilator of every subset (or left ideal) is a principal right ideal generated by a projection (R. Baer studied this condition earlier in his book "Linear Algebra and Projective Geometry"). Dropping the assumption of an involution in this definition, led Kaplansky to the concept of a Baer ring.

A *Baer ring* is defined to be a ring in which the right annihilator of any left ideal (or subset) is a right ideal, generated by an idempotent (e.g. all right self-injective von Neumann regular rings, von Neumann algebras, domains with unit, endomorphisms rings of semisimple modules). The concept of Baer rings was generalized to that of *quasi-Baer rings* by W. E. Clark in 1967 by replacing the 'left ideal' by a 'two-sided ideal' in the above definition (e. g. prime rings, and rings of matrices over Baer rings). An important fact that makes the quasi-Baer rings useful is that the quasi-Baer property is a Morita invariant property, while the Baer property is not. The theory of Baer and quasi-Baer rings has come to play an important role and major contributions to this theory have been made in recent years in the ring-theoretical setting.

Not much is known about these properties in a general module-theoretic setting. For example, a natural question that can be raised is: if $e2 = e$ is an idempotent in a (quasi-) Baer ring R, then does the right R-module eR possess any 'kind' of Baer or quasi-Baer properties?

In this talk, I will present the notions of the Baer and the quasi-Baer properties for arbitrary modules. Let M be an R-module and $S = End_R(M)$. We call M a *Baer module* if every right annihilator in M of any left ideal of S is generated by an idempotent of S. M is called a *quasi-Baer* module if the right annihilator in M of any ideal of S is generated by an idempotent of S. It is easy to see that, when $M = R_R$, the two notions coincide with the existing definitions of Baer and quasi-Baer rings, respectively. To exhibit examples of Baer modules we show that this property is satisfied by any nonsingular extending module, any semisimple $\mathbb{Z}$-module, any finitely generated torsion-free $\mathbb{Z}$-module, and any right ideal direct summand of a Baer ring. Examples of quasi-Baer modules include any projective R-module (in particular, any right ideal summand of R) over a quasi-Baer ring R, any nonsingular FI-extending module and any torsion-free abelian group. Other examples and results on (quasi-) Baer modules will be discussed.

Close binary stellar systems. Considerations on the constants of the stellar structure

Rodica Roman

For an independent determination of the apsidal constants k21 and k22, the Elliptical Restricted Three-Body Problem is resumed. In addition the tidal and rotational effects are considered. The corresponding results are coupled with the classical study of apsidal motion, and a system of two equations with the two unknowns k21 and k22 is established. Finally, the values of the individual apsidal motion constants for the binary system AG Persei are determined.

Artin's reciprocity law and Hasse's local-global principle

Peter Roquette

This talk will be of historical nature. It is based on the letters which were exchanged between Emil Artin and Helmut Hasse in the 1920s and 1930s. These letters are preserved in the University Library of Göttingen. We can see how Artin discovered his general reciprocity law and, in consequence, Hasse was able to develop his theory of local norm symbols. This led to the discovery of local class field theory and to the local-global principle which, in turn, completely changed our view of class field theory.

The details will be published in a paper which is in preparation.

On the solvability of linear variational equations

Ioan Roşca

The aim of this talk is to find necessary and sufficient conditions for solvability of the linear variational equations with applications to boundary value problems for partial differential equations and partial differential system of equations.

In the talk it is proved that if U is a Banach space and V is a reflexive Banach space and a is a real functional on $U \times V$ then the following statments are equivalent:

1) The functional a is a bilinear, continuous and weakly coercive form;

2) There exists a linear operator $S : V' \to U$ which maps V' onto U such that

$$a(Sf, v) = f(v) \quad \text{for all} \quad f \in V' \quad \text{and all} \quad v \in V$$

Also, it is proved that the Babuska-Brezzi condition for the form b and the weak coerciveeness of the form a on the kernel of b are necessary and sufficient conditions for the solvability of the linear variational equations with constraints.

The dynamics of the rational maps

Ionel Roventa

Based on joint work with **C. P. Niculescu**.

In this talk we prove the global asymptotic stability of a class of rational iterative processes. Our approach combine the presence of a group of symmetries with certain apriori estimates.

The aim of this talk is to discuss the asymptotic behavior of the following nonlinear iterative process

$$x_i = \frac{(1+w)x_{i-7}x_{i-6}x_{i-5}x_{i-4} + \prod_{i-3\leq p<q\leq i-1} x_p x_q}{\prod_{i-7\leq p<q<r\leq i-4} x_p x_q x_r + w x_{i-3}x_{i-2}x_{i-1}}, \quad i \geq 8$$

where the initial data $x_1, x_2, x_3, x_4, x_5, x_6, x_7$ and the parameter w are all positive. This process describes the dynamics of the map $T_w : \mathcal{M} \to \mathcal{M}$ that acts on

$$\mathcal{M} = \underbrace{(0,\infty) \times \cdots \times (0,\infty)}_{7 \text{ times}}$$

by the formula

$$T_w((x_1,x_2,x_3,x_4,x_5,x_6,x_7)) = (x_2,x_3,x_4,x_5,x_6,x_7, \frac{(1+w)x_1x_2x_3x_4 + x_5x_6 + x_5x_7 + x_6x_7}{x_1x_2x_3 + x_1x_2x_4 + x_1x_3x_4 + x_2x_3x_4 + wx_5x_6x_7}).$$

We actually prove that for $w \in [2,3]$, the point $C = (1,1,1,1,1,1,1)$ is globally asymptotically stable. In particular, C is the global attractor of the dynamical system associated to T_w.

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Filters, localizations and fractions in algebra of logic

Sergiu Rudeanu

The structures occurring in algebra of logic are usually lattices A endowed also with an implication $\rightarrow$. So both (lattice theoretical) filters and deductive systems do exist; the latter concept means a subset $D \subseteq A$ such that $1 \in D$ and $x, x \rightarrow y \in D \Longrightarrow y \in D$. Deductive systems are usually filters and in certain cases the two concepts coincide. In certain cases when this coincidence doesn't happen, another concept has been invented which is similar to or stronger than a filter and is equivalent to the concept of a deductive system. In the first part of this work we consider the possible implications and equivalences between these concepts and for each such implication or equivalence we establish sufficient conditions which ensure its validity.

Sample result: if an algebra $(A, \wedge, \rightarrow, 1)$ satisfies $x \wedge y \leq z \Longleftrightarrow x \leq y \rightarrow z$, then filters coincide with deductive systems and property $x \leq y \Longleftrightarrow x \rightarrow y = 1$ holds.

Following suggestions which come from ring theory, a series of papers by G. Georgescu, D. Buşneag, C.Dan, D. Piciu and F. Chirteş study localization algebras and algebras of fractions for bounded distributive lattices, Hilbert algebras and Hertz algebras, Heyting algebras, BL algebras and MV algebras, and Łukasiewicz-Moisil algebras. For each of these classes of algebras $\mathcal{A}$, the starting point is the set $M(\mathcal{F}, A)$ of multipliers, meaning certain functions $f : I \longrightarrow A$, where I runs over a family $\mathcal{F}$ of ideals of an algebra $A \in \mathcal{A}$. A large part of this research reveals quite similar techniques and results for the above classes of algebras.

In the second part of this work we suggest a unifying axiomatic approach able to eliminate redundancies. In the literature it is proved that $M(\mathcal{F}, A) \in \mathcal{A}$ for each class $\mathcal{A}$ under discussion, but it is easy to see that in fact each subset $M(I, A)$ of multipliers having a fixed domain $I \in \mathcal{F}$, belongs to $\mathcal{A}$ as well. This becomes an axiom in our approach, which shows that the rest of the theory is a matter of universal algebra and goes the same way for any class $\mathcal{A}$ which satisfies the axiom. Thus we recapture both the desired theorems in algebra of logic and several analogous results that had previously been obtained for rings and modules.

Special fixed point structures

Ioan A. Rus

Let X be a nonempty set and $(X, S(X), \mathbf{M})$ be a fixed point structure on X ([4]). In this paper we study the following problems:

Problem 1 ([4], pp. 32-36). Let $S_1(X) \subset P(X)$ be such that

$S_1(X) \supset S(X)$. Which are the fixed point structures $(X, S(X), M)$ with the following property:

$$S(X) = \{A \in S_1(X) |\ f \in \mathbf{M}(A) \Rightarrow F_f \neq \emptyset\}?$$

Problem 2 ([4], pp. 91-100). Which are the fixed point structures $(X, S(X), \mathbf{M})$ with the following property:

$$Y \in S(X),\ f, g \in \mathbf{M}(Y),\ f \circ g = g \circ f \Rightarrow F_f \cap F_g \neq \emptyset?$$

Problem 3 ([4], pp. 104-109). Which are the fixed point structures $(X, S(X), \mathbf{M})$ with the following property:

$$Y \in S(X),\ f, g \in \mathbf{M}(Y),\ f \circ g = g \circ f \Rightarrow C(f, g) \neq \emptyset?$$

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Approximations for reconstruction problems

Alin Marian Rusu

First I present the clasic model for solving the liniar inverse problem using the maximum entropy method and the way I use the convex optimization approach in solving maxentropic reconstruction problems. Starting with Jaynes's aproach I follow the connection with other approaches presenting major results in this field and also a paralel between Jaynes's approach and the approach of Dacunha and Gamboa.

On functionally completeness problems in provability intuitionistic logic

Andrei Rusu

Modal logics are traditionally extensions of classical logic with new operators, or modalities, whose operation is intensional. Modal logics are most commonly justified by the provision of an intuitive semantics based upon 'possible worlds', an idea originally due to Kripke. Kripke also provided a possible worlds semantics for intuitionistic logic, and so it is natural to consider intuitionistic logic extended with intensional modalities [1]. An intuitionistic modal logic that deal with foundations of mathematics is the provability intuitionistic logic I^{Δ} that was proposed by A. V. Kuznetsov in [2], [3]. Problems of functional completeness were investigated in the case of Boolean functions by E. Post [4], [5]. A. V. Kuznetsov [6] adapted the notions related to functional expressibility to the case of superintuitionistic logics. M. F. Ratsa [7] investigated the related problems in the case of intuitionistic logic, modal logics $S4$ and $S5$.

Let $\mathfrak{C} =< E; \&, \vee, \supset, \neg, \Delta >$ be the chain Δ-pseudo-Boolean algebra with traditionally defined operations.

Theorem 1. *There is a mapping f that is a formula realization of the algebra $\mathfrak{C}$ into the provability intuitionistic logic I^{Δ}.*

Theorem 2. *The classes of formulas, which conserve the respective predicates $x \neq \Delta 0$, $x \neq \Delta^2 0, \ldots$ constitutes a numerable collection of distinct two by two pre-complete in the logic L classes of formulas.*

Theorem 3. *The traditional formulation of the theorem of completeness with respect to functional expressibility in terms of a finite collection of pre-complete classes of formulas in the logic L does not exist, if $L \subseteq L\mathfrak{C}$.*

Theorem 4. *The class of formulas K_∞, which conserves on the algebra $\mathfrak{C}$ the relation $x \neq 1$, is non-tabular with respect to functional completeness in the logic $L\mathfrak{C}$.*

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Further properties of β- Pascu convex functions of order α

Grigore Sălăgean

Based on joint work with **H. Özlem Güney**.

Let $\mathbb{N} := \{1, 2, \cdots\}$; for $m, p \in \mathbb{N}$, $m \geq p+1$, let $\mathcal{A}(p, m)$ be the class of all p-valent analytic functions $f(z) = z^p + \sum_{n=m}^{\infty} a_n z^n$ defined on the open unit disk $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}$.

Let $\mathcal{T}(p, m)$ be the subclass of $\mathcal{A}(p, m)$ consisting of functions of the form

$$f(z) = z^p - \sum_{n=m}^{\infty} a_n z^n; \qquad a_n \geq 0 \ for \ n \geq m. \tag{15}$$

A function $f \in A(p, m)$ is a β-Pascu convex function of order α if

$$\frac{1}{p} Re \left\{ \frac{(1-\beta) z f'(z) + \frac{\beta}{p} z (z f'(z))'}{(1-\beta) f(z) + \frac{\beta}{p} z f'(z)} \right\} > \alpha \qquad (\beta \geq 0, 0 \leq \alpha < 1). \tag{16}$$

We denote by $\mathcal{TPC}(p, m, \alpha, \beta)$ the subclass of $\mathcal{T}(p, m)$ consisting of β-Pascu convex function of order α. Clearly, $\mathcal{TS}^*(\alpha) := \mathcal{TPC}(1, 2, \alpha, 0)$ is the class of starlike functions with negative coefficients of order α and $\mathcal{TC}(\alpha) := \mathcal{TPC}(1, 2, \alpha, 1)$ is the class of convex functions with negative coefficients of order α (studiedComments on the behavior of quasiconformal mappings in Ahlfors regular spaces by H. Silverman [2]).

Many interesting properties such as coefficient estimate, distortion theorems for the class $\mathcal{TPC}(p, m, \alpha, \beta)$ were given by Rosihan M. Ali at.al.[1]. We shall derive several interesting properties and characteristic of the $\delta-$neighborhood associated with the class $\mathcal{TPC}(p, m, \alpha, \beta)$.

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Free Moufang loops and alternative algebras

Nicolae Sandu

It is proved that any relatively free Moufang loop can be embedded in a loop of invertible elements of some alternative algebra.

A loop $(L, \cdot)$ is called *Moufang* it satisfies the law $(xy \cdot x)z = x(y \cdot xz)$. This law is satisfied also by alternative algebras. We remind that algebra $(A, +, \cdot)$ is *alternative* if the laws $(x, x, y) = 0$, $(y, x, x) = 0$ hold in it. The notion $(u, v, w) = uv \cdot w - u \cdot vw$ means the associator of elements $u, v, w \in A$.

It is well known that for an alternative algebra A with unit the set $U(A)$ of all invertible elements of A forms a Moufang loop with respect to multiplication. In [1] it is raised the question: is it true that any Moufang loop can be imbedded into a homomorphic image of a loop of type $U(A)$ for a suitable unital alternative algebra A? The equivalent version of this question is: whether the variety generated by the loops of type $U(A)$ is a proper subvariety of the variety of all Moufang loops? A more general question is raised in [2]: is it true that any Moufang loop can be imbedded into a loop of type $U(A)$ for a suitable unital alternative algebra A? In fact, the answer to this questions is negative: in [1] it is constructed a Moufang loop which is not imbedded into a loop of invertible elements of any alternative algebra.

However, this thesis gives a positive answer to the questions for free Moufang loops. Concretely, let L be a free Moufang loop, let F be a field and let FL be the loop algebra of L over F. This is a free F-module with the basis $\{g | g \in L\}$ and the product of the elements of this basis is determined as their product in loop L. Let H be a normal subloop of loop L. We denote the ideal of algebra FL, generated by the elements $1 - h$ $(h \in H)$ by ωH. If $H = L$, then ωL is called the *augmentation ideal* of algebra FL. By I we denote the ideal of FL generated by expressions $(a, b, c) + (b, a, c)$, $(a, b, c) + (a, c, b)$ for all $a, b, c \in L$. The quotient algebra FL/I is alternative and holds.

Theorem. *Let $(L, \cdot)$ be a free Moufang loop, let F be an arbitrary field and let $\varphi : FL \to FL/I$ be a natural homomorphism of algebra FL in the alternative algebra FL/I. Then the image $\varphi(L, \cdot) = (\overline{L}, \star)$ of loop $(L, \cdot)$ will be also a loop, and φ will be the isomorphism of these loops.*

We identify the loop $(\overline{L}, \star)$ with $(L, \cdot)$. Then every element in FL/I has the form $\sum_{q \in L} \lambda_q q$, $\lambda_q \in F$. Further for the alternative algebra FL/I we use the notation FL and we call them *"loop algebra"* (in inverted commas). Let H be a normal subloop of L. We denote the ideal of "loop algebra" FL, generated by the elements $1 - h$ $(h \in H)$ by ωH. If $H = L$, then ωL will be called the *"augmentation ideal"* (in inverted commas) of "loop algebra" FL. Let us determine the homomorphism φ of F-algebra FL by the rule $\varphi(\sum \lambda_q q) = \sum \lambda_q Hq$.

Proposition. *Let H be a normal subloop of free Moufang loop L and let FL and ωL are respectively "loop algebra" and "augmentation ideal" of L. Then*

1) $\omega H \subseteq Ker\varphi$*;*
2) $1 - h \in Ker\varphi$ *if and only if* $h \in H$*;*
3) $\omega L = \{\sum_{q\in L} \lambda_q q | \sum_{q\in L} \lambda_q = 0\}$*;*
4) the "augmentation ideal" ωL *is generated as* F*-module by elements of the form* $1 - q$ *(*$q \in L$*).*

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Triangulations, quasiregular mappings and differential geometry

Emil Saucan

Starting from the existence problem for quasimeromorphic mappings on manifolds, we explore the application of our previous results in this direction and of the methods employed, to the problem of convergence of curvature measures for piecewise flat manifolds. Applications are also considered, in particular to Regge Calculus. In addition, we further investigate the connection between quasiregular mappings and curvatures of PL manifolds.

On l-ideals of order lattice loop

Petru Sclifos

Definition 1. *Algebra* $G =< G, \bullet, /, \setminus >$ *of* $< 2, 2, 2 >$ *type, where the identities* $(x \bullet y)/y = x$*,* $(x/y) \bullet y = y$*,* $y \bullet (y\setminus x) = x$*,* $y\setminus(y \bullet x) = x$*,* $x/x = y\setminus y$ *hold true, is called a loop (see also [1, 2]).*

Element $e = x/x = y\setminus y$ of loop G meets the identity $e \bullet z = z \bullet e = e$, hence is an identity of loop G.

The mapping group of loop G, generated by all mappings of form $L_a : x \to a\bullet x$, $R_a : x \to x\bullet a$ is know as the group associated to loop G and will be denoted by $\mathfrak{M}(G)$. Mapping $\alpha \in \mathfrak{M}(G)$ is called inner, if $\alpha : e \to e$, where e is the unit of loop G. All inner mappings form a subgroup $\mathfrak{M}(G)$, known as the group of inner mappings of loop G and denoted by $\mathfrak{I}$. The group of inner mappings $\mathfrak{I}$ is generated by the mappings $T_x = L_x^{-1}R_x$, $L_{x,y} = L^{-1}_{x\bullet y}L_xL_y$, $R_{x\bullet y} = R^{-1}_{x\bullet y}R_yR_x$. If subloop H of loop G satisfies the condition $\alpha(H) = H$ for any inner mapping $\alpha \in \mathfrak{I}$, then H is called normal (invariant) in G [1,2].

Definition 2. *Algebra* $G =< G, \bullet, /, \setminus >$ *of* $< 2, 2, 2 >$ *type that satisfies the following conditions*
(i) algebra $< G, \bullet, /, \setminus >$ *is a loop,*
(ii) model $< G, \leq >$ *is partially ordered, well ordered, and respectively lattice ordered,*

(iii) $x \leq y \Leftrightarrow zx \bullet t \leq zy \bullet t$ *for any* $z, t \in G$ *is called a partially ordered, well ordered, and respectively lattice ordered loop* [5, 6, 7].

Theorem 1. *Normal subloop* A *of the lattice ordered loop* G *is an* l*-ideal if and only if for* $x \in G$ *and* $a \in A$ *inequality* $|x| \leq |a|$ *implies* $a \in A$.

Theorem 2. *If* α *is a mapping of the lattice ordered loop* G *in the partially ordered loop* Q *that satisfies the conditions:* $\alpha(a \bullet b) = \alpha(a) \bullet \alpha(b)$ *and* $\alpha(a \vee b) = \alpha(a) \vee \alpha(b)$ *(or* $\alpha(a \wedge b) = \alpha(a) \wedge \alpha(b)$*), then* α *is* 0*-homomorphism and its kernel* $Ker\alpha = A$ *is* l*-ideal in* G*, while quotient loop* G/A *is an ordered loop.*

Theorem 3. *Subloop* H *of the lattice ordered loop* G*, generated by the* l*-ideals of this loop, is a* l*-ideal. The intersection of* l*-ideals is a* l*-ideal.*

Theorem 4. *The* l*-ideals of the lattice ordered loop* G *form a distributive and Brown sublattice of the lattice of this loop's normal subloops.*

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The spectral properties of operators in approximate schemes

Ivan Secrieru

The general procedure to approximate solving of equation

$$A\varphi = f, \tag{1}$$

defined on infinite-dimensional spaces X and Y, consists in reduction of (1) to other equation

$$A_n\varphi_n = f_n, \tag{2}$$

formulated on finite-dimensional spaces X_n and Y_n,. The equation (2) is named the computational approximate scheme.

This reduction can be achieved by different methods. The construction of equation (2) using several finite-difference methods is studied in [1], [2]. In [4] this scheme is obtained by a projection method which depend of two basic systems in X. The choice of mesh or a bases are such made that the operator A_n has the main properties of operator A of exact equation. This fact is important in proof of existence of solution for the approximate equation,also in construction of efficient methods to their solutions.

In special, the spectral properties of operators A and A_n are used at the proof of convergence of approximate solutions to exact one. Also this properties are important for the stability of computational approximate schemes. The convergence and stability of finite difference method for boundary value problem are studied in [1] and [3]. In this case if the operator A is selfadjoint, then in (2) it is possible to choice in approximate equation such an operator A_n that has the reel spectrum.

For the integral equation

$$u(x) - \int_0^1 K(x,s)u(s)ds = f(x), \tag{3}$$

where $K(x,s)$ is a symmetric function it is possible, using an adequate quadrature formulas to construct the approximate equation with a symmetric operator K_n . In other case it is possible to obtain for (3) an approximate equation with the operator K_n with reel eigenvalues.

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An intrinsic study on a certain flow of an inviscid compressible fluid, with extension to some cases in magneto-plasma dynamics

Richard Selescu

Keywords: rotational flows, steady and unsteady flows, isentropic surfaces, flow of an electroconducting barotropic inviscid fluid in an external magnetic field

This work studies and clarifies some local phenomena in fluid mechanics as well as in magnetofluid dynamics. A model of a certain isoenergetic flow of an inviscid fluid is introduced, in order to establish a simpler form for the general PDE of the velocity potential. It consists mainly in using a new three-orthogonal system of curvilinear coordinates (one of them being tied to the local specific entropy value). The choice of this system (with two coordinate curves lying on the "isentropic" surfaces) enables the treatment of any 3-D flow (rotational, steady and unsteady) as a potential 2-D one, introducing a 2-D velocity "quasi-potential", specific to any isentropic

surface. The dependence of the specific entropy on this velocity "quasi-potential" was also established. On the above surfaces the streamlines are orthogonal paths of a family of lines of equal velocity "quasi-potential". This method can be extended to some special (but usual) cases in magneto-plasma dynamics (taking into account the flow vorticity effects, as well as those of the Joule-Lenz heat losses), considering a non-isentropic flow of a barotropic inviscid electroconducting fluid in an external magnetic field. There always are some space curves (Selescu) along which the equation of motion admits a first integral, making evident a new physical quantity - the Selescu's vector. For a fluid having an infinite electric conductivity, these curves are the isentropic lines of the flow, also enabling the treatment of any 3-D flow as a "quasi-potential" 2-D one.

Some remarks on the action of the operator of polarizartion

Romi Shamoyan

The so- called operator of polarization is a natural N- dimensional extention of the classical Bergman representation formula from the unit disk to polydisc, see for example [4,5].

I will use some chains of inequalities from [1], estimates for Stein- type maximal functions from [2] and inequalities for tent spaces from [3] to get new estimates from above for this operator in bidisc and polydisc. It will be shown that some estimates extend well known one dimensional inequalities for functions from various analytic spaces.

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Mathematical models for material damage

Meir Shillor

We present mathematical models for the processes of dynamic or quasistatic evolution of the mechanical state of a solid body and the development of material damage which results from internal compression or tension. As microcracks and microcavities open and groe the load carrying capacity of the material decreases, until possible failure. The novelty in these models is the parabolic inclusion for damage evolution.

The mathematical problems consist of a coupled system of a parabolic inclusion for the damage field and a hyperbolic (in the dynamci case) or elliptic (in the quasistatic case) systems for the displacements and stresses.

The modelling, numerical simulations, as well as the the existence of the unique local weak solution are discussed.

The cases of a damageable spring and damageable string are presented in more details.

On Besov-Dunkl spaces

Mohamed Sifi

By using Dunkl harmonic analysis on $\mathbb{R}^d$, we introduce L^p-function spaces that we call Besov-Dunkl spaces and we define Bochner-Riesz means and partial Dunkl integrals. We provide necessary and sufficient conditions, involving Bochner-Riesz means or partial Dunkl integrals, in order that a radial L^p-function belongs to a Besov-Dunkl space. We also provide characterization of these spaces by the Dunkl convolution.

Spectral problems in guided waves propagation in piezoelectric crystals subject to initial fields

Olivian Simionescu-Panait

Based on joint work with **I. Ana**.

The problems related to electroelastic materials subject to incremental fields superposed on initial mechanical and electric fields have attracted considerable attention last period. Soós and Simionescu studied in [8] the case of plane wave free propagation in 6-mm type crystals subject to initial fields, while in [1], [2] we analyzed the characteristics of the obtained waves. In [3] we studied the electrostrictive effect on plane wave propagation in isotropic solids subject to initial fields. In [4] we investigated the conditions of propagation of plane waves in cubic crystals subject to initial deformations and electric fields. In [6] we generalized the previous results, studying the problem of attenuated wave free propagation in an isotropic solid subject to initial electro-mechanical fields. Recent results on attenuated wave propagation in a cubic crystal, subject to initial fields, are described in [5], [7].

This work deals with the study of the coupling conditions for propagation of planar guided waves in a piezoelectric semi-infinite plane (*sagittal plane*). The piezoelectric material behaves linearly and without attenuation and the waveguide propagates in a normal mode. We suppose that the material is subject to initial electro-mechanical fields. If the sagittal plane is normal to a direct, resp. inverse dyad axis, we show that the fundamental system of equations decomposes for particular choices of the initial electric field. In this way we obtain mechanical and piezoelectric waves generalizing the classical guided waves from the case without initial fields (in particular, the Bleustein-Gulyaev wave). Furthermore, we obtain a similar decomposition of boundary conditions, which enable us to characterize the obtained guided waves.

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Recent research in cognitive science and neuroscience: is it relevant to mathematics learning?

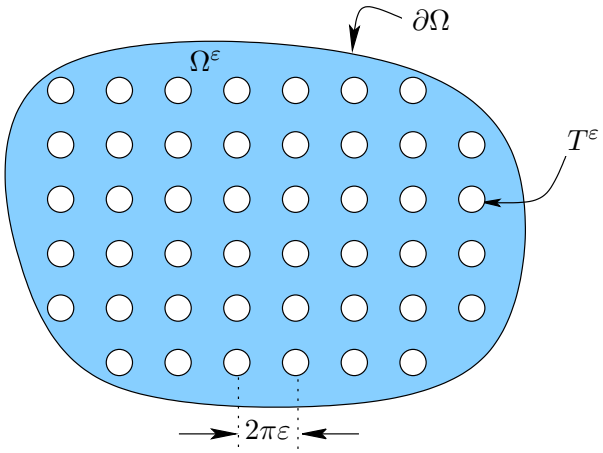
Florence Mihaela Singer

During the last three decades, a large body of research was devoted to analyzing infants' cognitive capacities. A series of experiments suggested that number representation in human has at least three components: one for recognizing numerosity limited up to four items, at a glance, without counting - subitizing; one for approximate numerosities; and the third for large exact numerosities, in which the natural language interferes. The findings in neuroscience and their adequate interpretation in relation with cognition might reshape the traditional ways of teaching and learning. The presentation will provide a description of a model for the dynamic infrastructure of mind that proved to be effective in developing tasks for efficient learning in mathematics. The infrastructure of the cognitive system consists in categories of mental operations foundational for learning that contain inborn components that are self-developing in the interaction mind-environment. The results of this study led to a methodology of learning based on activating the operational infrastructure of mind that enhanced students' flexibility of thinking and the capacity to focus on solving a variety of problems. Some concluding remarks will emphasize issues about how research might advance in connecting achievements in cognitive science and mathematics education.

Bloch waves homogenization of a Dirichlet problem in a periodically perforated domain

Loredana Smaranda

The general question that forms the focus of this work is the Bloch waves homogenization of Dirichlet's type-like problem in perforated domain. For any $\varepsilon > 0$, we consider the set Ω^ε obtained by removing from an open bounded set $\Omega \subseteq \mathbb{R}^N$ $(N \geq 2)$ a periodic network of balls in $\mathbb{R}^N$. The periodicity of the medium is $2\pi\varepsilon$ and the radius of balls is $r(\varepsilon)$. In this domain, we study the homogenization of the Poisson equation with homogeneous Dirichlet conditions on the boundary, including the boundaries of holes. We are going to consider that the hole size $r(\varepsilon)$ depends on the micro-structure size ε such that $r(\varepsilon)$ goes to zero more rapidly than the microstructure size.



$$\begin{cases} -\Delta u^\varepsilon = f & \text{in } \Omega^\varepsilon, \\ u^\varepsilon = 0 & \text{on } \partial\Omega, \\ u^\varepsilon = 0 & \text{on } \partial T^\varepsilon. \end{cases}$$

Using the Bloch waves decomposition technique, we can characterize the limit of the solution of previous problem, as ε goes to zero. We find that there exist different behaviors, depending on the way as the radius $r(\varepsilon)$ goes to zero, which are completely characterized by the technique we use. In the particular case, where $r(\varepsilon)$ converges to zero in a critical way, on the homogenized equation appears the so called "strange term", which is related to the asymptotic behavior of the first eigenvalue of a suitable spectral problem.

The method employed in the analysis of the asymptotic behavior with respect to the hole radius of the first eigenvalue of this problem, depends on the dimension of the space. In the two and three dimensional cases, we use the Green operator and two approximations of it based on the Schiffer-Spencer formula. The error of these approximations can be directly estimated in the L^2-norm. For the case $N \geq 4$ the method used in the previous proof does not work. Therefore, we have to introduce two essential changes. On one hand, we need to iterate the Green operator and on the other hand, it is necessary to work in appropriated L^p spaces in order to obtain the error estimates.

Using this asymptotic behavior and the Bloch waves technique, we find the exact value of the critical hole size, which separates the different behaviors, where the classical strange term may or may not appear in the homogenized equation.

Recent advances in contact mechanics: modelling and variational analysis of the models

Mircea Sofonea

Contact phenomena involving deformable bodies arise in industry and everyday life and play important roles in structural and mechanical systems. Owning to the complicated surface physics involved, they lead to new and nonstandard mathematical models. Considerable progress has been achieved recently in modelling and mathematical analysis of phenomena of contact and, as a result, a general Mathematical Theory of Contact Mechanics is currently emerging as a discipline on its own right. Its aim is to provide a sound, clear and rigorous background to the construction of models, their variational analysis as well as their numerical analysis.

In this lecture we give a review of our major results in the study of quasistatic contact problems involving elastic, viscoelastic and viscoplastic materials. We model the contact with Signorini's unilateral condition or with normal compliance, associated to frictionless or frictional conditions. We derive variational formulations of the models and present existence and uniqueness results for the weak solutions. The proofs are based on arguments of elliptic and evolutionary variational inequalities, time-discretization, regularization, compactness and fixed point. Also, we study the behavior of the solutions and prove various convergence results. Moreover, we consider numerical approximations of the models, derive error estimates for both semi-discrete and fully discrete schemes and present numerical simulations for two and three dimensional test problems.

On some extension and separation results in locally convex cones

Ligia-Adriana Sporiş

In this note, we investigate about some properties of locally convex cones embedded into a linear cone. We prove a few separation and extension results.

Small sample inference for cohort and case-control studies

Ana-Maria Staicu

Based on joint work with **Nancy Reid**.

Case-control studies are a primary tool for the study of factors related to disease incidence, particularly for rare disease. When the main objective is to draw inferences for the log odds ratio parameter, parametric inference is done by the "retrospective" likelihood and involves many nuisance parameters. Prentice & Pyke (1979), Roeder et al (1996) provide theoretical justification for using instead first order theory results along with a "prospective" analysis, which involves scalar nuisance parameter for unstratified studies. For the Bayesian analysis, recent findings by Seaman & Richardson (2004) reveal a specific prior for the equivalence of the two models. We investigate whether their prior is unique, when the model parameters are assumed independent, and furthermore we determine a larger class of priors for equivalence analysis of the two models. From a frequentist perspective we show the approximate equivalence of the Cox-Reid adjusted profile likelihood of prospective and retrospective models to second order. At higher order, the equivalence of the two models is examined only via a simulation study.

We conclude by presenting the results, in terms of p-values corresponding to assessing several hypothesis tests, when same data sets are analyzed prospectively as well as retrospectively by using higher order methods.

Multiple solutions for super linear p-Laplacian Neumann problems

Vasile Staicu

We consider the following Neumann problem driven by the p-Laplacian differential operator

$$\begin{cases} -div\left(\|Dx(z)\|^{p-2} Dx(z)\right) + \beta |x(z)|^{p-2} x(z) = f(z, x(z)) \text{ a.e. on } Z, \\ \frac{\partial x}{\partial n} = 0 \text{ on } \partial Z, \end{cases}$$

where $2 \leq p < \infty$, $Z \subseteq \mathbb{R}^N$ is a bounded domain with a C^2 boundary ∂Z, $\beta > 0$, $f(z, x)$ is a Caratheodory nonlinearity, and n is the outward unit normal on ∂Z.

Our goal is to establish the existence of multiple constant sign solutions and of nodal (sign-changing) solutions for this problem, when the nonlinearity f exhibits a $p-$ superlinear growth near infinity. This question was recently investigated in the framework of the Dirichlet boundary value problem by several authors. We mention the works of Bartsch-Liu [3], Garcia Azorero-Manfredi-Peral Alonso [8], Motreanu-Motreanu-Papageorgiou [9], Papageorgiou-Papageorgiou [11] for $p-$superlinear Dirichlet problems and by Carl-Perera [6], Zhang-Li [14], Zhang-Chen-Li [13] for $p-$linear Dirichlet problems. In contrast, the study of the Neumann case is lagging behind. There have been some multiplicity results, but under symmetry hypotheses on the nonlinearity, for low dimensional problems (i.e., $N < p$) and without any information concerning the sign of the solutions. We mention the works of Anello [2], Binding-Drabek-Huang [4], Bonnano-Candito [5], Filippakis-Gasinski-Papageorgiou [7], Motreanu-Papageorgiou [10], Wu-Tan [12] and the very recent work by the authors [1]. Only Binding-Drabek-Huang [4] provide information about the sign of their solutions for a nonlinearity of the form $f(t, x, \lambda) = \lambda a(z) |x|^{p-2} x + b(z) |x|^{q-2} x$, with a, $b \in L^\infty(Z)$, $\lambda \in \mathbb{R}$, $1 < p < N$, $1 < q < p^*$ (where $p^* = \frac{Np}{N-p}$ if $p < N$ and $p^* = +\infty$ if $p \geq N$). Using a combination of variational arguments based on critical point theory together with truncation techniques and the method of upper-lower solutions, we are able to show that the problem under consideration has five nontrivial smooth solutions, two positive, two negative and a nodal (sign-changing solution). In the semi linear case (p=2), employing tools from Morse theory, we produce a second nodal

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Duality for vector optimization problems

Cristina Stamate

For a vector optimization problem with set valued data we present weak and strong duality theorems, saddle point duality, stability and Kuhn-Tucker conditions and the link between vector and scalar duality theory.

On topological properties of solution set for a class of variational inequalities

Diana Stanciu

Based on joint work with **Miruna Beldiman**.
In this talk we establish the connectedness of the solution set for a class of weak vector variational inequalities, using the equivalence between this family and a family of scalar variational inequalities.

Normal families of ring FMO_{loc}-quasiregular mappings

Victoria Stanciu

We generalize the normality criterium obtained in the paper "Normal Families of BMO_{loc}-quasiregular Mappings", *Complex Variables*, **49** (2004), 10, 681-688, for BMO_{loc}-quasiregular mappings between Riemann surfaces to the class of ring FMO_{loc}-quasiregular mappings. The main result is, the following

Theorem *Let R and R' be two homeomorphic Riemann surfaces, R' is not conformally equivalent to either $\widehat{\mathbb{C}}$ or $\mathbb{C}$, $z_j \in R$, $\zeta_j \in R'$, $j = 0, 1$, $z_0 \neq z_1$, $\zeta_0 \neq \zeta_1$ and $Q \in FMO_{loc}(R)$. If $\mathcal{F}$ is a family of ring $Q(z) - qr$ mappings $f : R \to R'$ such that $f^{-1}(\zeta_j) = z_j$, $j = 0, 1$ then $\mathcal{F}$ is normal.*

Duality for multiobjective nonlinear fractional programming problems involving generalized d- type-I n-set functions

Ioan M. Stancu-Minasian

Based on joint work with **Andreea Mădălina Stancu**

Keywords: Duality, multiobjective programming, n-set functions, d-type-I n-set functions.

We establish duality results under generalized convexity assumptions for a multiobjective nonlinear fractional programming problem involving generalized d-type-I n-set functions. Our results generalize the results obtained by Preda [1] and Preda, Stancu-Minasian and Koller [2].

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Asymptotic approach of a fluid-structure interaction problem

Ruxandra Stavre

A non steady viscous flow in a thin channel with elastic walls is considered. We study three different cases:

1) The periodic case for a viscous fluid with constant viscosity,

2) The non-periodic case for a viscous fluid with constant viscosity,

3) The non-periodic case for a viscous fluid with variable viscosity.

The problem contains two small parameters. For various ratios of these parameters an asymptotic solution is constructed. In the last two cases, the asymptotic solution corresponding to the periodic case is modified by using a boundary layer method, in order to obtain a good approximation for the exact solution. In each case, the leading terms of the asymptotic solution are analised. The properties of the correctors (functions which modify the periodic solution) are established and, by using some *a priori* estimates, the error between the exact and the asymptotic solution is obtained.

Counting balanced polynomials over finite fields

Pantelimon Stănică

Based on joint work with **T.W. Cusick** and **Y. Li**

Under mild conditions on n, p, we give a lower bound on the number of n-variable balanced symmetric polynomials over finite fields $GF(p)$, where p is a prime number. The existence of nonlinear balanced symmetric polynomials is an immediate corollary of this bound. Furthermore, we conjecture that $X(2^t, 2^{t+1}\ell - 1)$ $(t, \ell \in \mathbb{Z})$ are the only nonlinear balanced elementary symmetric polynomials over $GF(2)$, where $X(d, n) = \sum_{i_1 < i_2 < \cdots < i_d} x_{i_1} x_{i_2} \cdots x_{i_d}$, and we prove various results in support of this conjecture.

Trichotomy in infinite dimensional spaces

Codruţa Stoica

Based on joint work with **M. Megan**.

The last years where characterized by an impressive development of evolution equations in infinite dimensional spaces, a greater importance being given to the study of asymptotic properties of evolution operators and cocycles over a semiflow respectively. The theory has seemingly reached a kind of maturity. We can talk about conditions that describe the exponential stability, the exponential instability as well as the dichotomy of evolution operators in Banach and Hilbert spaces.

As a natural generalization of the classical concept of dichotomy is the notion of trichotomy. In the study of trichotomy the basic idea is to obtain at any moment the decomposition of the space into three closed subspaces: a sable one, an unstable one and a neutral one. In the finite dimensional case noticeable results concerning trichotomy and exponential trichotomy were obtained by Sacker and Sell ([7]) as well as by Elaydi and Hajek ([1]).

The progress made in the last years in the study of asymptotic properties of cocycles over semiflows were used in the theory of evolution equations in infinite dimensional spaces. The approach from the point of ie of the asymptotic behavior of the associated evolution semigroup was essintial.

The concept of evolution cocycle over semiflows is more appropiate for the study of asymptotic behavior of evolution equations with the aid of evolution of evolution operators ([2 – 6]).

The present paper is aimed to present in an unitary way the trichotomy properties in continuous and discrete time in the uniform as well as the non-uniform case, in infinite dimensional spaces.

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Classes of operators similar to partial isometries and generalized inverses

Laurian Suciu

Keywords : Partial isometry, quasi-isometry, generalized inverse, Duggal transform, Aluthge transform, unitary dilation.

The present talk deals with operators similar to partial isometries. We get some (necessary and) sufficient conditions for the similarity to (adjoint of) quasinormal partial isometries, or more general, to power partial isometries. We illustrate our results on the class of n-quasi-isometries, obtaining that a n-quasi-isometry is similar to a power partial isometry if and only if the ranges $\mathcal{R}(T^j)$ $(1 \leq j \leq n)$ are closed. In particular, if $n = 2$ these conditions ensure the similarity to quasinormal partial isometries of Duggal and Aluthge transforms of 2-quasi-isometries. The case when a n-quasi-isometry is a partial isometry is also studied, and a structure theorem for a n-quasi-isometry which is a power partial isometry is given. We obtain some partial results related to the problems of Badea-Mbekhta [BM] concerning the similarity to partial isometries using the generalized inverses. A characterization for such a similarity is given in the terms of dilations similar to unitary operators, which leads to a new criterion for the similarity to an isometry.

Analytic functions of ρ-contractions

Nicolae Suciu

The classical theory of contractions is near to the analytic functions on the unit disc, by the von Neumann or Riesz-Dunford functional calculus. In fact, the von Neumann functional calculus can be defined for the class of polynomial bounded operators, but satisfactory results are especially obtained for the operators of class C_ρ (that is ρ-contractions) in the Sz. Nagy-Foias sense. We refer to this functional calculus for (uniformly) stable ρ-contractions (i.e. having the spectrum in the open unit disc), and in particular for strict ρ-contractions. In this context we give different forms of the von Neumann and Schwarz-Pick inequalities, as well as the classical Harnack inequalities. The last ones lead to a Harnack metric in the set of stable operators, and to a hyperbolic metric on the Harnack parts of such operators. Also, the stability of C_ρ classes under the functional calculus is studied.

The algorithms of Sister Celine and R. W. Gosper, Jr., for the hypergeometric summation

Eleutherius Symeonidis

By an "identity" in mathematics we usually mean an equation which is true without being obvious. The simplest kind of identities are those which can be verified by direct computation, as for instance

$$a^3+b^3+c^3-3abc=\frac{1}{2}(a+b+c)\left[(a-b)^2+(b-c)^2+(c-a)^2\right] \quad (a,b,c\in\mathbb{R})\,.$$

For other ones, like

$$\sum_{k=0}^{n}\binom{n}{k}=2^n \quad \text{or} \quad \sum_{k=0}^{n}\binom{n}{k}^2=\binom{2n}{n} \qquad (n\in\mathbb{N}\cup\{0\}) \tag{17}$$

this is not possible; here a mathematical proof is needed. The method of induction serves as a tool for proving some identities of this kind, but it is far from being always applicable. Therefore, there is a need of more adequate methods for establishing identities of the general form

$$\sum_{k=k_0}^{m}F(n,k)=f(n,k_0,m)\,. \tag{18}$$

In this talk we present two such methods. Sister Celine's one applies to sums of maximal range, that is when (18) takes the special form

$$\sum_{k=-\infty}^{\infty}F(n,k)=f(n) \qquad (n\in\mathbb{N}\cup\{0\})\,.$$

Here, the support of $k\mapsto F(n,k)$ — and consequently the sum — is usually finite. In (17) we have two examples of this situation. R. W. Gosper's algorithm handles the arbitrary case (18). Not only it gives a proof, but much more: Given the left side of (18), it discovers the right side if a "simple" expression $f(n,k_0,m)$ for the sum exists, otherwise it proves its non-existence. The database of candidates for $f(n,k_0,m)$ consists of the so-called hypergeometric sequences. An example of an identity for which Gosper's algorithm applies is the following:

$$\sum_{k=0}^{m}(-1)^k\binom{n}{k}=(-1)^m\left(1-\frac{m}{n}\right)\binom{n}{m}\,.$$

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Commuting systems of bounded operators, factorisation, and spectral correspondence

Mihai Şabac

An analogue of the classical spectral mapping theorem can be proved in circumstances more general than those offered by functional calculus. We discuss the case of "factorisation" of a bounded operator S by an n-tuple of bounded operators $(T_1, \ldots, T_n)$ when $\{S, T_1, \ldots, T_n\}$ is a commuting system acting on the same complex Banach space and the "factorisation" is given by the relation $S = T_1U_1 + \cdots + T_nU_n$, where $U_i \in \{S, T_1, \ldots, T_n\}'$, the commutant of $\{S, T_1, \ldots, T_n\}$.

Dirac fermions in curved space: spinorial invariance, densities and conservation laws

Ion Şandru

Based on joint work with **A. I. Şandru**.
In this talk, we study some properties regarding the spinorial invariance of Dirac's operators in context of curved spaces.

Differential operators that divide the wave operator. Applications in the mathematical physics

Ovidiu Ilie Şandru

Based on joint work with **I. Bacalu** and **I. Gruia**.

The notation of "differential operator that divides the wave operator" is mentioned in the works of the first author of this communication. The differential operators that divide the wave operator build a class of differential operators with special properties whose important role to the equation theory with partial derivatives and implicitly to the numerous applications in the field of the mathematical physics equations that was highlighted for the first time by the well-known Academician, Nicolae Teodorescu. The aim of this communication is to make known some of the applications of the operators from the class already mentioned. With this purpose a solving method of the Cauchy problems for some of the famous equations in the field of mathematical physics will be presented.

Mathematical proofs and algorithms

Doru Ştefănescu

We discuss the impact of the use of implementable algorithms on the philosophy and techniques of mathematical proofs, looking at relations between mathematical proof, constructive proof and algorithmic proof. We emphasize some aspects concerning the implicit function theorem and the existence of real roots for polynomials over reals with sing variations.

Traian Lalescu and the birth of algebra in Romania

Doru Ştefănescu

In this talk we discuss the contributions of Traian Lalescu to Algebra. During his PhD studies in Paris he published (1907, 1908) several research papers on Galois Theory and quadratic forms. These papers are arguably the first original contributions of a Romanian mathematician to Algebra.

Tiling and interaction

Gheorghe Ştefănescu

Tiling is an old and popular subject, which is and has been of interest not only for mathematicians, but also for general public. A marvelous presentation of the subject may be found in [2]. Recently, one can notice an increasing interest coming from discrete mathematics and computer science, mostly related to cellular automata, picture languages, logics, and complexity theories - see [3], for a recent survey.

Our interest comes from a different perspective: *interactive systems*. Interactive computing is emerging as a new paradigm in the new era of computer science which is dominated by the Internet, computing agents, embedded processors, autonomous robots, pervasive computers, etc. In this area, one may use two dimensional tiles, where one dimension represents the *space*, while the other represents the *time*.

In this talk we present a model, a core programming language, specification and analysis techniques appropriate for modeling, programming and reasoning about interactive computing systems. The model consists of *rv-systems* and is space-time invariant [4,5]. We also briefly describe the kernel programming language AGAPIA v0.1 [1], together with a type checking procedure, implementation studies, and verification techniques.

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Estimating the accuracy of some probabilistic algorithms. Applications

Ştefan Ştefănescu

In general it isnŠt so easy to analyze theoretically the accuracy of a probabilistic algorithm. For this reason the probabilistic problems are usually approached from a practical point of view by using computers and Monte Carlo techniques. The paper suggests how to improve a Monte Carlo algorithm when we have supplementary information. The weighting of the information is used too. The examples study how to estimate by simulation the volume of a complicated body or how to redirect a random search activity used in stochastic optimization. Finally, are proposed some generalizations of a classical Monte Carlo procedure to improve the accuracy of the p estimation.

Blow up for some critical semilinear wave equations

Daniel Tătaru

Based on joint work with **Wilhelm Schlag** (U. Chicago) and **Joachim Krieger** (Harvard).

We consider two semilinear wave equations, namely the wave-maps in 2+1 dimensions and the focusing $\Box u = u^5$ problem in 3+1 dimensions. Both have a conserved energy and are energy critical in the sense that the energy is scale invariant. We prove that blow-up can occur in finite time for arbitrarily smooth initial data.

Dislocation-based constitutive modelling at large plastic strains

Cristian Teodosiu

In recent years there has been a renewed interest in modelling the plastic anisotropy induced by the microstructural evolution during large deformations. This is mainly due to the progress achieved in investigating the underlying physical phenomena and in the use of the emerging multiscale modelling of materials. While statistical approaches have considerably evolved in this area, the most effective way of incorporating in the continuum mechanics the knowledge acquired at the micro- and mesoscale remains the introduction of the so-called internal state variables, which are some average macroscopic substitutes of the enormous amount and variety of microstructural features.

The aim of this lecture is to review some significant aspects of the dislocation modelling of the large deformation plasticity of single crystals and crystalline aggregates, by making use of the internal-variable approach.

For single-crystal plasticity, the most important internal variables are the scalar dislocation densities on various glide planes. Their evolution is governed by balance equations involving dislocation production and annihilation rates. On the other hand, dislocation interactions determine in a basically anisotropic way the slip rates and the evolution of the critical shear stresses.

Recently, such dislocation-based models of continuum plasticity have been employed for the finite-element simulation of inhomogeneously deformed crystalline aggregates. Such simulations help understanding the influence of the crystallographic mismatch across grain boundaries and of the difference in size between neighbouring grains on the heterogeneity of plastic deformation and on the strain localization and damage.

Going now to polycrystalline materials, one of the most striking features of the microstructural organization inside the grains is that dislocations evolve towards some steady-state microstructures, provided that a sufficient amount of monotonic deformation is allowed for along the same strain path. Reversed deformation and changes in strain path generally tend to the modification or dissolution of preformed microstructures and the formation of new ones that correspond to the last deformation mode. The last part of the lecture will focus on some recently-developed models of such processes, by means of tensor-valued internal variables associated to the directional strength and polarity of dislocation structures. The evolution equations of such variables have a work-hardening/recovery format, which is typical for the asymptotic approach of steady states, under constant thermomechanical loadings.

Polynomials and vanishing cycles

Mihai Tibar

We are concerned with a geometro-topological approach to the vanishing cycles appearing in non-proper fibrations, where the vanishing cycles do not correspond necessarily to singularities on the space. The aim is to conceive a bunch of topics—holomorphic germs, polynomial functions, pencils on quasi-projective spaces, meromorphic functions and non-generic Lefschetz pencils—as aspects of a single theory with vanishing cycles at its core.

Upscaling in diffusion problems in domains with semipermeable boundaries

Claudia Timofte

Keywords: homogenization, reactive flows, variational inequality, monotone graph.

The asymptotic behavior of the solutions of some nonlinear variational inequalities with highly oscillating coefficients modeling chemical reactive flows through the exterior of a domain containing periodically distributed reactive solid obstacles with semipermeable boundaries is analyzed. We focus on the case in which the obstacles are of the so-called *critical size* and we prove that the solution of such a boundary-value problem converges to the solution of a new problem, associated to an operator which is the sum of a standard homogenized one and extra zero order terms coming from the geometry and the nonlinearity of the problem.

The eigenvalues of a sum of selfadjoint operators

Dan Timotin

Based on joint work with **H. Bercovici** and **W.S. Li**.

The famous Horn conjecture concerning the eigenvalues of a sum of selfadjoint matrices has been solved in recent years by work of Klyachko, Totaro, Knutson and Tao. After presenting briefly these developments, we will discuss some extensions of these results to operators acting on infinite dimensional Hilbert spaces.

Time-constraining multiobjective fractional transportation problem

Alexandra Tkacenko

The multiobjective fractional programming models are of greater interest in our daily life. We are often concerned about the optimization of the ratios like the summary cost of the total transportation expenditures to the maximal necessary time to satisfy the demands, the total benefits or production values into time unit, the total depreciation into time unit and many other important similar criteria, what may appear in order to evaluate the economical activities and make the correct managerial decisions. These problems led to the multiple criteria transportation model of fractional type with identical denominators, where the "bottleneck" criteria appear as a "minmax" time constraining. In the proposed paper I studied the transportation problem of "bottleneck" type with multiple fractional criteria that is defined as follows:

$$\min z_1 = \frac{\sum_{i=1}^{m}\sum_{i=1}^{m} c_{ij}^1 x_{ij}}{\max\limits_{i,j}\{t_{i,j}/x_{i,j} > 0\}} \quad \min z_2 = \frac{\sum_{i=1}^{m}\sum_{i=1}^{m} c_{ij}^2 x_{ij}}{\max\limits_{i,j}\{t_{i,j}/x_{i,j} > 0\}}$$

...

$$\min z_r = \frac{\sum_{i=1}^{m}\sum_{i=1}^{m} c_{ij}^r x_{ij}}{\max\limits_{i,j}\{t_{i,j}/x_{i,j} > 0\}} \quad \min z_{r+1} = \max_{i,j}\{t_{i,j}/x_{i,j} > 0\} \tag{19}$$

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad \forall i = \overline{1,m} \quad \sum_{i=1}^{m} x_{ij} = b_j, \quad \forall j = \overline{1,n} \quad \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

where c_{ij}^k, $k = 1, \ldots, r$, $i = 1, \ldots, m$, $j = 1, \ldots, n$ correspond to the concrete interpretation of the respective criteria, a_i- availability at source i, b_j- requirement at destination j $x_{ij}-$ amount transported from source i to destination j.

In this paper We suggested reducing the model (1) to another multicriterial linear model, in order to solve this. We developed an iterative procedure of finding the basic efficient solutions of this. The theorems that prove the equivalence of the both models, meaning the common set of their basic efficient solutions are given. The proposed algorithm was tested on some examples.

Long-time stability of the implicit Euler scheme for the 2D Navier–Stokes equations

Florentina Tone

In this talk I will discuss the stability for all positive time of the fully implicit Euler scheme for the 2D Navier–Stokes equations. More precisely, I will discretize the Navier–Stokes equations in time using the implicit Euler scheme and with the aid of a discrete Gronwall lemma and of a discrete uniform Gronwall lemma, I will prove that the numerical scheme is unconditionally stable (uniformly in time).

Stability of mixing and rapid mixing for hyperbolic flows

Andrei Török

This is joint work with **Michael Field**, University of Houston, USA, and **Ian Melbourne**, University of Surrey, UK.

We study the mixing properties of hyperbolic flows (either Anosov flows, or the more general Axiom A flows). We obtain the first general results on the stability of rapid mixing (or even mixing) for Axiom A flows that holds in a C^r, as opposed to Hölder, topology.

Recall that for a flow $\Phi_t : \Lambda \to \Lambda$ that preserves a probability measure μ and $A, B \in L^2(\Lambda, \mu)$, the correlation function is

$$\rho_{A,B}(t) := \int_\Lambda (A \circ \Phi_t) B d\mu - \int_\Lambda A d\mu \int_\Lambda B d\mu, \qquad t \in \mathbb{R}.$$

The flow is mixing if $\rho_{A,B}(t) \to 0$ as $t \to \infty$, for each $A, B \in L^2(\Lambda, \mu)$. The flow is rapid mixing if $\rho_{A,B}(t) \to 0$ superpolynomially fast on sufficiently smooth observations A, B.

We show that amongst C^r Axiom A flows, $r \geq 2$, there is a C^2-open, C^r-dense set of flows for which each nontrivial hyperbolic basic set is rapid mixing.

About the spin structures on generalized twistor space

Adriana Turtoi

Le us consider $\mathbf{G}(M)$ the twistor space of generalized complex structures, [3], [4] on a $2n-$ dimensional manifold M as the associated bundle with the principal fibre bundle of positiv oriented frames of $TM \oplus T^*M$, with structure group $SO(2n, 2n)$ and standard fibre the homogeneous space $SO(2n, 2n)/U(n, n)$. Using the results of M Karoubi [5] which contains the topological obstructions for the existence of a spin structure on a pseudo- Riemannian manifold, we show that $\mathbf{G}(M)$ is $Spin(4n^2, 2n)$

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Non - standard properties in some classes of non - Newtonian fluids

Victor Ţigoiu

We consider the general class of third grade fluids with thermodynamical restrictions obtained in 1987. As it is well known, for this class of fluids (see Tigoiu 2001), that the rest state is asymptotically stable (this property assures the correctness of the model as a fluid model). Based on this result, this class has been investigated in various particular (simple and complex) flows. Two apparently surprising results have been obtained and presented in this paper, namely:

- the presence of secondary flows in non-circular (elliptic) pipes (a result proved by Coleman and Noll only for forth grade fluids);

- the propagation of discontinuity surfaces (like acceleration waves). It is well known (see for instance Truesdell) that in a linear, incompressible, viscous fluid the discontinuity surfaces does not propagate. The key of this result in a third grade fluid is due to the presence in the constitutive law of second order accelerations.

Such results cannot be putted into evidence with particular models like the classic model developed by Fosdick and Rajagopal, due to the absence (from the corresponding constitutive law) of above mentioned acceleration terms (in particular).

Nonlinear resolvents: supermedian functions

Corneliu Udrea

This work contains a study of the supermedian functions with respect to a nonlinear operator on a measure space. It is obtained the relation between the supermedian functions and the weak dominant functions. Afterwards it is introduced the supermedian functions with respect to a nonlinear sub-Markov resolvent and the corresponding class of dominant functions. This text completes the results from [8].

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Multi-time controllability, observability and Bang-Bang principle

Constantin Udrişte

Control problems for multi-time first order PDEs arise in many different contexts and ways. But the obstruction of complete integrability conditions (path independent curvilinear integrals) determines the mathematicians to study such problems only in the discrete context, though in this way they loose the geometrical character which is proper to the continuous approach.

In this talk, we study controllability, observability and bang-bang properties of multi-time completely integrable linear PDEs systems, overcoming the extant mathematical prejudices regarding the importance of a multi-time evolution of m-flow type. Our geometrical arguments show that each basic theorem has a correspondent in the case of single-time linear controlled ODEs system.

The main results include controllability criterions, the equivalence between controllability of a PDEs system and the observability of the dual PDEs system, the geometry of the control set, the extremality and multi-time bang-bang principle. All of these show that the passing from controlled single-time evolution (1-flow) to the controlled multi-time evolution (m-flow) is not trivial. Changing the geometrical language, the case of nonholonomic evolution can be recovered easily from our theory.

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Equilibrium principles for Pareto-Nash-Stackelberg games

Valeriu Ungureanu

We study various equilibria principles for strategic form games that involve both sequential decisions (Stackelberg game) and simultaneous decisions (Nash game, multi-criteria Pareto-Nash game) made by independent and interdependent players. Via notion of the *best response mapping graph* we define *unsafe and safe Stackelberg equilibria* for Stackelberg games, *pseudo and multi-stage Nash-Stackelberg equilibria* for Nash-Stackelberg games, and *Pareto-Nash-Stackelberg equilibria* for multi-criteria Pareto-Nash-Stackelberg games.

At every stage (level) of the Nash-Stackelberg game a Nash game is played. Stage profiles (joint decisions) are executed sequentially throughout the hierarchy as a Stackelberg game.

At every stage of the multi-criteria Pareto-Nash-Stackelberg game a multi-criteria Pareto-Nash game is played. Stage profiles are executed sequentially throughout the hierarchy.

Existence theorems are proved. Various properties are revealed. Some illustrative examples are given.

The carried out investigation continues and extends the Nash game research via Nash equilibrium as an element of the *best response mapping graphs intersection* [1,2]. It is significant that this research permits, among others, to look at a classical dynamic programming from a new perspective.

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Real Analyticity of Hausdorff dimension for higher dimensional graph directed Markov systems

Mariusz Urbanski

We will discuss the result that the Hausdorff dimension function of the limit sets of strongly regular, countable, conformal graph directed Markov systems living in higher dimensional Euclidean spaces $\mathbb{R}^d$, $d \geq 3$, and with an underlying finitely irreducible incidence matrix is real-analytic.

Product of quasivarieties of algebraic systems

Vasile Ursu

Let A_i, $i \in I$ be a totality of algebraic systems of arbitrary signatures. We complete the signature of every system with the functional symbols of projections and the predicational symbols (if lacking) of the characteristic functions of the Cartesian powers of this system's support set. The system we obtain in such a way will be referred to as enriched algebraic system and will be denoted by A_i.

The enriched Cartesian product of the algebraic systems A_i, $i \in I$ is an algebraic system $\otimes A_i$ with the basic set $A = \sqcap_{i \in I} A_i$, that has a basic m operation f^A and a basic n predicate p^A for every family of m operations $(f_i^{A_i} | i \in I)$ and every family of n predicates $(p_i^{A_i} | i \in I)$ and doesn't have other basic operations and predicates besides the aforementioned, defines by the following relations:

$$f^A(a_1, \ldots, a_m) = (f_i^{A_i}(a_1(i), \ldots, a_m(i) | i \in I),$$

$$p^A(a_i, \ldots, a_n) \Leftrightarrow \vee_{i \in I} p^{A_i}(a_1(i), \ldots, a_n(i)),$$

where $a_1, \ldots, a_m$ are elements of A.

Let Q_i, $i \in I$ be a totality of quasivarities of algebraic systems. There quasivarieties may be of various types.

The *product of quasivarieties* Q_i, $i \in I$ is defined as the quasivariety generated by all enriched Cartesian products $\otimes A_i$ of the algebraic systems A_i, $i \in I$. We will denote the product of quasivarieties Q_i, $i \in I$ by $\otimes Q_i$ and by $Q_1 \otimes \ldots \otimes Q_n$ if $I = \{1, \ldots, n\}$. The following results are obtained on the basis of some statements from the quasivariety theory [1,2]:

Theorem 1. *A subset B of the algebraic system $A = A_1 \otimes \ldots \otimes A_n$ is a subsystem of system A if and only if for any $i = 1, \ldots, n$ there exists such a subsystem B_i in system A_i that $B \cong B_1 \otimes \ldots \otimes B_n$.*

Theorem 2. *The congruence lattices $Con(A_1 \otimes \ldots \otimes A_n)$ and $Con(A_1 \times \ldots \times (A_n)$ are isomorphic.*

Theorem 3. *Epimorphism* $\varphi_i : A_1 \otimes \ldots \otimes A_n \to B$ *exists if and only if epimorphisms* $\varphi_i : A_i \to B_i$, $i = 1, \ldots, n$ *also exist, so that* $\varphi : A_1 \otimes \ldots \otimes A_n \to B$.

Corollary. *If* $Q_1, \ldots, Q_n$ *are quasivarieties (respectively, varieties) then the algebraic system* $A \in Q_1 \otimes \ldots \otimes Q_n$ *if and only if there exist such algebraic systems* $A_i \subseteq Q_i$, $i = 1, \ldots, n$ *that* $A \cong A_1 \otimes \ldots \otimes A_n$.

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Proof, explanation and example in the mathematical discource of some Romanian mathematicians: Dan Barbilian and Grigore C. Moisil

Dragoş Vaida

The distinction between syntactic and semantic methods has an interesting history via Bernays, Tarski, Robinson and others but has forcefully emerged from the important contributions of Kurt Goedel to the Foundations of Mathematics. The paper reports on two issues: Different ways of understanding the nature of the mathematical exercise and the works of D. Hilbert illustrated by the Romanian mathematicians D. Barbilian and Gr. C. Moisil; Trying to explain the references to theology due to Goedel with support ideas quoted from G. Frege but also from theologicians, e. g., V. Losky.

The beginning of computer science in Romania and the role of Professor Grigore C.Moisil

Dragoş Vaida

The first steps of informatics in Romania are closely linked to professor Gr. C. Moisil, the pioneer of this field in our country. Progress and development during 1957 and 1975 are due to his works and to his disciples contributions. The present paper aims to confirming the following two statements included in Gr. C. Moisil, 1970 paper, which read: "Computer science - our note, namely the papers, workshops and training carried out in our country - have their roots in some specific interests of the mathematicians" and "In march 1957 when the activity we will talk about has started, there was no computer working in our country".

The present paper deals with how computer science was born out from the mathematical researches. In time new institutions and outstanding names joined the common effort bringing along a valuable contribution to what informatics is today in Romania.

The second part of this paper presents with natural and reasonable details some main works published between 1957 and 1967.

The complete text of the above two parts are available in Romanian with the extended associated bibliography.

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Weak hamiltonian dynamical systems

Izu Vaisman

We propose a generalized Hamiltonian environment consisting of a g-isotropic subbundle E of $TM \oplus T^*M$, where M is a differentiable manifold, TM, T^*M are the tangent and cotangent bundle of M and g is the neutral metric defined by pairing. E is called a big-isotropic structure. Furthermore, E is integrable if it is closed by the Courant bracket. A function $f \in C^\infty(M)$ is Hamiltonian if there exists a vector field X_f such that $(X_f, df) \in E$ and f is weak-Hamiltonian if $(X_f, df) \in E^{\perp_g}$. The corresponding vector fields X_f are Hamiltonian, respectively weak-Hamiltonian, fields. We derive the expression of the weak-Hamiltonian vector fields and the integrability conditions if E satisfies a supplementary regularity condition. We show that the port-controlled Hamiltonian systems and constrained mechanics have a weak-Hamiltonian interpretation. We study symmetries and reduction for weak-Hamiltonian systems.

Unbounded subnormal operators

Florian-Horia Vasilescu

The class of bounded subnormal operators, that is, bounded Hilbert space linear operators having normal extensions on larger spaces, is fairly well known and understood. The Halmos-Bram criterion of subnormality, particularly simple and elegant, is a powerful tool in the study of this type of operators.

Once the boundedness hypothesis is given up, although the definition of subnormal operators makes perfectly good sense, many of the properties valid in the bounded case are either false or difficult to be obtained. In particular, the Halmos-Bram criterion fails and it is not too clear which would be an appropriate substitute for it.

We propose an approach to the class of unbounded subnormal operators using algebras of fractions of continuos functions and extension theorems of completely contractive maps, defined on such algebras.

On periodically correlated processes in complete correlated actions

Ilie Valuşescu

If $\mathcal{H}$ is an $\mathcal{L}(\varepsilon)$ - right module, in the general context of a complete correlated action $\{\mathcal{E}, \mathcal{H}, \Gamma\}$, an apropriate study for operatorial valued periodically Γ - correlated processes in $\mathcal{H}$ is given.

On some new results in the summation of numerical series

Andrei Vernescu

In the present work , using essentially the properties of the power series, we obtain the sums of certain numerical series of a special form. So, with the notation $\Omega_n = \frac{1\cdot 3\cdot 5\cdot \ldots \cdot (2n-1)}{2\cdot 4\cdot 6\cdot \ldots \cdot 2n}$, we find again the known result $\sum_{n=1}^{\infty} \frac{\Omega_n}{n} = \ln 4$ and we obtain the following new results:

$$\sum_{n=1}^{\infty} \frac{(-1)^n \Omega_n}{n} = 2\ln \frac{2}{1+\sqrt{2}}; \qquad \sum_{n=1}^{\infty} \frac{\Omega_n}{2n+1} = \frac{\pi}{2} - 1;$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n \Omega_n}{2n+1} = \ln(1+\sqrt{2}) - 1; \quad \sum_{n=1}^{\infty} \frac{\Omega_n}{n+1} = 1; \quad \sum_{n=1}^{\infty} \frac{(-1)^n \Omega_n}{n+1} = 2\sqrt{2} - 3.$$

In the final part, we obtain that, for real number $p > 0$, we have:

$$\sum_{n=1}^{\infty} \frac{\Omega_n}{n+p} = 2^{p-1} \frac{\Gamma^2(p/2)}{\Gamma(p)} - \frac{1}{p}.$$

Some final remarks close the work.

Application of Dirichlet L-series in the calculus of integrals

Radu-Octavian Vîlceanu

In this article are enunciated and demonstrated adding formulas for integrals of form

$$\int_0^1 \frac{\sum_{n=1}^{q-1} \chi_q(n)\, x^{n-1}}{1-x^q} \ln\left(\ln \frac{1}{x}\right) dx,$$

where χ_q is the odd character Dirichlet (mod q). These formulas are predicted through the utilization of a program in Maple having on the base the PSLQ algorithm of Ferguson. Then, are given the rigorous demonstrations using L-series of Dirichlet.

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Every point is critical

Costin Vîlcu

Based on joint work with **Imre Bárány, Jin-ichi Itoh**, and **Tudor Zamfirescu**

In the following, by *surface* we always mean a compact Alexandrov surface (with curvature bounded below and without boundary), as defined for example in [2].

For any surface S, denote by ρ its (intrinsic) metric, and by ρ_x the *distance function* from $x \in S$, given by $\rho_x(y) = \rho(x, y)$. Put $\mathrm{diam}(S) = \max_{y,z \in S} \rho(y, z)$ and call $\rho_x^{-1}(\mathrm{diam}(S))$ the *diametrally opposite set* of x, if it is not void.

A point $y \in S$ is called *critical* with respect to x, if for any direction v tangent to S at y there exists a segment from y to x whose tangent direction at y makes an angle $\alpha \leq \pi/2$ with v. Denote by Q_x the set of all critical points with respect to x, and put $Q_y^{-1} = \{x \in S : y \in Q_x\}$.

The survey [4] by K. Grove presents the principles of the critical point theory for distance functions, as well as some of its applications.

We proved [1] that every point on every surface is critical with respect to at least one (other) point of the surface. The lower bound $\mathrm{card} Q_y^{-1} \geq 1$ is sharp, and it is applied to characterize the orientable surfaces homeomorphic to the sphere. Then an upper bound is provided for $\mathrm{card} Q_y^{-1}$ in the case of orientable surfaces S of genus $g \geq 1$, $\mathrm{card} Q_y^{-1} \leq \max\{5, 8g - 4\}$, and it is applied to estimate the cardinality of diametrally opposite sets on S. Roughly speaking, our last result shows, for orientable surfaces, that *generically* the sets Q_y^{-1} have an odd number of elements. Some open problems will also be presented, if the time will allow.

Since every farthest point from x is critical with respect to x, our results also contribute to a description of farthest points H. Steinhaus has asked for (see §A35 in [3]).

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The non-anticipation of the asynchronous systems

Şerban E. Vlad

The asynchronous systems are the models of the asynchronous circuits from the digital electrical engineering. Let U be a set of $\mathbf{R} \to \{0,1\}^m$ functions. An *asynchronous system* is a multi-valued function f that associates to each cause $u \in U$ called *admissible input*, several effects $x \in f(u)$ called *possible states,* where $x : \mathbf{R} \to \{0,1\}^n$.

If $A \subset \mathbf{R}$ is an interval, we denote by $u_{|A}$ the restriction of u at A.

Here are some definitions of the *non-anticipation* of f:

a) $\forall u \in U$, $\forall x \in f(u)$, one of the following statements a.1), a.2) is true

a.1) x is constant,

a.2) u, x are both variable and we have

$\min\{t | u(t-0) \neq u(t)\} \leq \min\{t | x(t-0) \neq x(t)\}$;

b) $\forall t \in \mathbf{R}$, $\forall u \in U$, $\forall v \in U$,

$u_{|(-\infty,t)} = v_{|(-\infty,t)} \Longrightarrow \{x_{|(-\infty,t]} | x \in f(u)\} = \{y_{|(-\infty,t]} | y \in f(v)\}$;

c) $\forall t \in \mathbf{R}, \forall u \in U, \forall v \in U$,

$u_{|(-\infty,t)} = v_{|(-\infty,t)} \Longrightarrow \{x(t) | x \in f(u)\} = \{y(t) | y \in f(v)\}$;

d) $\exists d > 0, \forall t \in \mathbf{R}, \forall u \in U, \forall v \in U$,

$u_{|[t-d,t)} = v_{|[t-d,t)} \Longrightarrow \{x(t) | x \in f(u)\} = \{y(t) | y \in f(v)\}$;

e) $\forall t \in \mathbf{R}, \forall u \in U, \forall v \in U$,

$u_{|(-\infty,t]} = v_{|(-\infty,t]} \Longrightarrow \{x_{|(-\infty,t]} | x \in f(u)\} = \{y_{|(-\infty,t]} | y \in f(v)\}$;

f) $\forall t \in \mathbf{R}, \forall u \in U, \forall v \in U$,

$u_{|(-\infty,t]} = v_{|(-\infty,t]} \Longrightarrow \{x(t) | x \in f(u)\} = \{y(t) | y \in f(v)\}$;

g) $\exists d, \exists d', 0 \leq d \leq d'$ and $\forall t \in \mathbf{R}, \forall u \in U, \forall v \in U$,

$u_{|[t-d',t-d]} = v_{|[t-d',t-d]} \Longrightarrow \{x(t) | x \in f(u)\} = \{y(t) | y \in f(v)\}$.

We present also some definitions of the *non-anticipation** of f:

i) $\forall t \in \mathbf{R}, \forall u \in U, \forall v \in U$,

$(u_{|[t,\infty)} = v_{|[t,\infty)}$ and $\{x(t) | x \in f(u)\} = \{y(t) | y \in f(v)\}) \Longrightarrow$

$\Longrightarrow \{x_{|[t,\infty)} | x \in f(u)\} = \{y_{|[t,\infty)} | y \in f(v)\}$;

ii) $\forall t \in \mathbf{R}, \forall u \in U, \forall v \in U$,

$u_{|[t,\infty)} = v_{|[t,\infty)} \Longrightarrow \exists t' \in \mathbf{R}, \{x_{|[t',\infty)} | x \in f(u)\} = \{y_{|[t',\infty)} | y \in f(v)\}$;

iii) $\forall t \in \mathbf{R}, \forall u \in U, \forall v \in U$,

$(u_{|[t,\infty)} = v_{|[t,\infty)}$ and $\{x_{|(-\infty,t]} | x \in f(u)\} = \{y_{|(-\infty,t]} | y \in f(v)\}) \Longrightarrow$

$\Longrightarrow \exists t' \in \mathbf{R}, \{x_{|[t',\infty)} | x \in f(u)\} = \{y_{|[t',\infty)} | y \in f(v)\}$.

The purpose of our talk is to sketch the study of these concepts.

Conformal geometry and quasiregular mappings

Matti Vuorinen

Conformal geometry refers to the study of metrics that have some natural invariance properties. Some of these metrics have been used quite a lot in the theory of mappings which include but are not limited to quasiregular mappings of the Euclidean spaces. There seem to be a large number of open problems, some of which will formulated in the talk. This theory also has connections to the study of inequalities of special functions and numerical computation of the moduli of quadrilaterals.

Geometric prequantization of a generalized mechanical system

Dorin Wainberg

Keywords: generalized mechanical systems, symplectic geometry.

In this talk we try to understand some new properties of distributional symplectic geometry and generalized mechanical systems. The talk is divided up as follows. Section 1 presents some general facts on distributional symplectic geometry. In section 2 the central ideas of geometric prequantization are summarized. Section 3 contains the geometric prequantization of a generalized mechanical system.

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Value distribution theory related to number theory

Chung-Chun Yang

In the talk, concrete examples will be exhibited to illustrate how some of the recent open problems were raised by the speaker and his co-workers, as a result of further applications of Nevanlinna's value distribution theory to functional equations. Particularly, on the solvability or non-existence of meromorphic solutions of Diophantine type equations of the form: $P(z, f, g) \equiv 0$, where P denotes a polynomial in its arguments, x, f, and g. Moreover, generalized Manson's theorem for polynomials and its associated generalized *abc*-conjectures for number will be reported.

New status of spectral theory for Schroedinger equation in inverse problem approach

Boris N. Zakhariev

New breakthrough in quantum mechanics is connected with recent achievements in the inverse problem. With its help instead of about ten exactly solvable models which serve as a basis of the contemporary education there were received complete sets of such models. They particularly correspond to all partial possible elementary (!) variations of separate spectral parameters and suitable potential perturbations ΔV. The computer visualization of these models revealed unexpectedly the simplest fundamental constituents of these ΔV. So, we acquire the notion of what "bricks" it is possible in principle to construct objects with arbitrary given properties. This report on inverse problem quantum pictures is utmost intelligible: qualitative spectral problem " in mind" (visual intuition even without calculations).

Euler and the graph theory

Tudor Zamfirescu

Euler had a huge impact on mathematics, and not only. There are mathematical subfields in which he did the first steps. Such a field is the graph theory. Three graph-theoretical themes can be found among Euler's many papers. In this lecture I'll talk about them, and about their later development. We will be able to conclude that the least observed among these three themes was going to have the most important future impact on graph theory.

Some uses of the Fitzpatrick function in the study of maximal monotone operators

Constantin Zălinescu

S. Fitzpatrick introduced a convex function associated to a monotone operator in 1988. After more than ten years several mathematicians used this function in order to reobtain important results concerning maximal monotone operators in reflexive Banach spaces like the Minty-Rockafellar type characterization of the maximality or the maximality of the sum and of the composition with linear operators. Moreover, new results were obtained relative to convergence of certain type of sequences of monotone operators. In the last two years some results were obtained in the non reflexive case using the Fitzpatrick function. It is the aim of this talk to present some results on monotone operators obtained by myself and my collaborators using this approach.

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An α-fuzzy version of Hahn-Banach Theorem

Lemnaouar ZEDAM

Based on joint work with **Abdelkader Stouti**

The main objective of the paper is to prove an α-fuzzy version of the analytic form of Hahn-Banach theorem. As an application, the Hahn-Banach theorem for α-fuzzy bounded linear functionals on α-fuzzy normed linear spaces is obtained.