Deformations of Quantum Field Theories on de Sitter Spacetime

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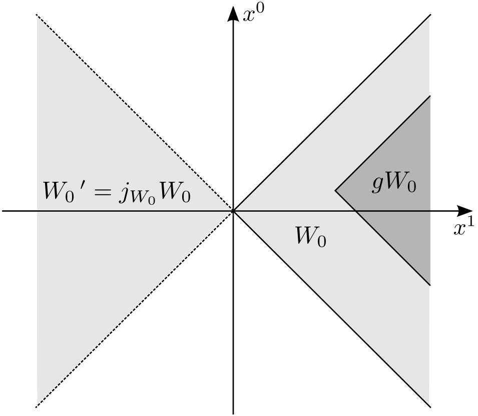
EU-NCG 4th Annual Meeting
Bucharest, 27.04.2011

Outline

- Motivation: QFT in terms of wedge triples
- de Sitter spacetime and wedges
- Wedge triples with global gauge symmetry
- Deformation by warped convolutions
- Conclusions and outlook

Motivation: QFT in terms of wedge triples

Minkowski space $(\mathbb{R}^4, \eta)$ with isometry group $\mathcal{P} = \mathrm{SO}(1,3)_0 \ltimes \mathbb{R}^4$



$$W_0 = \{x \in \mathbb{R}^4 : x^1 > |x^0|\}$$

A *wedge triple* $(\mathcal{A}_0, \mathcal{A}, \alpha)$ consists of C^*-algebras $\mathcal{A}_0 \subset \mathcal{A}$ and a strongly continuous action $\alpha : \mathcal{P} \to \mathrm{Aut}(\mathcal{A})$ satisfying

$$gW_0 \subset W_0 \;\Rightarrow\; \alpha_g(\mathcal{A}_0) \subset \mathcal{A}_0, \qquad \alpha_{j_{W_0}}(\mathcal{A}_0) \subset \mathcal{A}_0{}' \cap \mathcal{A}.$$

Motivation: QFT in terms of wedge triples

Proposition (Reconstruction of QFT from wedge triples)

Let $(\mathcal{A}_0, \mathcal{A}, \alpha)$ be a wedge triple. Then

$$W := gW_0 \mapsto \alpha_g(\mathcal{A}_0) =: \mathcal{A}(W)$$

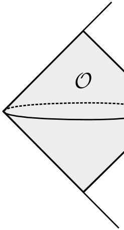
is a QFT in the sense of Haag and Kastler.

Conversely, every QFT (over wedges) gives rise to a wedge triple.

QFT over compact regions $\mathcal{O} = \bigcap_{W \supset \mathcal{O}} W$:

$$\mathcal{A}(\mathcal{O}) := \bigcap_{W \supset \mathcal{O}} \mathcal{A}(W)$$

is a QFT in the sense of Haag and Kastler.



Motivation: Deformation of wedge triples

Consider $(\mathcal{A}_0, \mathcal{A}, \alpha)$ in a covariant representation on a Hilbert space $\mathcal{H}$.

Warped Convolution:

$$A_Q := \frac{1}{(2\pi)^4} \int dv\, dv'\, e^{-ivv'} U(Qv) A U(-Qv) U(v'), \quad A \in \mathcal{A}_0 \text{ 'smooth'}$$

where U is a representation of $\mathbb{R}^4 \subset \mathcal{P}$ with spectrum condition and

$$Q = \begin{pmatrix} 0 & \kappa & 0 & 0 \\ \kappa & 0 & 0 & 0 \\ 0 & 0 & 0 & \kappa' \\ 0 & 0 & -\kappa' & 0 \end{pmatrix}, \quad \kappa \geq 0, \kappa' \in \mathbb{R}.$$

(close connection to Rieffel's deformation of C^*-algebras)

Theorem [Buchholz,Lechner,Summers:2010]

Let $(\mathcal{A}_0)_Q = \{A_Q : A \in \mathcal{A}_0\}''$. Then $((\mathcal{A}_0)_Q, \mathcal{A}, \alpha)$ is a wedge triple.

Associated QFT: non-trivial 2-particle scattering!

This talk

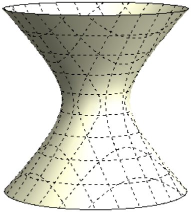
Questions:

- Are translations special?
- Do other Abelian subgroups of $\mathcal{P}$ yield 'interesting' deformations?
- What about gauge groups?
- Is this procedure also feasible on a curved spacetime?

Natural setting: QFT with gauge symmetry in de Sitter spacetime

de Sitter spacetime

Solution $(M, \mathbf{g})$ of Einstein equation with positive cosmological constant



$$M = \left\{ x \in \mathbb{R}^5 : (x^0)^2 - \sum_{j=1}^{4} (x^j)^2 = -1 \right\}$$

$$\mathbf{g} = \iota^* \eta, \ \iota : M \hookrightarrow \mathbb{R}^5$$

(maximally symmetric, globally hyperbolic)

- causal structure inherited from $(\mathbb{R}^5, \eta)$
- isometry group:
  $\mathrm{O}(1,4) \supset \mathrm{SO}(1,4)_0 =: \mathcal{L}_0$ (de Sitter group)
- covering: $\mathrm{Spin}(1,4) =: \widetilde{\mathcal{L}}_0 \xrightarrow{\pi} \mathcal{L}_0$

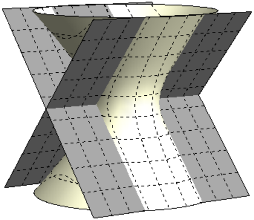
physically interesting: (inflationary) cosmology

de Sitter wedges

Causal closure of timelike geodesics (uniformly accelerated observers).

Equivalently:

- $W_0 = \{x \in \mathbb{R}^5 : x^1 > |x^0|\} \cap M$
- $\mathcal{W} = \{gW_0 : g \in \widetilde{\mathcal{L}}_0\}, \;\; gW_0 := \pi(g)W_0$



Properties:

- $W' \in \mathcal{W}$ and $W'' = (W')' = W$
- $\forall W \in \mathcal{W}$: $\exists \Gamma_W = \{\Lambda_W(t) \in \widetilde{\mathcal{L}}_0 : t \in \mathbb{R}\}$:

$$\Lambda_W(t)W = W, \quad \Lambda_{gW}(t) = g\Lambda_W(t)g^{-1}$$

  associated with Γ_W is a timelike Killing VF ξ_W
- $\forall W \in \mathcal{W}$: $\exists j_W \in \widetilde{\mathcal{L}}_0$:

$$j_W W = W', \quad j_{gW} = g j_W g^{-1}$$

de Sitter wedges

Lemma 1

Let $W \in \mathcal{W}$ and $\Lambda(t) \in \Gamma_W$, j_W be as before. Then

$$g\Lambda_W(t)g^{-1} = \Lambda_W(t)$$

for all $g \in \widetilde{\mathcal{L}}_0(W) = \{g \in \widetilde{\mathcal{L}}_0 : gW = W\}$ and

$$\Lambda_{W'}(t) = j_W \Lambda_W(t) j_W = \Lambda_W(-t)$$

Lemma 2

Let $W_1, W_2 \in \mathcal{W}$ and $W_1 \subset W_2$. Then $W_1 = W_2$.

Wedge triples for Bose/Fermi fields on de Sitter

C^*-algebras $\mathcal{F}_0 \subset \mathcal{F}$ and $\alpha : \widetilde{\mathcal{L}}_0 \to \mathrm{Aut}(\mathcal{F})$ strongly continuous action s.t.

$$\alpha_g(\mathcal{F}_0) = \mathcal{F}_0,\ \forall g \in \widetilde{\mathcal{L}}_0(W_0), \qquad \alpha_{j_{W_0}}(\mathcal{F}_0) \subset {\mathcal{F}_0}^{t\prime} \cap \mathcal{F}$$

with ${\mathcal{F}_0}^{t\prime} =$ 'twisted' commutant of $\mathcal{F}_0$
$\to$ (anti)commutation relations for Bose/Fermi fields

Gauge symmetry: G compact group, $\sigma : G \to \mathrm{Aut}(\mathcal{F})$ strongly cont.:

- $\sigma_h \circ \alpha_g = \alpha_g \circ \sigma_h, \quad \forall h \in G, g \in \widetilde{\mathcal{L}}_0$
- $\sigma_h(\mathcal{F}_0) = \mathcal{F}_0, \quad \forall h \in G$

Proposition (Reconstruction of QFT)

The map $gW_0 \mapsto \alpha_g(\mathcal{F}_0)$ is a QFT with global gauge symmetry (field net) in the sense of Doplicher, Haag and Roberts.

Example: free charged Dirac field ($G = \mathrm{U}(1)$)

Deformation of wedge triples

Let $(\mathcal{F}_0 \subset \mathcal{F}, \alpha, \sigma)$ be a wedge triple with $\mathrm{U}(1)$ gauge symmetry in a covariant representation on a Hilbert space $\mathcal{H}$.

Use the following $\mathbb{R}^2$-action (boosts + gauge symmetry)

$$(t, s) \mapsto \alpha_{\Lambda_{W_0}(t)} \circ \sigma_{e^{is}} = \tau^{\xi}_{t,s} : \mathcal{F} \to \mathcal{F}$$

for warped convolution.

Notation:

- $\xi = \xi_{W_0} =$ Killing vector field associated with W_0
- $\Lambda_\xi(t) = \Lambda_{W_0}(t)$
- $\mathrm{Ad}\, \mathbf{U}_\xi(t, s) = \tau^{\xi}_{t,s}$

Deformation of wedge triples

Definition

$$F_{\xi,\kappa} := \frac{1}{4\pi^2} \int_{\mathbb{R}^2 \times \mathbb{R}^2} dv\, dv'\, e^{-ivv'}\, \tau^{\xi}_{Qv}(F) \mathbf{U}_{\xi}(v'), \quad Q = \begin{pmatrix} 0 & \kappa \\ -\kappa & 0 \end{pmatrix},\ \kappa \in \mathbb{R}.$$

This integral exists for 'smooth' elements $F \in \mathcal{F}_0$ in an oscillatory sense.

Basic properties:

- $(F_{\xi,\kappa})^* = (F^*)_{\xi,\kappa}$
- $F_{\xi,\kappa} G_{\xi,\kappa} = (F \times_{\xi,\kappa} G)_{\xi,\kappa}$ with Rieffel product $\times_{\xi,\kappa}$
- If $[\tau^{\xi}_{v}(F), G] = 0,\ \forall v \in \mathbb{R}^2$, then $[F_{\xi,\kappa}, G_{\xi,-\kappa}] = 0$

Transformation properties:

- $\alpha_g(F_{\xi,\kappa}) = \alpha_g(F)_{g_*\xi,\kappa}$
- $\sigma_h(F_{\xi,\kappa}) = \sigma_h(F)_{\xi,\kappa}$

Deformation of wedge triples

Theorem

Let $(\mathcal{F}_0)_{\xi,\kappa} := \{F_{\xi,\kappa} : F \in \mathcal{F}_0 \text{ smooth}\}^{\|\cdot\|}$. Then $((\mathcal{F}_0)_{\xi,\kappa} \subset \mathcal{F}, \alpha, \sigma)$ is a wedge-triple, i.e,

a) $\alpha_g((\mathcal{F}_0)_{\xi,\kappa}) = (\mathcal{F}_0)_{\xi,\kappa}, \quad \forall g \in \widetilde{\mathcal{L}}_0(W_0)$

b) $\alpha_{j_{W_0}}((\mathcal{F}_0)_{\xi,\kappa}) \subset ((\mathcal{F}_0)_{\xi,\kappa})^{t\prime}$

c) $\sigma_h((\mathcal{F}_0)_{\xi,\kappa}) = (\mathcal{F}_0)_{\xi,\kappa}, \quad \forall h \in \mathrm{U}(1).$

proof: locality property b) (Bose case): let $F, G \in \mathcal{F}_0$

$$\alpha_{j_{W_0}}(G_{\xi,\kappa}) = \alpha_{j_{W_0}}(G)_{j_{W_0 *}\xi,\kappa} = \alpha_{j_{W_0}}(G)_{\xi,-\kappa}$$

As $[\tau_v^\xi(F), \alpha_{j_{W_0}}(G)] = 0, \ \forall v \in \mathbb{R}^2$ there follows

$$[F_{\xi,\kappa}, \alpha_{j_{W_0}}(G_{\xi,\kappa})] = [F_{\xi,\kappa}, \alpha_{j_{W_0}}(G)_{\xi,-\kappa}] = 0. \qquad \square$$

Deformation of wedge triples

Example of wedge-triples with gauge symmetry: selfdual CAR-algebras

$$\mathcal{F}_0 = \mathrm{CAR}(\mathcal{H}_0, C) \subset \mathrm{CAR}(\mathcal{H}, C) = \mathcal{F}$$

coming from $(\mathcal{H}_0 \subset \mathcal{H}, U, V)$.

Gauge transformations generated by charge operator (grading).

Results:

- deformed operators can be computed: $F : \mathcal{H}_n \to \mathcal{H}_{n+m}$

$$F_{\xi,\kappa} = \sum_{n\in\mathbb{Z}} U_\xi(\kappa n) F U_\xi(-\kappa(n+m)) E(n)$$

  $E(n) =$ projector on $\mathcal{H}_n$ (charge n subspace of $\mathcal{H}$)
- deformation fix-points for observables: $\mathbb{C} \cdot 1$
- unitary inequivalence of the associated nets for a variety of models

Conclusion and outlook

Used boosts and $\mathrm{U}(1)$ symmetry to define a warped convolution

- obtained deformed wedge triple
- inequivalence in a variety of models (e.g. free charged Dirac field)

Deformations with other Abelian groups

- pure gauge symmetry: trivial deformation
- subgroups of $\mathrm{SO}(1,4)_0$: deformation does not yield a wedge triple
  $\rightarrow$ modification of warping formula required

Outlook:

- applications in cosmology
- connection with [Dappiaggi,Lechner,M:2010]
  $\rightarrow$ deformations with non-Abelian groups, e.g. $\mathrm{SO}(3)$
- chiral conformal theories: affine group [Bieliavsky:2007]
- general deformation theory for wedge triples