Scaling limits of factorizing scattering models

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Bucharest, 28 April 2011

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Outline

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Models with factorizing scattering matrix

A specific class of quantum field theories; physical idea:

- Imagine a system of spin-0 bosons of mass $m > 0$ on 1+1 dimensional Minkowski space (1 spatial dimension)
- Two bosons (of different speed) will scatter – phase $S(\theta_1 - \theta_2)$.
  - $\theta_{1,2}$ are the rapidities of the particles: $p(\theta) = \begin{pmatrix} m\cosh\theta \\ m\sinh\theta \end{pmatrix}$.
- Multi-particle scattering is just a composition of subsequent 2-particle processes ("factorizing scattering matrix").

Task: Given a function S, construct a corresponding quantum field theory.

The scattering function S

The 2-particle scattering "matrix" S is a continuous function $\mathbb{R} + i[0, \pi] \to \mathbb{C}$, analytic in the interior, such that for $\theta \in \mathbb{R}$,

$$\overline{S(\theta)} = S(\theta)^{-1} = S(\theta + i\pi) = S(-\theta)\,.$$

Further conditions:

- S analytic and bounded $\mathbb{R} + i(-\kappa, \pi + \kappa)$ with some $\kappa > 0$. (Regularity)
- High energy limit: $\lim_{\theta \to \pm\infty} S(\theta)$ exists.

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Theorem: These are all examples.

Construction of QFTs with factorizing scattering matrix

- Given S, define a "deformed" free field theory.
  - Deformed annihilation and creation operators z, $z^\dagger$.

$$z(\theta_1)z^\dagger(\theta_2) = S(\theta_2 - \theta_1)\, z^\dagger(\theta_2)z(\theta_1) + \delta(\theta_1 - \theta_2) \cdot \mathbf{1}.$$

  - They act on an "S-symmetrized" Fock space $\mathcal{H}$.
- This allows us to define "fields",

$$\phi(x) = \int d\theta \Big(e^{ip(\theta)x} z^\dagger(\theta) + e^{-ip(\theta)x} z(\theta) \Big).$$

- $\phi(x)$ is not local at x, but in a wedge region $\mathcal{W}$ with tip at x.
- Define associated von Neumann algebra, $\mathfrak{A}(\mathcal{W})$.
- For double cone $\mathcal{O} = \mathcal{W}_1 \cap \mathcal{W}_2$: Set $\mathfrak{A}(\mathcal{O}) := \mathfrak{A}(\mathcal{W}_1) \cap \mathfrak{A}(\mathcal{W}_2)$.
- Result (Lechner 2006): $\mathfrak{A}(\mathcal{O})$ is large (cyclic vacuum).
- Scattering theory gives factorizing scattering matrix.

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- As $\lambda \to 0$, one expects the n-point functions of the massless free field, except for S factors:

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  - If $p > 0$, $\hat{p} > 0$, then $S(\theta_\lambda - \hat{\theta}_\lambda) \to S(\log p - \log \hat{p})$.

Scaling limit – Result

On further investigation, the limit theory looks like this:

- chiral: splits into two fields ϕ_L, ϕ_R on the left/right light ray
- ϕ_L, ϕ_R are one-dimensional fields, localized in half-lines.
- $[\phi_L(x), \phi_R(y)]_\pm = 0$ depending on $S(\infty)$
- translation and dilation covariant
- ϕ_L and ϕ_R are not free fields, but a kind of factorizing S model:
  - massless,
  - with the same function S as the massive model,
  - but with "pseudo-rapidity" $\beta = \log p$ instead of $\theta = \operatorname{arcsinh} p/m$.

Outline

Zamolodchikov-Faddeev algebra

- Zamolodchikov-Faddeev algebra (elements $z(\beta)$, $z^\dagger(\beta)$):

$$z(\beta_1)z(\beta_2) = S(\beta_1 - \beta_2)\, z(\beta_2)z(\beta_1)\,,$$
$$z^\dagger(\beta_1)z^\dagger(\beta_2) = S(\beta_1 - \beta_2)\, z^\dagger(\beta_2)z^\dagger(\beta_1)\,,$$
$$z(\beta_1)z^\dagger(\beta_2) = S(\beta_2 - \beta_1)\, z^\dagger(\beta_2)z(\beta_1) + \delta(\beta_1 - \beta_2)\cdot \mathbf{1}.$$

- "Fock space" $\mathcal{H}$ spanned by n-particle vectors,

$$\psi_n = \int d^n\beta\, f(\beta_1,\ldots,\beta_n)\, z^\dagger(\beta_1)\ldots z^\dagger(\beta_n)\Omega.$$

- Representation of the translation-dilation-reflection group:

$$U(T_x)z^\dagger(\beta_1)\ldots z^\dagger(\beta_n)\Omega = \exp(ie^{\beta_1+\ldots+\beta_n}x)z^\dagger(\beta_1)\ldots z^\dagger(\beta_n)\Omega,$$
$$U(D_\lambda)z^\dagger(\beta_1)\ldots z^\dagger(\beta_n)\Omega = z^\dagger(\beta_1+\lambda)\ldots z^\dagger(\beta_n+\lambda)\Omega,$$
$$U(j)z^\dagger(\beta_1)\ldots z^\dagger(\beta_n)\Omega = z^\dagger(\beta_n)\ldots z^\dagger(\beta_1)\Omega.$$

Wedge-local fields

- With $\hat{f}_\pm(\beta) = \pm i\, e^\beta \int dx\, f(x)\, \exp(\pm i e^\beta x)$, define

$$\phi(f) := z^\dagger(\hat{f}_+) + z(\hat{f}_-), \quad \phi'(f) := U(j)\phi(f^j)U(j).$$

- The fields are half-line local:

$$[\phi(f), \phi'(g)] = 0 \text{ if } \operatorname{supp} f \subset (a, \infty),\ \operatorname{supp} g \subset (-\infty, a).$$

- Consider associated von Neumann algebras,

$$\mathfrak{M}(a,\infty) = \{\exp i\phi(f) \mid \operatorname{supp} f \subset (a,\infty)\}''$$
$$\mathfrak{M}(-\infty,a) = \{\exp i\phi'(f) \mid \operatorname{supp} f \subset (-\infty,a)\}'' = \mathfrak{M}(a,\infty)'.$$

- The fields / the net of algebras are covariant under the translation-dilation-reflection representation U.

Local algebras?

- Now we can define local algebras for finite intervals $I = (a, b)$:

$$\mathfrak{A}(a, b) := \mathfrak{M}(a, \infty) \cap \mathfrak{M}(-\infty, b).$$

- This gives a consistent local net of algebras, translation-dilation-reflection covariant.
- Define $\mathfrak{A}(I)$ for unbounded intervals by taking unions and closure.
  - $\mathfrak{A}(a, \infty) \subset \mathfrak{M}(a, \infty)$, inclusion may be proper.

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- Question: How large are the $\mathfrak{A}(a, b)$?
- Question: Is this a conformal model?

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Existence of conformal symmetry?

Is our model conformally covariant?

- More precisely: Does the net of interval algebras $I \mapsto \mathfrak{A}(I)$ extend to a net on the circle, covariant under an extension of U to the Möbius group?

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  - Argument: Every dilation covariant theory is conformally covariant.
  - This would give strong restrictions on the models (classification by conformal charge).
- But in this generality, the statement is false.
  - Counterexamples known (Buchholz/Schulz-Mirbach).
  - It is true if a local energy density exists.
  - But here, it's not clear whether any local observables exist.
- What is the case in our situation?

The locally generated Hilbert space

For any interval $I \subset \mathbb{R}$, let us consider the space

$$\mathcal{H}_{\text{loc}} := \overline{\mathfrak{A}(I)\Omega}.$$

Lemma

The space $\mathcal{H}_{loc}$ is independent of I, and invariant under U.

This follows from Reeh-Schlieder type arguments.

$\mathcal{H}_{\text{loc}}$ is the largest space on which we can expect a conformal extension.

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$\mathfrak{A}$ has the Bisognano-Wichmann property on $\mathcal{H}_{\text{loc}}$.

- That is, the modular group of $\mathfrak{A}(0,\infty)$ is the dilation group.
- Follows because the original half-line algebras $\mathfrak{M}(0,\infty)$ have this property.

Extension of the net

Due to the Bisognano-Wichmann property, we can apply a general result for translation-dilation covariant nets (Guido/Longo/Wiesbrock 1998).

Theorem

The representation $U\lceil\mathcal{H}_{loc}$ extends to a strongly continuous unitary representation of $\mathrm{PSL}(2,\mathbb{R})$ on $\mathcal{H}_{loc}$, and $I \mapsto \mathfrak{A}(I)$ extends to a local net on the circle, conformally covariant under this representation.

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This could mean:

- Many local observables ($\mathcal{H}_{\text{loc}} = \mathcal{H}$) & conformal symmetry,
- No conformal symmetry and no local observables ($\mathcal{H}_{\mathrm{loc}} = \mathbb{C}\Omega$, $\mathfrak{A}(a, b) = \mathbb{C}\mathbf{1}$),
- Or anything inbetween.

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Size of the local observables in our case

How large are $\mathcal{H}_{\rm loc}$ and $\mathfrak{A}(a, b)$ in our case?

- For general S, this is unknown – local operators very inexplicit even for $m > 0$.
- Lechner's argument for $m > 0$ does not apply for $m = 0$.
- Let us have a look at some simple examples that we *can* control.

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- Let us have a look at some simple examples that we *can* control.

- The simplest example is $S = 1$.
  - This is identical to the free $U(1)$ current
  - $\mathcal{H}_{\rm loc} = \mathcal{H}$
  - We have large local algebras $\mathfrak{A}(a, b)$, vacuum is cyclic for them.
  - Conformal symmetry ($c = 1$)
- This is an entirely trivial case, but good to keep in mind.

The critical Ising model

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- The next simple example is $S = -1$.
  - expectation: critical Ising model, generated by a chiral Fermi field, $c = 1/2$
  - But how can this be seen here?
- Consider the following field:

$$\psi(x) := \frac{1}{\sqrt{2\pi}} \int d\beta\, e^{\beta/2} \left(\sqrt{i}\, e^{ie^{\beta} x} z^{\dagger}(\beta) + \frac{1}{\sqrt{i}} e^{-ie^{\beta} x} z(\beta) \right) .$$

- It turns out that ψ is an antilocal Fermi field.
- $T(x) = :\psi(x)\partial_x\psi(x):$ is the energy density of our model, and relatively local to the halfline algebras $\mathfrak{M}$.
- Local algebras $\mathfrak{A}(I)$ are precisely those generated by $T(x)$.
- $\mathcal{H}_{\text{loc}} = \mathcal{H}_{\text{e}}$ (even particle number vectors)

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- Recall that in the massive Ising model, many local observables exist:
  - Even operators – squares of wedge fields (Buchholz/Summers)
  - Odd operators – known to exist by abstract arguments (Lechner); heuristically given by infinite sums of wedge fields or $z, z^\dagger$ (Schroer/Truong)
  - Vacuum Ω is cyclic for double cone algebras.
- In the massless case, we see (roughly):
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  - Odd local operators fail to exist – $\mathcal{H}_{\text{loc}} = \mathcal{H}_{\text{e}}$
- What does this mean for general S?
  - Expectation (following Zamolodchikov & Zamolodchikov 1992): Scaling limit should be identical to $S = \pm 1$, depending on $S(\infty)$.
  - But their arguments are not applicable in our context.

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Results & Open Questions

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- The scaling limit of the massive factorizing scattering models are massless, chiral factorizing scattering models.
- The chiral components can be defined in the algebraic framework.
  - Observables localized in half-lines
  - Translation-dilation-reflection symmetry
- For $S = \pm 1$, one gets the expected conformal models ($c = 1$, $c = 1/2$)
- For general S, we might have
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Open points:

- Can we determine the size of local algebras if S is not constant?
- In which sense are the models interacting? Can one measure S?