Thermal states in conformal QFT

(joint work with P. Camassa, R. Longo and M. Weiner)

Yoh Tanimoto

Department of Mathematics, University of Rome “Tor Vergata”

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Introduction: KMS states

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$$f_{x,y}(t) = \varphi(x\alpha_t(y)), f_{x,y}(t+i\beta) = \varphi(\alpha_t(y)x).$$

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$T = \frac{1}{\beta}$ is called the **temperature** of φ.

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$\mathcal{A} = M$, a von Neumann algebra,φ: a faithful normal state, σ^φ: the modular automorphism. φ is a β-KMS state with $\beta = -1$.

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- There is a vector Ω invariant under $\mathrm{PSL}(2,\mathbb{R}) \subset \mathrm{Diff}(S^1)$.

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Dilation covariance: correspondence between KMS states in different temperatures.

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We consider always $\beta = 1$.

Geometric KMS state

Fact: Bisognano-Wichmann property

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Theorem (The geometric KMS state)

The state $\omega \circ \mathrm{Exp}$ is well defined and a KMS state with respect to translation.

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Theorem (Uniqueness of KMS state)

Any completely rational net admits only the geometric KMS state.

Thermal completion

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In completely rational case, the thermal completion is an irreducible conformal **extension** of the original net with finite index.

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Any KMS state φ on a completely rational maximal net $\mathcal{A}$ is $\varphi = \varphi_{\mathrm{geo}} \circ \gamma$ where $\gamma = \pi_\varphi \circ \pi_{\varphi_{\mathrm{geo}}}^{-1}$ is an automorphism of $\mathcal{A}|_{\mathbb{R}_+}$.

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Theorem

Any two-dimensional completely rational conformal net admits only the geometric state.