Deformations of conformal field theories and the problem of asymptotic completeness

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Motivation

- Asymptotic completeness in QFT attracted much attention during the last two decades:
  (a) Non-relativistic QED [Spohn 97, Dereziński-Gérard 99, Fröhlich-Griesemer-Schlein 04]
  (b) Local, relativistic QFT (massive models in 1+1 dim.) [Lechner 08]
- A new class of interacting wedge-local, relativistic QFTs has been constructed. [Grosse, Lechner, Buchholz, Summers 07-10]
  (a) It contains massless models.
  (b) We will show that (in 1+1 dim.) some of these massless models are asymptotically complete.

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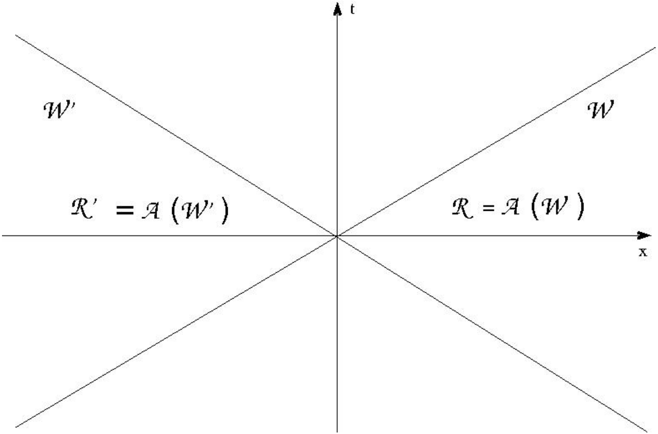
Outline

Wedge-local QFT



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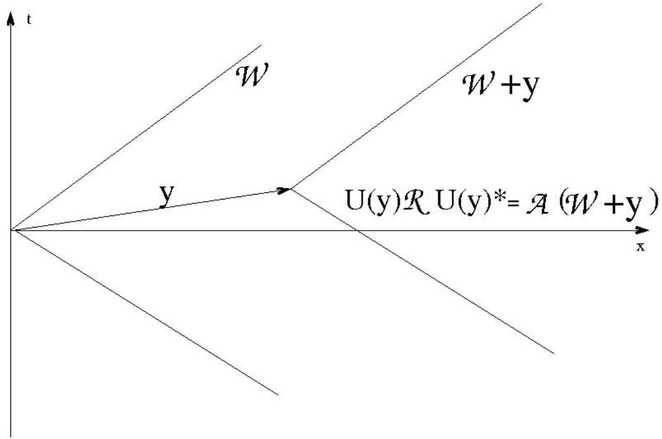
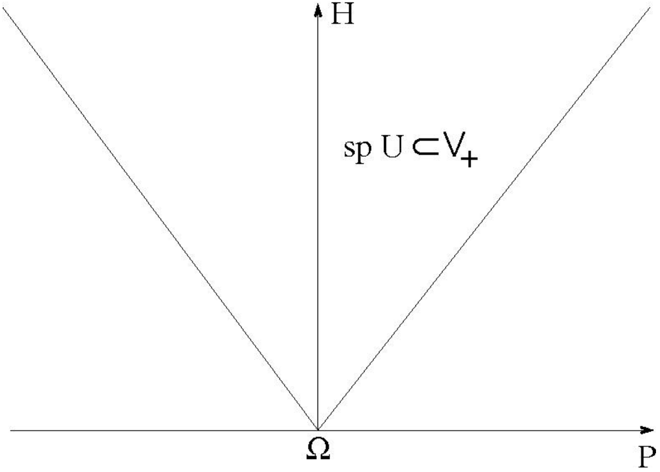


$$U(y)\mathcal{R}U(y)^* \subset \mathcal{R}$$

Wedge-local QFT



Borchers triple

Definition

A Borchers triple $(\mathcal{R}, U, \Omega)$ w.r.t. $\mathcal{W}$ consists of

- a von Neumann algebra $\mathcal{R} \subset B(\mathcal{H})$;
- a unitary representation $\mathbb{R}^2 \ni x \to U(x)$ s.t.

$$\alpha_x(\mathcal{R}) = U(x)\mathcal{R}U(x)^{-1} \subset \mathcal{R} \text{ for } x \in \mathcal{W},$$
$$\mathrm{sp}\,\mathrm{U} \subset V_+;$$

- a vacuum vector Ω, invariant under U, which is cyclic w.r.t. $\mathcal{R}$ and $\mathcal{R}'$. (We assume that Ω is a unique invariant vector).

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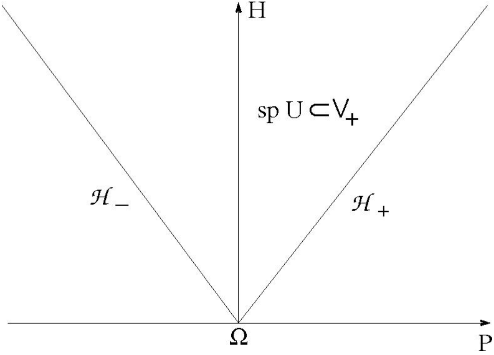
Single-particle spaces

- We are interested in theories of massless particles.
- $\mathcal{H}_\pm = \ker(H \mp P)$ - single-particle spaces.
- These are not particles in the sense of Wigner.



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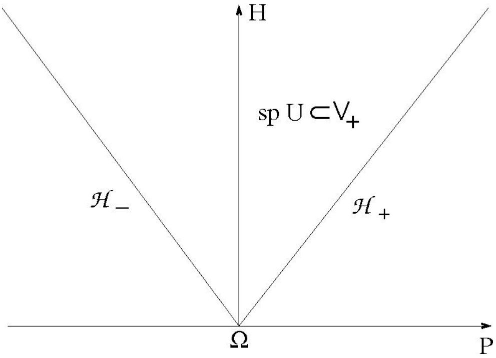

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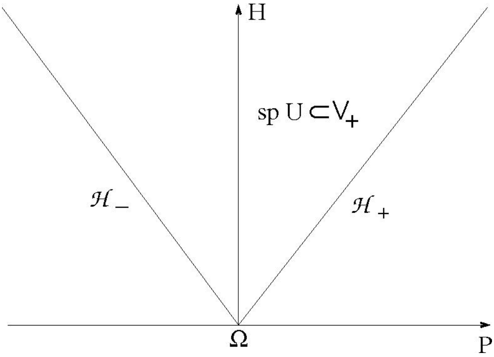
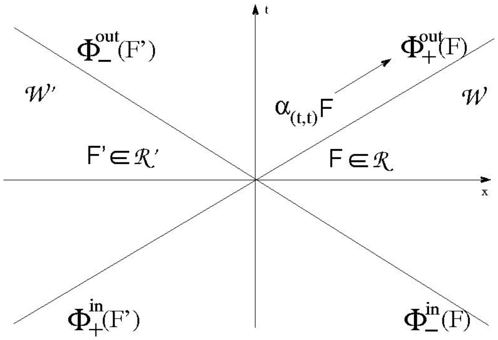


Asymptotic fields

- Scattering theory for such particles in local theories developed in [Buchholz 75]
- We generalize this theory to the wedge-local case:



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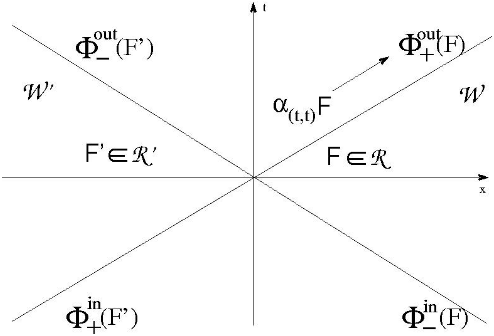

Asymptotic fields

Proposition

Let $F \in \mathcal{R}$. Then the limits

$$\Phi_+^{\mathrm{out}}(F) := s\text{-}\lim_{T\to\infty} \frac{1}{\ln|T|} \int_T^{T+\ln|T|} dt\ \alpha_{(t,t)}(F),$$

$$\Phi_-^{\mathrm{in}}(F) := s\text{-}\lim_{T\to-\infty} \frac{1}{\ln|T|} \int_T^{T+\ln|T|} dt\ \alpha_{(t,-t)}(F)$$

exist and are elements of $\mathcal{R}$. Operators $\Phi_-^{\mathrm{out}}(F')$, $\Phi_+^{in}(F')$, where $F' \in \mathcal{R}'$, are constructed analogously.

Asymptotic fields

Proposition

The asymptotic fields Φ_{+}^{out}, Φ_{-}^{in} *satisfy, for any* $F \in \mathcal{R}$, $x \in \mathcal{W}$,

(a) $\alpha_x(\Phi_{+}^{\mathrm{out}}(F)) = \Phi_{+}^{\mathrm{out}}(\alpha_x(F))$,

(b) $\alpha_x(\Phi_{-}^{\mathrm{in}}(F)) = \Phi_{-}^{\mathrm{in}}(\alpha_x(F))$,

(c) $\Phi_{+}^{\mathrm{out}}(F)\mathcal{H}_{+} \subset \mathcal{H}_{+}$,

(d) $\Phi_{-}^{\mathrm{in}}(F)\mathcal{H}_{-} \subset \mathcal{H}_{-}$,

Analogous relations hold for Φ_{-}^{out}, Φ_{+}^{in}.

Scattering states

Definition

Given $F \in \mathcal{R}$, $F' \in \mathcal{R}'$, we create two single-particle states:

$$\Psi_+ = \Phi_+^{\mathrm{out}}(F)\Omega \in \mathcal{H}_+, \quad \Psi_- = \Phi_-^{\mathrm{out}}(F')\Omega \in \mathcal{H}_-.$$

The corresponding (outgoing) scattering state is given by

$$\Psi_+ \overset{\mathrm{out}}{\times} \Psi_- := \Phi_+^{\mathrm{out}}(F)\Phi_-^{\mathrm{out}}(F')\Omega$$

and it depends only on Ψ_+, Ψ_-.

Remark 1: For arbitrary $\Psi_\pm \in \mathcal{H}_\pm$, the scattering state $\Psi_+ \overset{\mathrm{out}}{\times} \Psi_-$ is constructed by an approximation procedure.

Remark 2: The incoming scattering states $\Psi_+ \overset{\mathrm{in}}{\times} \Psi_-$ are obtained analogously, using Φ_+^{in}, Φ_-^{in}.

Scattering states

Proposition

For any $\Psi_\pm, \Psi'_\pm \in \mathcal{H}_\pm$, there holds:

(a) $(\Psi_+ \overset{\mathrm{out}}{\times} \Psi_-, \Psi'_+ \overset{\mathrm{out}}{\times} \Psi'_-) = (\Psi_+, \Psi'_+)(\Psi_-, \Psi'_-)$,

(b) $U(x)(\Psi_+ \overset{\mathrm{out}}{\times} \Psi_-) = (U(x)\Psi_+) \overset{\mathrm{out}}{\times} (U(x)\Psi_-)$, *for* $x \in \mathbb{R}^2$.

Analogous relations hold for the incoming scattering states.

Thus the asymptotic spaces $\mathcal{H}^{\mathrm{in}} = \mathcal{H}_+ \overset{\mathrm{in}}{\times} \mathcal{H}_-$ and $\mathcal{H}^{\mathrm{out}} = \mathcal{H}_+ \overset{\mathrm{out}}{\times} \mathcal{H}_-$ have a tensor product structure.

Scattering matrix

Definition

The scattering matrix $S : \mathcal{H}^{\rm out} \to \mathcal{H}^{\rm in}$ is given by

$$S(\Psi_+ \overset{\rm out}{\times} \Psi_-) = \Psi_+ \overset{\rm in}{\times} \Psi_-.$$

We say that:
(a) a theory is interacting, if S is not a multiple of identity;
(b) a theory is asymptotically complete, if $\mathcal{H}^{\rm in} = \mathcal{H}^{\rm out} = \mathcal{H}$.

Preliminaries on deformations

- Let $(\mathcal{R}, U, \Omega)$ be a Borchers triple (with scattering matrix S) and

$$Q_\kappa = \begin{pmatrix} 0 & \kappa \\ \kappa & 0 \end{pmatrix}$$

- Then, one can construct a deformed Borchers triple $(\mathcal{R}_{Q_\kappa}, U, \Omega)$, (with scattering matrix S_κ). [Grosse, Lechner, Buchholz, Summers, 07-10].

Question: What is the relation between S_κ and S?

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Deformation procedure

Definition

Given $F \in \mathcal{R}^\infty$, one can define the "warped convolution"

$$\begin{aligned} F_{Q_\kappa} &= \lim_{\varepsilon\to 0} \frac{1}{(2\pi)^n} \int dx\, dy\, f(\varepsilon x, \varepsilon y) e^{-ixy} \alpha_{Qx}(F) U(y) \\ &= \int dE(q)\, \alpha_{Q_\kappa q}(F) \in B(\mathcal{H}), \end{aligned}$$

where E is the spectral measure of (H, P). We set

$$\mathcal{R}_{Q_\kappa} = \{\, F_{Q_\kappa} \,|\, F \in \mathcal{R}^\infty \,\}''$$

Theorem (Buchholz-Lechner-Summers)

Let $\kappa > 0$. If $(\mathcal{R}, U, \Omega)$ is a Borchers triple w.r.t. $\mathcal{W}$, then $(\mathcal{R}_{Q_\kappa}, U, \Omega)$ is also a Borchers triple w.r.t. $\mathcal{W}$. Moreover, $(\mathcal{R}')_{-Q_\kappa} \subset (\mathcal{R}_{Q_\kappa})'$.

Scattering states of the deformed theory

Main Theorem

For any $\Psi_\pm \in \mathcal{H}_\pm$ *the following relations hold*

$$\Psi_+ \overset{\text{out}}{\times}_\kappa \Psi_- = e^{-i\frac{1}{2}\kappa(H^2-P^2)}(\Psi_+ \overset{\text{out}}{\times} \Psi_-),$$
$$\Psi_+ \overset{\text{in}}{\times}_\kappa \Psi_- = e^{i\frac{1}{2}\kappa(H^2-P^2)}(\Psi_+ \overset{\text{in}}{\times} \Psi_-).$$

Proof of the main theorem

Proof. Let $F \in \mathcal{R}^\infty$, $F' \in (\mathcal{R}')^\infty$. Then $F_{Q_\kappa} \in \mathcal{R}_{Q_\kappa}$, $F'_{-Q_\kappa} \in (\mathcal{R}_{Q_\kappa})'$.

$$\begin{aligned}
\Psi_+ \overset{\text{out}}{\times}_\kappa \Psi_- &= \Phi_+^{\text{out}}(F_{Q_\kappa})\Phi_-^{\text{out}}(F'_{-Q_\kappa})\Omega \\
&= \int dE(q)\, \Phi_+^{\text{out}}(\alpha_{Q_\kappa q}(F))\Phi_-^{\text{out}}(F')\Omega \\
&= \int dE(q)\, (U(Q_\kappa q)\Psi_+) \overset{\text{out}}{\times} \Psi_- \\
&= \int dE(q)\, e^{-i\frac{1}{2}\kappa(H+P)(q^0-q^1)}(\Psi_+ \overset{\text{out}}{\times} \Psi_-) \\
&= e^{-i\frac{1}{2}\kappa(H^2-P^2)}(\Psi_+ \overset{\text{out}}{\times} \Psi_-). \qquad \square
\end{aligned}$$

Scattering matrix of the deformed theory

Corollary

Let S be the scattering matrix of $(\mathcal{R}, U, \Omega)$ and let S_κ be the scattering matrix of $(\mathcal{R}_{Q_\kappa}, U, \Omega)$. Then

$$S_\kappa = e^{i\kappa(H^2 - P^2)} S.$$

Remark: If $(\mathcal{R}, U, \Omega)$ is asymptotically complete, non-interacting and $\mathrm{sp}\,\mathrm{U} = V_+$, then $(\mathcal{R}_\kappa, U, \Omega)$ is asymptotically complete and interacting.

Local nets on $\mathbb{R}$.

Definition

A local net of von Neumann algebras on $\mathbb{R}$, denoted by $(\mathcal{A}_0, U_0, \Omega_0)$, consists of

- a map $\mathbb{R} \supset I \to \mathcal{A}_0(I) \subset B(\mathcal{H})$ s.t.

$$\mathcal{A}_0(I) \subset \mathcal{A}_0(J) \text{ for } I \subset J$$
$$[\mathcal{A}_0(I), \mathcal{A}_0(J)] = 0 \text{ for } I \cap J = \varnothing;$$

- a unitary representation $\mathbb{R} \ni s \to U_0(s)$ s.t.

$$\mathrm{sp}\, \mathrm{U}_0 \subset \mathbb{R}_+$$
$$U_0(s)\mathcal{A}_0(I)U_0(s)^{-1} = \mathcal{A}_0(I+s) \;\text{ for } s \in \mathbb{R};$$

- a unique vacuum vector Ω_0, invariant under U_0, which is cyclic for any $\mathcal{A}_0(I)$.

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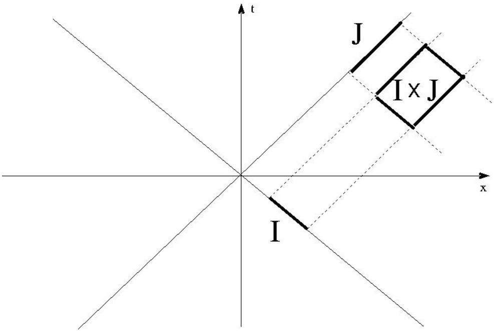
Chiral nets on $\mathbb{R}^2$.



Chiral nets on $\mathbb{R}^2$.

Definition

A chiral net of von Neumann algebras on $\mathbb{R}^2$ is given by

$$\begin{aligned} \mathcal{A}(I \times J) &:= \mathcal{A}_0(I) \otimes \mathcal{A}_0(J) \\ U(t,x) &:= U_0((\sqrt{2})^{-1}(t-x)) \otimes U_0((\sqrt{2})^{-1}(t+x)) \\ \Omega &:= \Omega_0 \otimes \Omega_0 \end{aligned}$$

Remark: Setting $\mathcal{R} = \bigvee_{I \times J \subset \mathcal{W}} \mathcal{A}(I \times J)$, we obtain a Borchers triple $(\mathcal{R}, U, \Omega)$ with some scattering matrix S.

Asymptotic fields in chiral theories

Proposition

For any $A_1 \in \mathcal{A}(I)$, $A_2 \in \mathcal{A}(J)$ there holds

$$\begin{aligned} \Phi_+^{\mathrm{out/in}}(A_1 \otimes A_2) &= A_1 \otimes (\Omega_0 | A_2 \Omega_0) 1, \\ \Phi_-^{\mathrm{out/in}}(A_1 \otimes A_2) &= (\Omega_0 | A_1 \Omega_0) 1 \otimes A_2. \end{aligned}$$

Corollary

Any theory given by a chiral net is asymptotically complete and non-interacting.

Proof. $\Phi_+^{\mathrm{out/in}}(A_1 \otimes 1) \Phi_-^{\mathrm{out/in}}(1 \otimes A_2)\Omega = A_1\Omega_0 \otimes A_2\Omega_0$ □

Conclusions

- There exist relativistic theories of interacting, massless particles in two-dimensional spacetime, which are asymptotically complete.
- These theories can be constructed by deformations of chiral nets of local algebras.
- Open question: Do the deformed theories contain local observables?
- Future direction: Particle aspects of CFT.
  Preliminary results:
  (a) Existence of infraparticles in charged sectors of chiral CFT.
  (b) Asymptotic completeness for such infraparticles.
  (c) Superselection of infraparticle's velocity.

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