Biset functors, saturated fusion systems, and modular representation theory

Report 2013

January 31, 2014

The collaboration of the French team, composed of Serge BOUC and Radu STANCU, and the Romanian team, composed of Andrei MARCUS and Constantin-Cosmin TODEA continued in 2013 with the visit of the French team in Cluj, during the second half of April 2013. The mathematical discussion was axed around the concepts of biset functors, saturated fusion systems and block algebras.

We had very fruitful discussions around the generalization of Mislin's theorem to fusion systems. For a subgroup H of a group G this theorem says that an isomorphism on mod-p cohomology induced by the inclusion $H \leq G$ implies the control of p-fusion of H in G. Constantin-Cosmin TODEA was already very advanced in his work on an algebraic proof of this theorem and he recently posted a preprint on this subject on the ArXiv, (arXiv:1311.3944). Similar ideas on this subject and some straightforward properties of cohomology of fusion systems are now published by Constantin-Cosmin TODEA in Journal of Algebra.

Another subject discussed during this visit is the extension of simple cohomological Mackey functors. Serge Bouc and Radu Stancu have an older preprint on this subject giving a presentation by generators and relations of the extension algebra of some simple cohomological functors over a field of characteristic p. While discussing in Cluj they understood that it is enough to prove the results over the primitive field $\mathbb{F}_p$ and, by abstract nonsense, generalize them to any field of characteristic p. This simplifies greatly the arguments used in the proofs.

Finally, the Romanian and French teams continued their discussion on the application of biset functors, domain that has Serge Bouc as one of the world specialists, to modular representation theory. In particular we tried to understand connections between the representation functors of a block and of its Brauer correspondent. One question was how to characterise the Mackey functors that are also biset functors. The answer to this question is given by Serge Bouc in his 'Fused Mackey functors' paper. We intend to continue the investigation in this area.

Both Romanian and French teams participated to the 6th Algebra Symposium, organized at the University Babes-Bolyai, from 19 to 20 April 2013. Andrei Marcus was among the organisers and Serge Bouc, Constantin-Cosmin Todea and Radu Stancu were among the speakers. For the members of the French team this was a great opportunity to interact with other participants at the conference.

Our French-Romanian team does not have yet publications or pre-publications in this project but the members of the team have the following accepted publications in 2013, which are related to the research theme of our current collaboration.

- Bouc, Serge; A conjecture on B-groups. Math. Zeitschrift 274 Issue 1-2 (2013), 367-372
- Bouc, Serge; Fused Mackey functors. to appear in Geometriae Dedicata.
- Bouc, Serge; Stancu, Radu; Thévenaz, Jacques; Vanishing evaluations of simple functors. J Pure Appl Algebra 218 (2014) 218-227
- Bouc, Serge; Stancu, Radu; Thévenaz, Jacques; Simple biset functors and double Burnside ring. J. Pure Appl. Algebra 217 (2013), no. 3, 546-566.
- Ragnarsson, Kári; Stancu, Radu; Saturated fusion systems as idempotents in the double Burnside ring. Geom. Topol. 17 (2013), no. 2, 839-904.
- Todea, Constantin-Cosmin; A theorem of Mislin for cohomology of fusion systems and applications to block algebras of finite groups. arXiv:1311.3944
- Todea, Constantin-Cosmin; On cohomology of saturated fusion systems and support varieties. J. Algebra 402 (2014), 83-91.
- Coconeţ, Tiberiu; Todea, Constantin-Cosmin; The extended Brauer quotient for N-interior G-algebras. J. Algebra 396 (2013), 10-17.