APPLICATION

LABORATOIRE EUROPÉEN ASSOCIÉ CNRS FRANCO-ROUMAIN
2014

Title of the Project: *Inverse scattering for inhomogeneous media*

Participants

- Renata Bunoiu, Maître de Conférences, Institute Elie Cartan de Lorraine UMR7502
- Ingrid Beltiţă, CS III Institute of Mathematics of the Romanian Academy

1. Scientific Project

Inverse scattering for inhomogeneous media. Consider a layer in $\mathbb{R}^{n+1}$ $n \geq 2$, of the form $\Omega = \mathbb{R}^n \times (0, \pi)$. The acoustic wave propagation in a static inhomogeneous fluid, with no external forces, that occupies Ω and has constant density and the variable speed of sound c, is governed by the wave equation for the acoustic field,

$$\partial_t^2 U - c^2 \Delta U = F,$$
$$U(t, \cdot) = 0 \text{ on } \partial\Omega.$$

Here c is assumed to be a positive real function on Ω. Due to the invariance principle, this reduces further to the study of a special class of solutions for the equation

$$-c\Delta(cu) - k^2 u = 0 \text{ in } \Omega, \quad u = 0 \text{ on } \partial\Omega. \tag{1}$$

We will consider the scattering and inverse scattering problem for the equation (1) when $n \geq 2$, under certain conditions on c, namely that for every $y \in [0, \pi]$ and x tends ∞ in $\mathbb{R}^n$, we have

$$c(x, y) \to 1$$

sufficiently fast.

A similar problem in the Schrödinger case has been considered in [4] in the context of inverse scattering problems for ocean acoustics. In their case the potential is assumed to be compactly supported, and the data is in fact recovered from the Dirichlet-to-Neumann map, which is shown to be linked with a certain scattering data. The compactness condition on the potential is dropped in [2], where potential is determined from the scattering matrix at energies tending at infinity. We will follow this approach, by using methods in [1] to overcome the extra difficulty coming from the fact that our operator is a second order perturbation of the Laplacian with Dirichlet conditions on Ω.

The first step is to consider the direct problem and study spectral and scattering properties of the self-adjoint operator associated with equation (1). To this end we will use methods based on Mourre's conjugate operator method (see [3] and [1]) to obtain a good description of the scattering

matrix for the pair of Hamiltonians H and H_0, where H_0 is given by the Laplacian with Dirichlet conditions on Ω, and H is defined by

$$Hu = -c\Delta(cu) \quad \text{for } u \in L^2(\Omega) \text{ such that } cu \in H^2(\Omega) \cap H_0^1(\Omega).$$

Using the informations thus obtained on the scattering matrix, we will then consider the inverse scattering problem, namely, the problem of finding the speed of sound c from the knowledge of the scattering matrix at a set of energies that tend to infinity. The main difficulty in our problem comes from the existence of thresholds both for H_0 and H, which complicates the construction of exponentially growing solutions for the problem (1).

We will also study the case when $n = 1$. Here the difficulty comes from the fact that it leads to a problem on the line, which requires more data to be solved uniquely. In this case we may allow c to have two different limits when $x \to \pm\infty$, yet we will need to impose a stronger condition on the rate of convergence.

References

[1] I. Beltiţă, Inverse scattering in a layered medium. *Comm. Partial Differential Equations* **26** (2001), no. 9-10, 1739–1786.

[2] M. Cristofol, P. Gaitan, V. Iftimie, Inverse problems for the Schrödinger operator in a layer. *Rev. Roumaine Math. Pures Appl.* **50** (2005), no. 2, 153–180.

[3] Y. Dermenjian, M. Durand, V. Iftimie, Spectral analysis of an acoustic multistratified perturbed cylinder. *Comm. Partial Differential Equations* **23** (1998), no. 1-2, 141–169.

[4] M. Ikehata, G.N. Makrakis, G. Nakamura, Inverse boundary value problem for ocean acoustics using point sources. *Math. Methods Appl. Sci.* **27** (2004), no. 12, 1367–1384.

[5] C.H. Wilcox, *Sound propagation in stratified fluids.* Applied Mathematical Sciences **50**. Springer-Verlag, New York, 1984.

2. Activities to be supported by the project

We propose 4 research visits to IMAR and IECL, with financial support needed for travel expenses, and accommodation and local expenses:

- One research visit to Bucharest of Renata Bunoiu, 11-18 February 2015.

Required financial support 1000 €, as follows:

- 400 € for travel expenses (flight, train and bus tickets)
- 600 € for accommodation and local expenses.

- One research visit to Metz of Ingrid Beltita, 1 week, 2015

Required financial support 1000 €, as follows:

- 400 € for travel expenses (flight and train tickets)
- 600 € for accommodation and local expenses.

- One research visit to Bucharest of Renata Bunoiu, 1 week, 2016.

Required financial support 1000 €, as follows:

- 400 € for travel expenses (flight, train and bus tickets)
- 600 € for accommodation and local expenses.

- One research visit to Metz of Ingrid Beltita, 1 week, 2016.

Required financial support 1000 €, as follows:

- 400 €for travel expenses (flight and train tickets)

- 600 € for accommodation and local expenses.

Total required financial support: ≈ 4000 €.

3. Curricula Vitae

Renata Bunoiu

Surname : Bunoiu (married Schiltz)
First name: Renata Béatrice
Age : 44
Nationality : French
Marital status : married, two children
Professional address: Institute Elie Cartan, Université de Lorraine - Metz. Ile du Saulcy, F-57045, Metz, cedex 01, FRANCE
E-mail: renata.bunoiu@univ-lorraine.fr
Position : Maître de Conférences, Université de Lorraine - Metz, since September 1998
Education 1997 PhD in Mathematics, Université de Metz (trés honorable)

Thesis: *Sur quelques problémes mathématiques en mécanique des fluides.*
Supervisor Prof. Jeannine Saint Jean Paulin. 1994 D.E.A. de Mathématiques (mention bien), supported by a Tempus stipendium, at Université de Metz
Thesis: *Sur les équations de bioconvection.*
Supervisor Prof. Jeannine Saint Jean Paulin, Université de Metz.

Publications.

(1) D. Polişevski, R. Schiltz-Bunoiu, A. Stănescu, Homogenization cases of heat transfer in structures with interfacial barriers, To appear in *Bull. Math. Soc. Sci. Math. Roumanie.*
(2) R. Bunoiu, G. Cardone, S. Nazarov Scalar boundary value problems on junctions of thin rods and plates. I. Asymptotic analysis and error estimates. *ESAIM: Mathematical Modelling and Numerical Analysis* **48** (2014), no. 5, 1495-1528.
(3) D. Borisov, R. Bunoiu, G. Cardone Waveguide with non-periodically alternating Dirichlet and Robin conditions: homogenization and asymptotics. *Zeitschrift für angewandte Mathematik und Physik* **64** (2013), no. 3, 439-472.
(4) R. Bunoiu, G. Cardone. C. Perugia Unfolding method for the homogenization of Bingham flow. In *Modelling and Simulation in Fluid Dynamics in Porous Media*, J.A. Ferreira, S Barbeiro, G. Pena, M.F. Wheeler (Eds.), Springer Proceedings in Mathematics and Statistics **28**, New York, 107-122, 2013.
(5) D. Borisov, R. Bunoiu, G. Cardone, Homogenization and asymptotics for a waveguide with an infinite number of closely located small windows. Problems in mathematical analysis. No. 58. *J. Math. Sci.* (N. Y.) **176** (2011), no. 6, 774–785.
(6) R. Bunoiu, G. Cardone, T. Suslina, Spectral approach to homogenization of an elliptic operator periodic in some directions. *Math. Methods Appl. Sci.* **34** (2011), no. 9, 1075–1096.

(7) D. Borisov, R. Bunoiu, G. Cardone, On a waveguide with an infinite number of small windows. *C. R. Math. Acad. Sci. Paris* **349** (2011), no. 1-2, 53–56.
(8) D. Borisov, R. Bunoiu, G. Cardone, On a waveguide with frequently alternating boundary conditions: homogenized Neumann condition. *Ann. Henri Poincaré* **11** (2010), no. 8, 1591–1627.
(9) D. Polişevski, R. Schiltz-Bunoiu, Heat conduction through a first-order jump interface. In *New trends in continuum mechanics*, M. Mihăilescu- Suliciu, 225–230, Theta Ser. Adv. Math **3**, Theta, Bucharest, 2005,
(10) D. Polişevski, R. Schiltz-Bunoiu, D. Polisevski, Diffusion in an intermediate model of fractured porous media, *Bul. Ştiinţ., Univ. Piteşti*, Ser. Mat. Inf. **10** (2004), 99-106.
(11) R. Bunoiu, S. Kesavan, Asymptotic behaviour of a Bingham fluid in thin layers. *J. Math. Anal. Appl.* **293** (2004), no. 2, 405-418.
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(13) R. Bunoiu, Nonlinear viscous flow through a thin slab in the lubrication case. *Rev. Roumaine Math. Pures Appl.* **45** (2000), no. 4, 577–591 (2001).
(14) R. Bunoiu, J. Saint Jean Paulin, Linear flow in porous media with double periodicity. *Portugaliae Mathematica*, **56** (1999), 221–238.
(15) R. Bunoiu, J. Saint Jean Paulin, Fluide á viscosité non linéaire dans un domaine de faible épaisseur dans le cas de lubrification. *C. R. Acad. Sci. Paris*, t. **323**, Série I (1996), 1097–1102.

Other publications.

(1) R. Bunoiu, Sur les caractéristiques hydrauliques d'un module de stockage. Rapport interne Commissariat à l'énergie atomique, 1996.
(2) R. Bunoiu, Sur un probléme mathématique en mécanique des fluides. *Rev. de l'Assoc. Femmes et Mathématiques* (1997).
(3) R. Bunoiu, K. Taous, On a thermal problem in domains with cuts. Proceedings of the conference *Homogenization and applications to material sciences, September 15-19, 2001, Timişoara*, Editura Universităţii de Vest, Timisoara, 2001.

Preprints.

• I. Beltiţă, R. Bunoiu, Inverse scattering for the 1-D Helmholtz equation, preprint arXiv:math/0511401, submitted.

Lectures at international meetings.

(1) 28.05-31.05 1996 28-ème Congrés National d'Analyse Numérique, La Londe les Maures, France.
(2) 20.07-03.08 1996 Optimisation of nonlinear systems and of free boundaries - International Workshop, Constanta, Roumanie.
(3) 30.01-01.02 1997 Forum des Jeunes Mathématiciennes, Paris.
(4) 15.09-20.09 1997 Analysis and Control of Differential Systems - International Workshop, Constanta, Roumanie.

(5) 31.08-04.09 1998 4-ème Colloque Franco-Roumain, Metz.
(6) 11.02-31.02 1999 Summer school in Partial Differential Equations, Temuco, Chile.
(7) 22.03-24.03 1999 International Conference on Optimisation,Trèves, Germany.
(8) 12.04-16.04 1999 Congrès GAMM, Université de Metz.
(9) 17.05-21.05 1999 31-Congrès National d'Analyse Numérique, Bonascre, France.
(10) 15.09-19.09 2001 Homogenization and Applications to Material Sciences, Timisoara.
(11) 02.09-06.09 2002 6-ème Colloque Franco-Roumain, Perpignan, France.
(12) 10.10-11.10 2002 Interregional Congress in Mathematics, Namur, Belgium.
(13) 13.10-14.10 2005 Interregional Congress in Mathematics, Liège, Belgium.
(14) 24.07-28.07 2006 Asymptotic Behaviour in Fluid Mechanics, EPFL, Lausanne, Suisse.
(15) 28.08-02.09.2006 8-ème Colloque Franco-Roumain, Chambéry, France.
(16) 12.09-14.09.2011 Workshop on Fluid Dynamics in Porous Media, Coimbra, Portugal.
(17) 21.05-25.05.2012 7-the European Conference on Elliptic and Parabolic problems, Gaeta, Italie.
(18) 8.10-9.10.2013 3-rd Workshop on the Multiscale Models and Methods, Saint-Etienne, France.
(19) 26.05-30.05.2014 8-the European Conference on Elliptic and Parabolic Problems, Gaeta, Italy.
(20) 4.09-5.09.2014 BECASIM meeting, Lille, France.

Ingrid Alma Beltiţă

PERSONAL DATA.

- Name: Ingrid Alma Beltiţă
- Date of birth: 16 November 1970
- Address: Institute of Mathematics "Simion Stoilow" of the Romanian Academy, PO Box 1–764, Bucharest, Romania.
  e-mail: Ingrid.Beltita@imar.ro
  webpage: http://www.imar.ro/ ibeltita/

- Present position: Senior researcher 3, Institute of Mathematics "Simion Stoilow" of the Romanian Academy, since 2005.
- Previous employments:
  - 1995–1999 : Research Assistant, Institute of Mathematics "Simion Stoilow" of the Romanian Academy.
  - 1999–2005 : Researcher, Institute of Mathematics "Simion Stoilow" of the Romanian Academy

EDUCATION.

- Ph. D. in Partial Differential Equations, University of Bucharest (2005). Thesis: *Inverse scattering problems for stratified media*. Advisor Prof. V. Iftimie

RESEARCH INTERESTS.

- harmonic analysis, nilpotent Lie groups, partial differential equations, inverse problems.

EXPERIENCE.

- 19 research papers, 5 survey and proceedings papers, 4 preprints.

Lectures in International Meetings.

- *Local smoothing results for the backscattering transform.* Programme "Inverse Problems and Applications", Institut Mittag-Leffler (14 January - 15 May 2013).
- *Symbol compositions for the Weyl-Pedersen calculus on nilpotent Lie groups.* Colloque à la mémoire de Georges Pinczon, *Algèbres et super-algèbres de Lie et quantification par déformation.* Dijon, 24 - 26 octobre 2012.
- *Boundedness properties of the Weyl-Pedersen calculus*, Seminar on Harmonic Analysis, Bucharest, 21-22 September 2012.
- *Weyl-Pedersen calculus on coadjoint orbits of nilpotent Lie groups.* 9th International Conference on Harmonic Analysis and Partial Differential Equations, El Escorial, June 11-15, 2012.
- *Weyl-Pedersen calculus on coadjoint orbits of nilpotent Lie groups.* 10-ème Coll. Franco-Roumaine de Mathématiques Appliquées, 26–31 August 2010, Poitiers.
- *Magnetic Weyl calculus on coadjoint orbits of some semidirect products of Lie groups.* Generalized Functions GF2009, Wien, 31 August- 04 September 2009.
- *Local smoothing for the backscattering transform.* Second International Conference on Pseudo-Differential Operators and Related Topics, Växjö, 23–27 June 2008.
- *L^2-Sobolev estimates for the backscattering transformation.* 10th Quantum Mathematics International Conference, QMath10 Conference, Moieciu, September 10–15, 2007.
- *Multilinear estimates in backscattering theory*: Conference Operator Theory, Analysis and Mathematical Physics OTAMP 2006, 15–22 June 2006, Lund.
- *Inverse scattering problems for Schrödinger operators with magnetic fields.* Workshop Quantum Hamiltonians with Magnetic Fields, Bucharest, 8 –14 September 2002.
- *Mourre theory and inverse scattering problems for layered media*: Mini-Workshop in Mathematical Physics, Århus University, May 29, 2001.
- *On an abstract radiation condition*: Workshop Spectral and Scattering Theory and Related Fields, RIMS Kyoto, December 2000.
- *Inverse scattering problems for layered media*: Summer School New Analytic and Geometric Methods in Inverse Problems, July 2000, Heriot-Watt University, Edinburgh.
- *Spectral theory for Schödinger operators with boundary conditions on a half-space*: Colloque "New results in quantum mechanics and related topics" (September 1998, Bucharest)

Short courses.

- *Local smoothing for the backscattering transform*, ICMAT Madrid 2011, Special Trimester on Inverse Problems: Theoretical and Numerical Aspects of Inverse Problems and Scattering Theory.
- *Inverse problem of conductivity*, Ålborg, 2001.

Various.

- "Spiru Haret" Prize for Mathematics of the Romanian Academy, 2009, together with Anders Melin for the work in backscattering theory.

LIST OF PUBLICATIONS.

Papers in peer-reviewed journals.

(1) I. Beltiţă, D. Beltiţă, Inverse-closed algebras of integral operators on locally compact groups. To appear in Annales Henri Poincaré, doi: 10.1007/s00023-014-0320-x.

(2) I. Beltiţă, D. Beltiţă, Boundedness for Weyl-Pedersen calculus on flat coadjoint orbits. To appear in Int. Math. Res. Notices, doi: 10.1093/imrn/rnt225.

(3) I. Beltiţă, D. Beltiţă, On the differentiable vectors for contragredient representations. *C. R. Math. Acad. Sci. Paris* **351** (2013), no. 13-14, 513-516.

(4) I. Beltiţă, M. Măntoiu, Rieffel deformation and twisted crossed products. *Int. Math. Res. Notices.* **2014** (2014), 551-567.

(5) I. Beltiţă, D. Beltiţă, Faithful representations of infinite-dimensional nilpotent Lie algebras. *Forum. Math.* First published on line 10.1515/forum-2012-0085, September 2012.

(6) I. Beltiţă, D. Beltiţă, On differentiability of vectors in Lie group representations. *J. Lie Theory* **21** (2011), no. 4, 771-785.

(7) I. Beltiţă, D. Beltiţă, Algebras of symbols associated with the Weyl calculus for Lie group representations. Monatsh. Math. 167 (2012), no. 1, 13-33.

(8) I. Beltiţă, D. Beltiţă, Modulation spaces of symbols for representations of nilpotent Lie groups. *Journal of Fourier Analysis and Applications* **7** (2011), no. 2, 290-319.

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(11) I. Beltiţă, D. Beltiţă, Uncertainty principles for magnetic structures on certain coadjoint orbits. *Journal of Geometry and Physics* **60** (2010), no. 1, 81-95.

(12) I. Beltiţă, D. Beltiţă, Magnetic pseudo-differential Weyl calculus on nilpotent Lie groups. *Annals of Global Analysis and Geometry* **36** (2009), no. 3, 293-322.

(13) I. Beltiţă, A. Melin, The quadratic contribution to the backscattering transform in the rotation invariant case. *Inverse Problem and Imaging* **4** (2010), no. 4, 619-630.

(14) I. Beltiţă, A. Melin, Analysis of the quadratic term in the backscattering transformation. *Math. Scand.* **105**, no. 2 (2009), 218-234.

(15) I. Beltiţă, A. Melin, Local smoothing for the backscattering transform. *Commun. Part. Diff. Equations* **34** (2009), no. 1–3, 233-256.

(16) I. Beltiţă, H.D. Cornean, On a theorem of Arne Persson. *Cubo* **6** (2004), no. 2, 1-14.

(17) I. Beltiţă, Inverse scattering in a layered medium, *Commun. Partial Differ. Equations* **26** (2001), no. 9-10, 1739-1786.

(18) I. Beltiţă, Inverse scattering in a layered medium, *C.R. Acad Sci. Paris*, Sér. I, **329** (1999), 927-932.
(19) I. Beltiţă, Spectral theory for Schrödinger operators with boundary conditions on a half-space, *Rev. Roum. de Math. Pures et Appl.* **43** (1998), no. 7-8, 659–683.

Survey papers and proceedings.

(1) I. Beltiţă, D. Beltiţă, M. Pascu, Boundedness for pseudo-differential calculus on nilpotent Lie groups. In *Geometric Methods in Physics. XXXI Workshop Białowieża, Poland, June 24–30, 2012*, Kielanowski, P.; Ali, S.T.; Odesskii, A.; Odzijewicz, A.; Schlichenmaier, M.; Voronov, T. (Eds.) Birkhäuser, Trends in Mathematics, 87–97.
(2) I. Beltiţă, D. Beltiţă, On Weyl calculus in infinitely many variables. P. Kielanowski, V. Buchstaber, A. Odzijewicz, M. Schlichenmaier, Th. Voronov (eds.), *XXIX Workshop on Geometrical Methods in Physics*, AIP Conf. Proc., Amer. Inst. Phys., 1307, Melville, NY, 2010, pp. 19-26.
(3) I. Beltiţă, D. Beltiţă, A survey on Weyl calculus for representations of nilpotent Lie groups. S.T.Ali, P. Kielanowski, A. Odzijewicz, M. Schlichenmaier, Th. Voronov (eds.), *Proceedings of the XXVIII Workshop on Geometric Methods in Physics*, AIP Conf. Proc., Amer. Inst. Phys., Melville, NY, 2009.
(4) I. Beltită, A. Melin, Multilinear singular integral operators in backscattering. In *Mathematical Modeling of Wave Phenomena: 2nd Conference on Mathematical Modeling of Wave Phenomena*. AIP Conference Proceedings, Volume **834**, pp. 225-233, 2006.
(5) I. Beltiţă, On an abstract radiation condition. In RIMS Kukyuroku 1028, Spectral and Scattering Theory and Related Topics (2001).

Preprints:

(1) I. Beltiţă, D. Beltiţă, Coadjoint orbits of stepwise square integrable representations. Preprint arXiv: 1408.1857.
(2) I. Beltiţă, D. Beltiţă, M. Măntoiu, Quantization and dequantization via square-integrable families of operators. Preprint arXiv:1203.6347.
(3) I. Beltiţă, D. Beltiţă, On Kirillov's lemma for nilpotent Lie algebras. Preprint arXiv: 1308.3632.
(4) I. Beltită, A. Melin, L^1-estimates for the cubic term of the backscattering transform. Institute Mittag-Leffler Report No. 20, 2012/2013, Spring.
(5) I. Beltiţă, R. Bunoiu, Inverse scattering for the 1-D Helmholtz equation. Preprint arXiv:math/0511401