

Bisets, fusion systems, and modular representation theory

Report 2012

December 21, 2012

The collaboration of the French team, composed of Serge BOUC and Radu STANCU, started this year with the visit of the Romanian team, composed of Andrei MARCUS and Cosmin TODEA in Amiens for one week, from the 10th to the 16th of September. As presented in our collaboration project, the mathematical discussion was axed around the concepts of bisets, fusion systems and block algebras.

We studied a recent article by Yaraneri [14]. In this article, for a biset functor M on a class of finite groups X , the author constructs bijections between some classes of maximal (respectively, simple) subfunctors of M and some classes of maximal (respectively, simple) $K\text{Out}(H)$ -submodules of the evaluation of M at a group in X . We aim to adapt the ideas in this article to the case where X is the set of subgroups of a given finite p -group D and the bisets involved are the one given by the conjugation morphisms in the fusion category of a block having D as defect.

Another direction of our mathematical discussion is based on the papers of Kawata, Okuyama and Uno cited in the bibliography. In [5, 6] Kawata study the Auslander-Reiten quiver of a block of a group algebra and prove that if a connected component of this quiver contains a trivial source lattice then the tree class of the stable part of this connected component is A_∞ . In [7, 8] Okuyama and Uno study the distribution of vertices of indecomposable modules in non-periodic components of the quiver and show that there are only very few distinct vertices in such a component. We aim to understand which part of this information could come from the fusion structure of the block and what is the relation with the associated biset functors.

Our French-Romanian team does not have yet publications or pre-publications in this project but the members of the team have the following accepted publications in 2012, related to the research theme of our current collaboration.

- Bouc, Serge; The slice Burnside ring and the section Burnside ring of a finite group. *Compos. Math.* 148 (2012), no. 3, 868-906.
- Bouc, Serge; Stancu, Radu; ThĀŠvenaz, Jacques; Simple biset functors and double Burnside ring. *J. Pure Appl. Algebra* 217 (2013), no. 3, 546-566.
- Ragnarsson, Kári; Stancu, Radu; Saturated fusion systems as idempotents in the double Burnside ring. To appear in *Geometry and Topology*.
- Todea, Constantin-Cosmin; Symmetric algebras, skew category algebras and inverse semigroups. *J. Algebra* 369 (2012), 226-234.

References

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- [4] S. Kawata, The modules induced from a normal subgroup and the Auslander-Reiten quiver, *Osaka J. Math.* 27 (1990), 265-269.
- [5] S. Kawata, On Auslander-Reiten components for certain group modules, *Osaka J. Math.* 30 (1993), 137-157.
- [6] S. Kawata, On Auslander-Reiten components and splitting trace lattices for integral group rings. *J. Algebra* 359 (2012), 69-79.
- [7] T. Okuyama and K. Uno, On vertices of Auslander-Reiten sequence, *Bull. London Math. Soc* 22 (1990), 153-158.
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- [13] E. Yaraneri, Ergün. Clifford theory for Mackey algebras. *J. Algebra* 303, No. 1, 244-274 (2006).
- [14] E. Yaraneri, Ergün. Composition factors of functors. *J. Algebra* 335, No. 1, 113-162 (2011).