- PROJET DE RECHERCHE LEA -
BIFURCATION SETS AND HOLOMORPHIC ARCS

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Let $f : \mathbb{K}^n \to \mathbb{K}^p$, for $n > p > 0$ and $\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$, be a non-constant polynomial maps. It is well-known that f is a locally trivial topological fibration over the complement of the *bifurcation set* $B(f)$, also called *the set of atypical values*. The atypical values may come from the critical values but also from the asymptotic behaviour of the fibres. One can easily see this in the example $f(x, y) = x + x^2y$, where the value $0 \in \mathbb{K}$ is not critical but there is no trivial fibration in any neighbourhood of 0.

A complete characterization of $B(f)$ is available only in the case $n = 2$ and $p = 1$, see [Su] for $\mathbb{K} = \mathbb{C}$ and [TZ] for $\mathbb{K} = \mathbb{R}$. One has therefore imagined various ways to approximate $B(f)$, essentially through the use of *regularity conditions* at infinity. For $p = 1$ and $\mathbb{K} = \mathbb{C}$, Broughton worked with a Palais-Smale type condition called *tameness* [Br], extended by Némethi-Zaharia [NZ] and Rabier [Ra]. Parusinski used the *Malgrange condition* [Pa], which is a Łojasiewicz type condition at infinity, extended more recently to $p > 1$ in [Ra] and [KOS]. Siersma and Tibăr worked with the *t-regularity* (also called t-equisingularity) [ST], which is a type of non-characteristic condition at infinity, recently extended in [DRT].

In this project we shall consider the setting of polynomial maps $\mathbb{R}^n \to \mathbb{R}^{n-1}$ and $\mathbb{C}^n \to \mathbb{C}^{n-1}$. For some regular value a, we propose to find a criterion with "if and only if" as whether a is atypical or not. This would be an extension of the above cited results proved in [Su] and [TZ] for $n = 2$. This is also related to a question posed by Gurjar concerning an extension of some results of Zaidenberg [Za] in dimension $n = 2$.

In a recent preprint, J. Kollár and A. Némethi [KN] initiated the study of holomorphic arcs in complex spaces. These are holomorphic functions defined on a neighborhood of the closed unit disk $\overline{\mathbb{D}}$ with values in the closed complex space X. The topology on the space of hololomorphic arcs is defined as follows : we start with a metric d on X and we define a distance between two arcs ϕ and ψ by $\sup\{d(\phi(z), \psi(z) : z \in \overline{\mathbb{D}}\}$. A holomorphic arc is called short if the only point mapped into the singular locus of X is the origin.

We would like to study the structure of holomorphic arcs and short holomorphic arcs for 2-dimensional isolated singularities for which all the irreducible components of the exceptional divisor of a resolution of singularties are rational.

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