Title of the Project: Homogenization of a Thermal Problem with Flux Jump

PARTICIPANTS

- Renata Bunoiu, Maître de Conférences, Institut Élie Cartan de Lorraine, (Unité Mixte de Recherche 7502 du CNRS), Université de Lorraine - Metz, France.
- Claudia Timofte, Professor, Department of Theoretical Physics, Mathematics, Optics, Plasma and Lasers, Faculty of Physics, Bucharest University, Romania.

1 Brief Description of the Scientific Project

Homogenization of a Thermal Problem with Flux Jump

The goal of our project is to analyze, using homogenization techniques, the effective thermal transfer in a periodic composite material formed by two constituents, separated by an imperfect interface where both the temperature and the flux have a jump.

In the last decades, the study of the macroscopic properties of heterogeneous composite materials which exhibit imperfect contact between their constituents has been a subject of major interest for mathematicians, physicists, engineers, etc. In particular, the problem of thermal transfer in such heterogeneous media has attracted the attention of a broad category of researchers, due to the fact that the macroscopic properties of a composite can be affected by the imperfect bonding between its constituent components. This imperfect contact can be generated by various causes: the presence of a thin interphase, chemical processes, the presence of impurities at the boundaries, the interface damage, etc.

Let $\Omega$ be an open bounded material body in $\mathbb{R}^n (n \geq 2)$, with a Lipschitz-continuous boundary $\partial \Omega$. We assume that $\Omega$ is formed by two constituents, $\Omega_1^\varepsilon$ and $\Omega_2^\varepsilon$, representing two materials with different thermal characteristics, separated by an imperfect interface $\Gamma^\varepsilon$. We also assume that the phase $\Omega_1^\varepsilon$ is connected and reaches the external fixed boundary $\partial \Omega$ and that $\Omega_2^\varepsilon$ is not connected. Actually, $\Omega_2^\varepsilon$ is the union of domains of size $\varepsilon$, periodically distributed in $\Omega$ with periodicity $\varepsilon$ ($\varepsilon$ is a small real parameter related to the characteristic size of the two constituents).

Since the pioneering work [1], where a thermal problem in a two-component composite with interfacial barrier was studied for the first time by using asymptotic expansions, many mathe-
matical studies were performed, in order to rigorously justify the convergence results. Various mathematical methods were used: the energy method in [7], the two-scale convergence method in [4] and more recently the unfolding method for periodic homogenization in [3]. The main common point of all these studies is the fact that at the interface between the two components the flux of the temperature is continuous, the temperature field has a jump and the flux is proportional to this jump. Several cases are studied, following the order of magnitude with respect to the small parameter $\varepsilon$ of the resistance generated by the imperfect contact between the constituents. Moreover, the case when both components are connected is studied, too.

The main novelty brought by our project consists in allowing the presence of a jump in the flux across the imperfect interface, too. Following the ideas presented in [5] and [6], we will define suitable interface conditions corresponding to our problem. More precisely, the problem that we will study is the following one:

$$
\begin{align*}
-\text{div} \left( A^\varepsilon \nabla u^1_\varepsilon \right) &= f & \text{in } \Omega^1_\varepsilon, \\
-\text{div} \left( A^\varepsilon \nabla u^2_\varepsilon \right) &= f & \text{in } \Omega^2_\varepsilon, \\
A^\varepsilon \nabla u^1_\varepsilon \cdot \nu &= \varepsilon^\alpha h^\varepsilon (u^1_\varepsilon - u^2_\varepsilon) & \text{on } \Gamma^\varepsilon, \\
A^\varepsilon \nabla u^2_\varepsilon \cdot \nu &= \varepsilon^\alpha h^\varepsilon (u^1_\varepsilon - u^2_\varepsilon) + \varepsilon^\beta g^\varepsilon & \text{on } \Gamma^\varepsilon, \\
u^1_\varepsilon &= 0 & \text{on } \partial \Omega.
\end{align*}
$$

Here, $\alpha$ and $\beta$ are real parameters, $\alpha \leq 1$, $\nu$ is the unit outward normal to $\Omega^2_\varepsilon$, $f \in L^2(\Omega)$, $h$ and $g$ are positive bounded $Y$-periodic functions ($Y$ being the unit cell in $\mathbb{R}^n$) and

$$
h^\varepsilon(x) = h \left( \frac{x}{\varepsilon} \right), \quad g^\varepsilon(x) = g \left( \frac{x}{\varepsilon} \right) \text{ a.e. on } \Gamma^\varepsilon.
$$

Also, the matrix $A$ is supposed to be bounded, uniformly elliptic and

$$
A^\varepsilon(x) = A \left( \frac{x}{\varepsilon} \right) \text{ a.e. in } \Omega.
$$

Under the above hypotheses, problem (1) is well-posed.

Proving suitable energy estimates and using the unfolding operators introduced in [2] and developed in [3], we shall pass to the limit, with $\varepsilon \to 0$, in the corresponding variational formulation of problem (1) in order to get the homogenized problem. Various limit problems will be obtained, the most interesting one being the case corresponding to $\alpha = -1$ and $\beta = 1$. Corrector results will be obtained, too.

Both participants to this project already worked on problems with interfacial barriers and published independently (alone or with co-authors) a series of papers on this topic (see the articles written in bold in the lists of publications).
References


2 Activities to be Supported by the Project

We propose two research visits, with financial support needed for travel expenses, accommodation and local expenses:

- One research visit (10 days) to Metz of Claudia Timofte, 2015.

  **Required financial support** 1400 €, as follows:
  
  • 400 € for travel expenses (flight, train and bus tickets)
  • 1000 € for accommodation and local expenses.

- One research visit (7 days) to Bucharest of Renata Bunoiu, 2016.

  **Required financial support** 1100 €, as follows:
  
  • 400 € for travel expenses (flight, train and bus tickets)
  • 700 € for accommodation and local expenses.

**Total required financial support:** ≈ 2500 €.
3 Curricula Vitae

RENATA BUNOIU

Surname: Bunoiu (married Schiltz)
First Name: Renata Béatrice
Age: 44
Nationality: French
Marital Status: married, two children
Professional Address: Institut Elie Cartan, UMR 7502, Université de Lorraine, Ile du Saulcy, F-57045, Metz, cedex 01, FRANCE
E-mail: renata.bunoiu@univ-lorraine.fr
Position: Maître de Conférences, Université de Lorraine - Metz, since September 1998

Education
1997 PhD in Mathematics, Université de Metz (très honorable)
1994 D.E.A. de Mathématiques (mention bien), Université de Metz.

Research Interests
- Homogenization theory
- Asymptotic analysis in thin domains
- Theory of waveguides
- Spectral theory
- Scattering theory

Publications

OTHER PUBLICATIONS

PREPRINTS
• R. BUNOIU, R. PRECUP, "Vectorial approach to coupled nonlinear Schrödinger systems under nonlocal Cauchy conditions" (www.becasim.math.cnrs.fr), submitted.
CLAUDIA TIMOFTÉ

SURNAME: Timofte
FIRST NAME: Claudia
AGE: 50
NATIONALITY: Romanian
MARITAL STATUS: divorced, one child
PROFESSIONAL ADDRESS: Department of Theoretical Physics, Mathematics, Optics, Plasma and Lasers, Faculty of Physics, Bucharest University, Bucharest, P.O. Box MG-11, Romania
E-MAIL: claudia.timofte@g.unibuc.ro
POSITION: Professor, Bucharest University, since February 2008

EDUCATION

1996 PhD in Mathematics, IMAR, Bucharest, Romania
1988 Master of Science, Bucharest University, Faculty of Mathematics, Bucharest, Romania.

RESEARCH INTERESTS

• Homogenization theory
• Macrotransport processes
• Probabilistic methods in fluid dynamics
• Relaxation methods for optimization problems
• Upscaling in chemical reactive processes in porous media
• Mathematical models in biology and in crystallography

PUBLICATIONS

PAPERS IN REFEREED JOURNALS

• C. Timofte, ”Upscaling in Nonlinear Thermal Diffusion Problems”, PAMM, WILEY VCH Verlag, Volume 7, Issue 1. Published Online: Dec 12 2008.


• C. Timofte, "A macrotransport paradigm for thermal Taylor dispersion processes in periodic media", Romanian Reports in Physics, 52 (5-6-7), pp. 441-446, 2000.


PAPERS IN PROCEEDINGS OF INTERNATIONAL CONFERENCES

• C. Timofte, "On the effective behavior of some chemical reactive flows through porous media”, French-Romanian Colloquium on Applied Mathematics, Craiova, Romania, 2004.