Spectral analysis for convolution operators on groups

Rafael Tiedra de Aldecoa
Université de Cergy-Pontoise

We consider operators H_μ of convolution with measures μ on locally compact groups. We characterize the spectrum of H_μ by constructing auxiliary operators whose kernel contain the pure point and singular subspaces of H_μ, respectively. The proofs rely on commutator methods.

Construction of ground states and resonances in nonrelativistic Quantum Electrodynamics

Volker Bach
Universität Mainz

In the lecture I will present several algorithms to iteratively construct the ground state and resonance eigenvectors for atoms coupled to the (UV-cutoff) quantized radiation field (with UV cutoff), corresponding to the atomic ground and excited states.

The ground state construction is joint work with J. Froehlich and A. Pizzo, the resonance construction is joint work A. Pizzo and M. Shoufan.

The Aharonov-Bohm Effect in Three Dimensions and Time-Dependent Inverse Scattering Theory

Miguel Ballesteros
University of Helsinki

We compute the high-energy limit of the scattering operator for the Schrödinger equation with magnetic field in the exterior of a finite number of tori, using time-dependent methods. In the case of one torus we give a method for the reconstruction of the flux of the magnetic field across a cross section of the torus modulo 2π. Equivalently, we determine, modulo 2π, the difference in phase for two particles that travel to infinity, when one goes inside the hole and the other outside of it. In the general case the information that we can obtain on the fluxes, and on the differences of phases, depends in the relative location of the tori.

This is joint work with Ricardo Weder.

L^2-Sobolev estimates for the backscattering transform

Ingrid Beltiţă
IMAR

Consider the Scrödinger operator $H_v = -\Delta + v$ in $\mathbb{R}^n$, where the dimension $n \geq 3$ is odd and $v \in L^q(\mathbb{R}^n, \mathbb{R})$ is compactly supported. By definition, the backscattering transform Bv of v is, modulo a smooth term, the real part of the Fourier transform of the backscattering amplitude of H_v; then $v \to B(v)$ is an entire analytic mapping in $L^q_{\rm cpt}(\mathbb{R}^n)$. The $N : th$ term $B_N(v)$ in the Taylor expansion of $B(v)$ at $v = 0$ is given by a N-linear singular operator B_N. The main interest lies in the recovering of v, or the singularities of v, from $B(v)$, and this motivates the study of continuity properties of B_N in various (weighted) Sobolev spaces. The main result of this talk shows that $B_N(v)$, $N \geq 2$, is more regular then v, whenever v belongs to the L^2-Sobolev space $H_{(\sigma)}(\mathbb{R}^n)$, with $\sigma > (n-3)/2$. Moreover it turns out that, when $\sigma > (n-2)/2$, v and $B(v)$ differ by a $H_{(\sigma+\epsilon)}(\mathbb{R}^n)$-function, with $0 \leq \epsilon < 1$.

This is joint work with Anders Melin.

The integrated density of states in strong magnetic fields

Phillippe Briet
Université du Sud Toulon - Var

We consider three-dimensional Schrödinger operators with constant magnetic field and ergodic electric potentials. We study the asymptotic behaviour of the integrated density of states in strong magnetic fields, distinguishing between the asymptotics far from the Landau levels, and asymptotics near a given Landau level.

Flow equations for operators and non-autonomous evolution equations

Jean-Bernard Bru
Fakultät für Physik, Universität Wien

The so-called "flow equations for operators" are non-linear operator value first order differential equations. They were formally defined independently by Brockett in 1991 for symmetric real matrices to solve linear programming problems and by Wegner in 1994 to obtain diagonal Hamiltonians. The main interest of these differential equations is to realize an appropriate isospectral flow on a self-adjoint operator acting on Hilbert spaces. However, their mathematical foundations are still missing and rigorous results in that direction will be discussed in this talk.

A structural approach to unambiguous state discrimination

Dagmar Bruss
Düsseldorf University

Distinguishing two non-orthogonal states in an optimal way is a fundamental problem in quantum information theory. If one is not allowed to make an error, this task is called unambiguous state discrimination (USD). The optimal solution of USD (i.e. the one that minimizes the probability of an inconclusive outcome) is not yet generally known. We show that the optimal measurement is unique, and derive the solution for the four-dimensional case. Our approach is structural in the sense that it allows to understand the properties of different regimes of the solution.

This is joint work with Matthias Kleinmann and Hermann Kampermann.

The effect of time-dependent coupling on non-equilibrium steady states

Horia D. Cornean
Ålborg University

Consider (for simplicity) two one-dimensional semi-infinite leads coupled to a quantum well via time dependent point interactions. In the remote past the system is decoupled, and each of its components is at thermal equilibrium. In the remote future the system is fully coupled. We define and compute the non equilibrium steady state (NESS) generated by this evolution. We show that when restricted to the subspace of absolute continuity of the fully coupled system, the state does not depend at all on the switching. Moreover, we show that the stationary charge current has the same invariant property, and derive the Landau-Lifschitz and Landauer-Büttiker formulas.

This is joint work with H. Neidhardt (Berlin) and V.A. Zagrebnov (Marseille).

Manipulating entanglement

Nilanjana Datta
Cambridge University

Entanglement plays a fundamental role in quantum information processing and is regarded as a valuable, fungible resource, The practical ability to transform (or manipulate) entanglement from one form to

another is useful for many applications. Usually one considers the entanglement manipulation of states which are multiple copies of a given bipartite entangled state and requires that the fidelity of the transformation to (or from) multiple copies of a maximally entangled state approaches unity asymptotically in the number of copies of the original state. The optimal rates of these protocols yield two asymptotic measures of entanglement, namely, entanglement cost and distillable entanglement.

It is not always justified, however, to assume that the entanglement resource available, consists of states which are multiple copies, i.e., tensor products, of a given entangled state. More generally, an entanglement resource is characterized by an arbitrary sequence of bipartite states which are not necessarily of the tensor product form. In this seminar, we address the issue of entanglement manipulation for such general resources and obtain expressions for the corresponding entanglement cost and distillable entanglement.

New self-adjointness results for Dirac operators via Hardy-Dirac inequalities

Maria Esteban
Université Paris - Dauphine

In this talk I will present recent results with M. Loss in which distinguished selfadjoint extension of Dirac operators are constructed for a class of potentials including Coulombic ones up to the critical case, $-|x|^{-1}$. The method uses Hardy-Dirac inequalities and quadratic form techniques.

Dirichlet networks squeezed to graphs, with a bent tube example

Pavel Exner
Doppler Institute for Mathematical Physics and Applied Mathematics, Prague

The problem discussed in this talk comes from efforts to understand approximation of quantum graph Hamiltonians by Laplacians on "fat graphs". After reviewing the background and known result in both the Neumann and Dirichlet setting we discuss how quantum graphs with nontrivial spectral properties can be obtained from squeezed Dirichlet networks. To illustrate the propose strategy we work out the simplest nontrivial example, a family of bent tubes giving a graph of one vertex and two edges, or a two-parameter family of generalized point interactions on the line.

Universoccurrencence of Anderson localization

François Germinet
Université de Cergy-Pontoise

We shall review recent results concerning localization for random Schroedinger operators. This will include, in particular, localization for Poisson potentials (impurities are located in the space according to a Poisson process) as well as a for the Anderson model with any non degenerate probability distribution (this is a new result).

Positive commutators, Fermi golden rule and the spectrum of the 0 temperature Pauli-Fierz Hamiltonians. The spectral translation approach

Sylvain Golenia
Universität Erlangen-Nürnberg

In this talk, we consider 0 temperature Pauli-Fierz Hamiltonians under the assumption of the Fermi-Golden-Rule and show the absence of eigenvalue and a limit absorption principal at energy 0. As a tool, we use the extention of the Mourre's theory developed by Skibsted on one side and by Georgescu-Gerard-Möller on the other side. We compare our results with the one obtained by Bach-Froehlich-Sigal-Soffer and by Derezinski-Jaksic.

Fredholm determinants and counting statistics

Gian Michele Graf
ETH Zürich

Two electrodes, filled with independent electrons, are joined by a quantum dot. During some time interval, electrons are pumped through the dot. The object of interest is the statistical distribution of the number of electrons transferred between the leads in the process. We review the Levitov-Lesovik determinant expressing the generating function of the moments of that counting statistics. We discuss a convenient regularization, or normal ordering, of this infinite dimensional determinant, thereby ensuring it becomes a Fredholm determinant. We then discuss how the same result can be obtained without recourse to ad hoc regularizations by computing the counting statistics in the GNS representation appropriate to the Fermi sea, or to a positive temperature state.

On the Bardeen-Cooper-Schrieffer model of superfluidity for general pair interactions

Christian Hainzl
University of Alabama, Birmingham

Motivated by experiments on cold atoms we present a rigorous study about the BCS model of weakly interacting Fermi gases. We give a precise characterization of the class of potentials giving rise to superfluidity, i.e. for which the non-linear BCS gap-equation attains a non-trivial solution. Among other properties we further present an exact asymptotic formula for the critical temperature $T_c(\lambda V)$ in the weak coupling limit. By means of spectral theory we are able to improve the formula found in the Physic's literature. The talk is intended to be a review of several papers.

Tunnel effect for Fokker-Planck type operators and applications

Frédéric Hérau
Université de Reims
In a series of papers with M. Hitrik, C. Stolk and J. Sjöstrand we studied operators of Fokker-Planck type in the low temperature limit. In this talk we shall speak about the tunnel effect in some simple cases and explain the context of this study with some applications and models.

Some Mathematical Remarks on the Feynman Path Integral for the Non-relativistic Quantum Electrodynamics

Wataru Ichinose
Shinshu University

We give the mathematical definition of the Feynman path integral for the non-relativistic quantum electrodynamics under the constraint condition, following Feynman 1950 and Feynman and Hibbs 1965 by the time-slicing method. In addition, the Feynman path integral is also defined without the constraint condition, which may be completely new. It is proved that these two Feynman path integrals above are equal before taking the limit of the discretization parameter.

In the present paper the Fourier coefficients of electromagnetic potentials are quantized, which is a familiar method in physics, and photons with largemomentum are arbitrary cut off. Our study is different from the one for the model using the Fock spaces.

Eigenvalues and resonances - some results and comments

Arne Jensen
Ålborg University

I will review some of the work with G. Nenciu on perturbation of embedded eigenvalues, concentration on eigenvalues embedded in the continuum proper, and comment on the relation with other results in this area.

Repeated interaction quantum systems: deterministic and random

Alain Joye
Université de Grenoble

Consider a quantum system of reference interacting in sequence with the successive elements of an infinite chain of quantum sub-systems. When the elements of the chain and the interactions between these elements and the reference system are identical, we speak of deterministic repeated interaction quantum systems. We will also consider cases where the elements of the chain and/or the successive interactions with the reference system are random and we mainly focus on the dynamics of observables on the reference system. We show that the states on these observables almost surely converge asymptotically in time to a deterministic state, the properties of which will be discussed.

This is joint work with L. Bruneau and M. Merkli

The cubic non-linear Schrödinger equations in two dimensions

Rowan Killip
UCLA

I will outline a recent result on this equation as a model for technology that has been developed for dealing with nonlinear equations at critical regularity. The emphasis will be on the important properties of the linear Schrödinger equation that lie at the core of the argument.

This is joint work with T. Tao and M. Visan.

Lyapunov exponents and singular continuous spectrum

Frédéric Klopp
Université Paris 13

In this talk, we discuss the above exceptional sets and, in particular, the pointwise existence of the Lyapunov exponent for the ergodic family $\psi(n+1)+\psi(n-1) = 2\lambda e^{i\pi\omega/2}\cos(\pi(n\omega+\theta))\psi(n)$, $n \in \mathbb{Z}$. Here, $\lambda \in \mathbb{R}^+$ is a fixed coupling constant, $\omega \in (0,1)$ is a fixed frequency, and $\theta \in [0,1)$ is the ergodic parameter numbering the equations. This is actually the Almost Mathieu equation with zero spectral parameter (and complex coupling constant). This model is related to various self-adjoint models via a cocycle representation. The existence of the Lyapunov exponent and the behavior of the solution can be described quite explicitly. The techniques developed in this study can be naturally generalized to study difference almost periodic Schrödinger operators with general real analytic potentials.

This talk is based on joint work with A. Fedotov.

The periodic magnetic Schrödinger operators spectral gaps and tunneling effect

Yuri A. Kordyukov
Istitute of Matematics RAS, Ufa

Let M be a noncompact oriented manifold of dimension $n \geq 2$ equipped with a properly discontinuous action of a finitely generated, discrete group Γ such that M/Γ is compact. Suppose that $H^1(M,\mathbb{R}) = 0$.

Let g be a Γ-invariant Riemannian metric and $\mathbf{B}$ a real-valued Γ-invariant closed 2-form on M such that $\mathbf{B} = \mathbf{dA}$ for some real-valued 1-form $\mathbf{A}$ on M.

Consider a Schrödinger operator with magnetic potential $\mathbf{A}$:

$$H^h = (ih\,d + \mathbf{A})^*(ih\,d + \mathbf{A})$$

(here $h > 0$ is a semiclassical parameter, which is assumed to be small).

For any $x \in M$, denote by $B(x)$ the anti-symmetric linear operator on the tangent space T_xM associated with the 2-form $\mathbf{B}$. Consider the function $\mathrm{Tr}^+ B$ on M defined as $\mathrm{Tr}^+(B(x)) = \frac{1}{2}\,\mathrm{Tr}([B^*(x) \cdot B(x)]^{1/2})$.

Put $b_0 = \min\{\mathrm{Tr}^+(B(x)) : x \in M\}$.

Theorem. *Assume that there exist a (connected) fundamental domain $\mathcal{F}$ and a constant $\epsilon_0 > 0$ such that $\mathrm{Tr}^+(B(x)) \geq b_0 + \epsilon_0$ for any $x \in \partial\mathcal{F}$. Then, for any natural N, there exists $h_0 > 0$ such that*

$$\sigma(H^h) \cap [0, h(b_0 + \epsilon_0)]$$

has at least N gaps for any $h \in (0, h_0)$.

Theorem. *Under the assumptions of the previous theorem, suppose that there exists a zero $\bar{x}_0$ of B, $B(\bar{x}_0) = 0$, such that*

$$c^{-1}|x - x_0|^k \leq \mathrm{Tr}^+(B(x)) \leq c|x - x_0|^k$$

for all x in some neighborhood of x_0 with some $c > 0$ and some integer $k > 0$. Then, for any natural N, there exist $C > 0$ and $h_0 > 0$ such that

$$\sigma(H^h) \cap [0, Ch^{\frac{2k+2}{k+2}}]$$

has at least N gaps for any $h \in (0, h_0)$.

The proofs are based on the study of the tunneling effect for the corresponding quantum system.

This is a joint work with B. Helffer.

Resonances for Schrödinger operator with periodic plus compactly supported potentials

Evgeny Korotyaev
Humboldt Universität zu Berlin

We consider the Schrödinger operator with a periodic plus the compactly supported potentials on the half line (and the real line). We prove the following results:

1. a forbidden domain for the resonances is specified,
2. the distribution of resonances in the disk with large radius is determined,
3. the asymptotics of real resonances and eigenvalues in the gap are determined at high energy,
4. some properties of real resonances and eigenvalues in a gap are obtained.

Dynamical evolution of self-gravitating relativistic matter in Hartree theory

Enno Lenzmann
Massachusetts Institute of Technology

Pseudo-relativistic Hartree- and Hartree-Fock equations equations have recently found a significant application as effective descriptions for the dynamical evolution of self-gravitating, relativistic matter. Based

upon this physical motivation, I will present results that aim at understanding the qualitative behavior of solutions for these model equations. More precisely, I will discuss solitary wave solutions and their stability, as well as finite-time blowup of solutions. Such a singular breakdown indicates the "gravitational collapse" of the physical system (e. g. a white dwarf) modeled by this equation. These blowup results substantiate the intuitive picture of collapsing stellar matter that exceeds a critical total mass (Chandrasekhar's theory).

If time permits, I will discuss some future applications and open problems.

Part of the material covered in my talk is joint work with Jürg Fröhlich (ETH Zürich) and Lars Jonsson (KTH Stockholm).

The thermodynamic limit of quantum Coulomb systems: a new approach

Mathieu Lewin
Université de Cergy-Pontoise

I will review the methods for proving the existence of a thermodynamic limit for quantum systems composed of electrons and nuclei, like ordinary matter. I will also present a new approach (joint work with Christian Hainzl and Jan Philip Solovej) which provides a general setting for the study of many different quantum systems.

Quantum large deviations

Christian Maes
Katolische Universitet Leuven

We discuss the question of large deviations in quantum equilibrium systems. A first approach wants to describe the statistics in the outcome of macroscopic measurements. We discuss the question of equivalence of ensembles and we show a relation between the large deviation property and the boundedness of an entanglement length. A second approach consists in a quantum generalization of the Varadhan integral lemma of classical large deviations to a noncommutative context. Finally, we discuss the meaning of current fluctuations in a nonequilibrium setting.

This is joint work with Wojciech De Roeck and Karel Netocny.

Entanglement of formation for some special two-mode Gaussian states

Tudor A. Marian
University of Bucharest

We develop an insightful approach to the problem of evaluating the entanglement of formation for a two-mode Gaussian state. When its covariance matrix is in a scaled standard form, the corresponding pure-state decompositions are shown to be superpositions of a squeezed vacuum state and a classical Gaussian one. We establish the connection between the properties of the latter and the optimal squeezed vacuum state determining the entanglement of formation. The general case reduces to an algebraic system hard to exploit. However, for some especially interesting states we found its solution retrieving previous results in a unitary manner.

This is joint work with Paulina Marian.

Scattering theory for Jacobi operators with quasi-periodic background

Johanna Michor
Imperial College, London

Jacobi operators can be viewed as the discrete analogue of Sturm-Liouville operators. We will develop direct and inverse scattering theory for Jacobi operators which are short range perturbations of quasi-periodic finite-gap background operators. We derive the corresponding Gel'fand-Levitan-Marchenko

equation and find minimal scattering data which determine the perturbed operator uniquely. In addition, we show how the transmission coefficients can be reconstructed from the eigenvalues and one of the reflection coefficients.

Then we apply this knowledge to solve the associated initial value problem of the Toda hierarchy via the inverse scattering transform.

This talk is based on joint work with Iryna Egorova (Kharkov) and Gerald Teschl (Vienna).

Schrödinger operators with random δ magnetic fields

Takuya Mine
Kyoto Institute of Technology

We consider the Schrödinger operators on $\mathbf{R}^2$ with the magnetic field given by a constant plus random δ magnetic fields of the Poisson-Anderson type. We give a sufficient condition for each Landau level to be an infinitely degenerated eigenvalue, in terms of the average of the flux per unit area

Random colourings of quasiperiodic graphs: ergodic and spectral properties

Peter Müller
Universität Göttingen

We study randomly coloured graphs whose vertex sets are infinite, uniformly discrete subsets of finite local complexity in multi-dimensional Euclidean space. We construct the appropriate ergodic dynamical systems, explicit characterize ergodic measures and prove an ergodic theorem. For covariant operators of finite range defined on those graphs, we show existence and self-averaging of the integrated density of states, as well as non-randomness of the spectrum. The main result establishes Lifshits tails at the lower spectral edge of the graph Laplacian on percolation subgraphs for not too large bond probabilities inside the non-percolating phase. Among others, the proof requires exponential decay of the cluster-size distribution for percolation on rather general graphs.

This is joint work with Christoph Richard.

On Eisenbud's and Wigner's R-matrix A general approach

Hagen Neidhardt
Weierstraß-Institut für Angewandte Analysis und Stochastik, Berlin

The main objective of this paper is to give a rigorous treatment of Wigner's and Eisenbud's R-matrix method for scattering matrices of scattering systems consisting of two selfadjoint extensions of the same symmetric operator with finite deficiency indices. In the framework of boundary triplets and associated Weyl functions an abstract generalization of the R-matrix method is developed and the results are applied to Schrödinger operators on the real axis.

This is joint work with Jussi Behrndt (Technische Universität Berlin), Elena Roxana Racec (Technische Universität Cottbus and University of Bucharest, Paul Nicolae Racec (Institut für Angewandte Analysis und Stochastik, Berlin, and National Institute of Materials Physics, Bucharest Magurele) and Ulrich Wulf (Technische Universität Cottbus).

On the defocusing Ablowitz-Ladik equation

Irina Nenciu
Courant Institute

In this talk we will describe various properties of a completely integrable system, the Ablowitz-Ladik (AL) equation, through its connection to the theory of orthogonal polynomials on the unit circle. In particular, we will concentrate on its associated Poisson structure using a functional analytic approach; this in turn will allow us to define a multi-Hamiltonian structure for the AL hierarchy. As an important

consequence, we show that, in some of the new Poisson brackets, the classical map taking orthogonal polynomials on the unit circle into orthogonal polynomials on the real line becomes a symplectic mapping taking certain of the AL flows into flows from the Toda lattice hierarchy.

Semiclassical techniques and mean field limits

Francis Nier
Université Rennes 1

This joint work with Z. Ammari reviews the links between the mean field limit of bosonic dynamics and the semiclassical asymptotics. Recent results about the propagation of chaos in a mean field limit are intimately related with older ones about the propagation of coherent states. Accurate asymptotic expansions can be explicitly given while working with the good set of observables. Extending each of the three Wick, Anti-Wick and Weyl, quantizations to the infinite dimensional case has its own interest. Although this question has a long history, the relationships with the standard semiclassical or pseudo-differential techniques in finite dimension have not yet been exhausted. An improved description of the mean field limits can be derived from a good understanding of the different possible extensions of the semiclassical calculus.

This is joint work with Z. Ammari.

On short-range perturbations of spin-orbit Hamiltonians

Konstantin Pankrashkin
Humboldt Universität zu Berlin

We give an estimate from below for the number of eigenvalues for spin-orbit Hamiltonians (Rashba-or Dresselhaus-type) perturbed by short range potentials. It apperas that negative perturbations lead in general to an infinite numer of eigenvalues below the continuous spectrum, and this effect holds even for perturbations supported by zero-measure sets.

Classes of infinite order pseudodifferential operators

Mihai Pascu
Institute of Mathematics of the Romanian Academy

Pseudodifferential operators which symbol has an exponential growth with respect to the phase variable can be defined as operators acting on spaces of ultradistributions. They generalize the ultradifferential operators studied by H. Komatsu in the paper "Ultradistributions I. Structure theorems and a characterization", J. Fac. Sci. Univ. Tokyo, 20 (1973), 25-105. We present some classes of pseudodifferential operators, acting on Gevrey ultradistributions and we investigate the possibilities of defining such operators on Roumieu type spaces of tempered ultradistributions.

Fluctuation algebra for coupled fermionic systems

Yan Pautrat
Université Paris Sud

Following a joint project with Jaksic and Pillet we study the linear response of coupled fermionic systems close to thermodynamic equilibrium. The cornerstones of linear response are the Kubo formula, the Onsager reciprocity relations and the central limit theorem for fluctuations of observables.

In this talk we describe a general central limit theorem which allows to describe fluctuations of flux observables, is constructed. We show that these fluctuations have a classical joint law if and only if the system is at thermal equilibrium; in this case we obtain a rigorous fluctuation-dissipation theorem.

A Beals type criterion for pseudodifferential operators with a magnetic field

Radu Purice
IMAR

In a series of papers we have elaborated a covariant pseudodifferential calculus associated to the algebra of quantum observables of a system in a non-homogenuous magnetic field. In this talk I shall very briefly present the main ideas of this calculus, some of the main results that we obtained and I shall insist on a twisted variant of the Beal's criterion, formulated and proved in an algebraic framework that does not involve any given representation in a Hilbert space.

This is joint work with Viorel Iftimie, Marius Măntoiu and Serge Richard.

Linear response theory for open two-terminal quantum semiconductor systems

Paul N. Racec
Weierstrass Institute for Applied Analysis and Stochastics
and National Institute of Materials Physics, Bucharest Magurele

We show that a linear response theory in the frame of random phase approximation (RPA) can be applied to open quantum systems under a small, time-dependent bias superimposed to a stationary source-drain bias. The main ingredients of our linear response theory are: i) the unperturbed system is considered the stationary system under source-drain bias and is described in the frame of the Landauer Büttiker formalism, taking into account the Coulomb interaction in the Hartree approximation; ii) an ansatz for the statistical operator for the unperturbed system is presented, so that the expectation values for the particle density and density current are in agreement with the Landauer-Büttiker formalism; iii) the response functions (chare-charge and current-charge) can be calculated in linear approximation using the wave functions of the unperturbed system.

In such a way we can provide explicit RPA expressions for the quantum admittance of effectively one-dimensional open quantum systems. As a first example the case of a quantum capacitor is considered, for which low frequency experimental data are available. A comparison of the low-frequency expansion with the complete RPA-expression shows that for a quantum capacitor a simple classical equivalent circuit with frequency-independent elements does not describe satisfactorily the quantum-admittance with increasing the frequency.

This is joint work with E. R. Racec (Technische Universität Cottbus and University of Bucharest) and U. Wulf (Technische Universität Cottbus) and IHP/BTU JointLab, Cottbus.

On the spectrum of magnetic Dirac operators with Coulomb-type perturbations

Serge Richard
Université Lyon 1

During this talk, I will present the spectral analysis of singular matrix valued perturbations of 3-dimensional Dirac operators with variable magnetic field of constant direction. Under suitable assumptions on the magnetic field and on the perturbations, we will obtain a limiting absorption principle, prove the absence of singular continuous spectrum in certain intervals and state properties of the point spectrum. Constant, periodic as well as diverging magnetic fields are covered, and Coulomb potentials up to the physical nuclear charge $Z < 137$ are allowed. The importance of an internal-type operator (a 2-dimensional Dirac operator) will also be revealed.

Fourier integral operators with complex phase, a mathematical justification of the semiclassical Herman-Kluk propagator

Vidian Rousse
Freie Universität Berlin

We build a class of Fourier integral operators with complex-valued phase associated to a canonical transform of the Euclidean phase-space. The presented construction extends the Wick-quantization of pseudodifferential operators. We will focus on their application to evolution equations of the Schrdinger type and thus provide the first mathematical justification of the well-known Herman-Kluk propagator in the semiclassical context. The proof mainly relies on a L^2-boundedness result for those FIO analogous to the Calderón-Vaillancourt theorem.

On μ- scale invariant linear relations

Adrian Sandovici
Department of Mathematics, University of Bacău

The concept of μ-scale invariant operator with respect to a unitary transformation in a separable Hilbert space is extended to the case of linear relations (multi-valued linear operators). It is shown that if S is a nonnegative linear relation which is μ-scale invariant for some $\mu > 0$, then its adjoint S^* and its extremal nonnegative selfadjoint extensions are also μ-scale invariant.

Vortices and spontaneous symmetry breaking in rotating Bose gases

Robert Seiringer
Princeton University

We present a rigorous proof of the appearance of quantized vortices in dilute trapped Bose gases with repulsive two-body interactions subject to rotation. Starting from the many-body Schrödinger equation, we show that the ground state of such gases is, in a suitable limit, well described by the nonlinear Gross-Pitaveskii equation. In the case of axially symmetric traps, our results show that the appearance of quantized vortices causes spontaneous symmetry breaking in the ground state.

The mathematical model of the scattering operator in the stepwise waveguides

Ilya A.Shereshevskii
Institute for Physics of Microstructures RAS, Nizhnii Novgorod

The problem of modes transformation in non-homogeneous waveguides (as well as quantum and classical) has the very long history, and extensive list of physics and mathematics publications was devoted to this problem during last century. Nevertheless, the well-known and widely used term "scattering matrix" seems to be not yet mathematically well-defined in the waveguide problems, and, correspondingly, the mathematical properties of the scattering operator in waveguide (in any sense) are not well described. Such description is, however, necessary to understanding the processes in the quantum, electrodynamics and acoustics waveguides, and, in particular, to developing good numerical algorithms for waveguides simulation.

I consider in this work the mathematical model of the stepwise waveguide and demonstrate that the investigation of this model is, in essential, the problem of the theory of the selfadjoint extensions of the symmetric operators in the Hilbert spaces. In such approach the scattering operator appears naturally as the parameter of the selfadjoint extensions of some symmetric operator and, hence, the scattering operator is, from the very beginning, the operator in appropriate Hilbert space, namely, the deficiency space of the initial symmetric operator. This allows us to investigate the properties of the "scattering matrix" in frame of the operator theory.

Besides the construction of the scattering operator, we obtain in this way the description of some of its important properties. Some of them seem to be rather unexpected, as, for example, the fact that this operator may be, in general, unbounded in the Hilbert space of the sequences of modes amplitudes with usual scalar product. Also it is interesting that, under some conditions, the scattering operator may be approximated (in some well-defined sense) by finite-dimensional operators, which is scattering operators for appropriate "finite dimensional waveguide". This approximation conserves most of important properties of such operators, such as the flow conservation low, and can be used to numerical calculation of the scattering matrix for various types of waveguides.

I consider also some algebraic properties of the family of scattering operators, which allow us to construct new operators from existing ones. These properties are well-known in the finite dimensional case, and are very useful for investigation of multi-step waveguides.

The two spectra inverse problem for Jacobi matrices. Necessary and sufficient conditions.

Luis Octavio Silva Pereyra
Universidad Nacional Autonoma de Mexico

The talk presents necessary and sufficient conditions for two sequences to be the spectra of two elements of a rank-one perturbation family of a Jacobi operator. The cases of singular and bounded perturbations are discussed.

The results presented here were obtained jointly with R. Weder.

Two-body scattering at low energies

Erik Skibsted
Institut for Matematiske Fag, Århus Universitet

We give an account of various recent results obtained with Jan Dereziński on low-energy scattering for a class of long-range potentials containing the attractive Coulombic one. This includes the construction of wave operators of Isozaki–Kitada type diagonalizing the *whole* continuous part of the Hamiltonian. The corresponding S–matrix is strongly continuous (although not differentiable) at zero energy. We derive a relationship to the analogous Dollard type constructions, and show that the location of the singularities of the scattering kernel $S(\lambda)(\omega,\omega')$ experiences an abrupt change at $\lambda = 0$. Thus, for example, for the purely Coulombic case the set of singularities jumps (as the energy goes down) from the set of coinciding outgoing and incoming angles, $\omega = \omega'$, to the set of oppositely oriented angles, $\omega = -\omega'$, reflecting the fact that the classical orbits at zero energy in this case are parabolas.

Asymptotics of clusters of eigenvalues for suitable perturbations of the hydrogen atom

Carlos Villega Blas
Universidad Nacional Autonoma de Mexico

Let us consider the Schrödinger Hamiltonian operator $H_\hbar = H_0 + \epsilon Q_\hbar$ where H_0 is the Hamiltonian of the hydrogen atom (with the Planck parameter $\hbar$ included), $Q_\hbar$ is a pseudodifferential operator of order zero uniformly bounded in $\hbar$ and $\epsilon = O(\hbar^\delta)$ with $\delta > 1$. We show that by taking $\hbar = 1/N$ with N a natural number, the family of operators $H_{1/N}$ has well defined clusters of eigenvalues around the number $-1/2$. We obtain the limiting eigenvalue distribution of such clusters when N goes to infinity. The result involves the averages of the principal symbol of $Q_\hbar$ along the clasical orbits with fixed energy $E = -1/2$ in the phase space of the Kepler problem. The collision orbits are considered in our analysis.

Inverse scattering at a fixed energy

Ricardo Weder
University of Helsinki

We prove that the averaged scattering solutions to the Schrödinger equation with short-range electromagnetic potentials (V, A) where $V(x) = O(|x|^{-\rho}), A(x) = O(|x|^{-\rho})$, $|x| \to \infty, \rho > 1$, are dense in the set of all solutions to the Schrödinger equation that are in $L^2(K)$ where K is any connected bounded open set in $\mathbf{R}^n$, , $n \geq 2$, with smooth boundary.

We use this result to prove that if two short-range electromagnetic potentials (V_1, A_1) and (V_2, A_2) in $\mathbf{R}^n$, $n \geq 3$, have the same scattering matrix at a fixed positive energy and if the electric potentials V_j and the magnetic fields $F_j := \mathrm{curl} A_j, j = 1, 2$, coincide outside of some ball they necessarily coincide everywhere.

In a previous paper of Weder and Yafaev the case of electric potentials and magnetic fields in $\mathbf{R}^n$, $n \geq 3$, that are asymptotic sums of homogeneous terms at infinity was studied. It was proven that all these terms can be uniquely reconstructed from the singularities in the forward direction of the scattering amplitude at a fixed positive energy.

The combination of the new uniqueness result of this paper and the result of Weder and Yafaev implies that the scattering matrix at a fixed positive energy uniquely determines electric potentials and magnetic fields that are a finite sum of homogeneous terms at infinity, or more generally, that are asymptotic sums of homogeneous terms that actually converge, respectively, to the electric potential and to the magnetic field.

A Carleman type estimate for the Dirac operator and its application to inverse problems

Leo Tzou
University of Washington

Recently there has been increasing interest in inverse problems involving the Dirac operator

$$P_0(D) = \begin{pmatrix} 0 & \sigma \cdot D \\ \sigma \cdot D & 0 \end{pmatrix},$$

where $D = -i\nabla$ and $\sigma = (\sigma_1, \sigma_2, \sigma_3)$ is a vector of Pauli matrices with

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \ \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

In this talk we consider the matrix valued differential operator

$$P_V(D) := P_0(D) + V$$

and the homogenous boundary value problem associated with this operator. The question we ask is, knowing only certain information about the solutions on the boundary, what information can we recover about the matrix V?

Under infinitely smooth assumptions, a version of this question was answered in the paper by Nakamura et al. However, to do this under the minimal amount of regularity assumptions ($W^{1,\infty}$) on V requires us to use a completely different method of approach - one requiring a Carleman-type estimate for the matrix valued differential operator.

In this talk we discuss the idea behind this work and possible future research in this direction.

This work is done with the collaboration of Mikko Salo, University of Helsinki

Boson Gas with BCS Interaction

Valentin A. Zagrebnov
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and Centre de Physique Théorique, Marseille

We give a two parameter variational formula for the grand-canonical pressure of the BCS-Bose model (*Pair Boson Hamiltonian*) for attractive and repulsive BCS interaction. We show that exact solution of this model is described by quasi-free states and that it allows a coexistence of single-particle and pair condensations for the BCS attraction. In the case of repulsion there is no pair condensation and the single-particle condensation transforms into type III generalized Bose-Einsten condensationion.

This lecture is based on the joint projects with J. V.Pulé (UC Dublin) and A.F. Verbeure (KU Leuven)

Stability of pseudorelativistic systemsof n identical particles with permutational symmetry account

Grigorii Zhislin
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Let $H(n)$ be the energy operator of the system $Z(n)$, consisting of $n$identical pseudorelativistic particles with short-range interparticle interaction $V = V(|r_i - r_j|)$ and let $B(n,k)$ be the space of the system $Z(n)$ states, for which permutation symmetry with respect to permutations of particles coordinates is determined by Young schemes with scolumns, where $s = 1, 2, \dots k$ and k is an arbitrary fixed integer. We denote by $H(n,k)$ the restriction of the operator $H(n)$ to the space $B(n,k)$ and by $H(n,k;P)$ the operator $H(n,k)$ after introducing of the relative coordinates and fixing the total momentum $P$of the system $Z(n)$ particles. In this talk we prove that under some conditions on V there is exist such infinite sequence of the integers $1 < n_1 < n_2 < \dots < n_t < \dots$, $n_t = n_t(k)$ that

i) the system $Z(n_t)$ has the stable state from $B(n,k)$;

ii) the energy operator $H(n_t,k;P)$ has nonempty discrete spectrum for some values $P = P(n_t,k)$;

iii) for some constant $C$and all t, $n_{t+1} : n_t < C$.

Local smoothing in the presence of lots of trapping

Maciej Zworski
University of California, Berkeley

The Schrödinger propagator, $\exp(-it\Delta)$, on $\mathbf{R}^n$ is unitary on any Sobolev space so regularity is not improved in propagation. Remarkably, and as has been known for about 20 years, the regularity improves when we integrate in time and cut-off in space:

$$\int_0^T \|\chi \exp(-it\Delta)u\|^2_{H^{1/2}} dt \leq C\|u\|^2_{L^2}\,. \quad \chi \in C^\infty_{\rm c}(\mathbf{R}^n)\,,$$

and this much exploited effect is known as *local smoothing*.

The same is true on many other noncompact manifolds under *nontrapping* assumptions. In fact, any trapping (e.g. presence of closed geodesics) will destroy local smoothing. Using recent results obtained by Stéphane Nonnenmacher and the speaker we show that local smoothing with $H^{1/2}$ replaced by $H^{1/2-\epsilon}$ can be obtained under the assumptions on the dimension of the trapped set, or more generally on the topological pressure of the classical flow.