Research project submitted to be considered by LEA Math-Mode
Strichartz estimates for the Schrödinger equation on trees/graphs and applications

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1 Description of the project

Let us consider the linear Schrödinger equation (LSE):

$$\begin{cases} iu_t + u_{xx} = 0, \, x \in \mathbb{R}, \, t \neq 0, \\ u(0,x) = \varphi(x), \, x \in \mathbb{R}. \end{cases} \tag{1.1}$$

The linear equation (1.1) is solved by $u(x,t) = S(t)\varphi$, where $S(t) = e^{it\Delta}$ is the free Schrödinger operator. The linear semigroup has two important properties. First, the conservation of the L^2-norm:

$$\|S(t)\varphi\|_{L^2(\mathbb{R})} = \|\varphi\|_{L^2(\mathbb{R})} \tag{1.2}$$

and a dispersive estimate of the form:

$$|(S(t)\varphi)(x)| \leq \frac{1}{(4\pi|t|)^{1/2}}\|\varphi\|_{L^1(\mathbb{R})}, \ x \in \mathbb{R}, \ t \neq 0. \tag{1.3}$$

The space-time estimate

$$\|S(\cdot)\varphi\|_{L^6(\mathbb{R},\, L^6(\mathbb{R}))} \leq C\|\varphi\|_{L^2(\mathbb{R})}, \tag{1.4}$$

due to Strichartz [4], is deeper. It guarantees that the solutions of system (1.1) decay as t becomes large and that they gain some spatial integrability. Inequality (1.4) was generalized by Ginibre and Velo [3]. They proved the mixed space-time estimates, well known as Strichartz estimates:

$$\|S(\cdot)\varphi\|_{L^q(\mathbb{R},\, L^r(\mathbb{R}))} \leq C(q,r)\|\varphi\|_{L^2(\mathbb{R})} \tag{1.5}$$

for the so-called admissible pairs (q, r):

$$\frac{2}{q} + \frac{1}{r} = \frac{1}{2}. \tag{1.6}$$

In our research project we want to consider the Schrödinger equation on a network formed by the edges of a tree/graph. In the particular case of a tree having the property that each internal node has two children nodes and the last generation of edges is formed by infinite strips L. Ignat has obtained similar estimates to those mentioned above (1.5). In previous joint works [2, 1] the authors extended the results to the case of a general tree with Kirchhoff's connection and more general to connections of δ-type.

The main goal of our project is to extend the class of trees/graphs where these dispersive properties of Strichartz type hold and to apply these estimates to solve some nonlinear Schrödinger equations on the considered network.

The problems we address here enter in the framework of quantum graphs. The name quantum graph is used for a graph considered as a one-dimensional singular variety and equipped with a differential operator. These quantum graphs arise as simplified models in mathematics, physics, chemistry, and engineering (e.g., nanotechnology and microelectronics), when one considers propagation of waves through a quasi-one-dimensional system that looks like a thin neighborhood of a graph. We can mention in particular the quantum wires and thin waveguides. We refer to the survey paper of Kuchment, *Quantum graphs: an introduction and a brief survey*, 2008, and the references therein for more information on this topic.

2 Budget needed for the project

During the duration of this project, the two researchers involved in its development plan to make two visits. The expenses are detailed below.

1. Visit of Valeria Banica to Institute of Mathematics "Simion Stoilow" - one week in 2014
   (a) travel= 300 €
   (b) accommodation=7 x 60€=420€
   (c) daily allowance= 7x35 €=245 €
   (d) TOTAL=965€
2. Visit of Liviu Ignat to Université d'Evry- two weeks in 2014
   (a) travel= 300 €
   (b) accommodation=14 x 90€=1260€
   (c) daily allowance= 14x35 €=490 €
   (d) TOTAL=2050€
3. TOTAL = 3015 €.

References

[1] V. Banica and L. I. Ignat. Dispersion for the schr̈dinger equation on the line with multiple dirac delta potentials and on delta trees. *submitted to APDE.*

[2] V. Banica and L. I. Ignat. Dispersion for the Schrödinger equation on networks. *J. Math. Phys.*, 52(8):083703, 14, 2011.

[3] J. Ginibre and G. Velo. The global Cauchy problem for the nonlinear Schrödinger equation revisited. *Ann. Inst. H. Poincaré Anal. Non Linéaire*, 2(4):309–327, 1985.

[4] R.S. Strichartz. Restrictions of Fourier transforms to quadratic surfaces and decay of solutions of wave equations. *Duke Math. J.*, 44:705–714, 1977.