Report on the project *Strichartz estimates for the Schrödinger equation on trees/graphs and applications*

V. Banica and L. Ignat

Participants. Valeria Banica (Université d'Evry) and Liviu Ignat (Institute of Mathematics "Simion Stoilow")

1 Description of the project

Let us consider the linear Schrödinger equation (LSE):

$$\begin{cases} iu_t + u_{xx} = 0,\ x \in \mathbb{R},\ t \neq 0, \\ u(0,x) = \varphi(x),\ x \in \mathbb{R}. \end{cases} \tag{1.1}$$

The linear equation (1.1) is solved by $u(x,t) = S(t)\varphi$, where $S(t) = e^{it\Delta}$ is the free Schrödinger operator. The linear semigroup has two important properties. First, the conservation of the L^2-norm:

$$\|S(t)\varphi\|_{L^2(\mathbb{R})} = \|\varphi\|_{L^2(\mathbb{R})} \tag{1.2}$$

and a dispersive estimate of the form:

$$|(S(t)\varphi)(x)| \leq \frac{1}{(4\pi|t|)^{1/2}}\|\varphi\|_{L^1(\mathbb{R})},\ x \in \mathbb{R},\ t \neq 0. \tag{1.3}$$

The space-time estimate

$$\|S(\cdot)\varphi\|_{L^6(\mathbb{R},\, L^6(\mathbb{R}))} \leq C\|\varphi\|_{L^2(\mathbb{R})}, \tag{1.4}$$

due to Strichartz [3], is deeper. It guarantees that the solutions of system (1.1) decay as t becomes large and that they gain some spatial integrability. Inequality (1.4) was generalized by Ginibre and Velo [2]. They proved the mixed space-time estimates, well known as Strichartz estimates:

$$\|S(\cdot)\varphi\|_{L^q(\mathbb{R},\, L^r(\mathbb{R}))} \leq C(q,r)\|\varphi\|_{L^2(\mathbb{R})} \tag{1.5}$$

for the so-called admissible pairs (q,r):

$$\frac{2}{q} + \frac{1}{r} = \frac{1}{2}. \tag{1.6}$$

In our research project we want to consider the Schrödinger equation on a network formed by the edges of a tree/graph. In the particular case of a tree having the property that each internal node has two children nodes and the last generation of edges is formed by infinite strips L. Ignat has obtained similar estimates to those mentioned above (1.5). The main idea behind its result is that solving the linear Schrödinger equation on such a structure could be reduced to solve an equation with finitely many piecewise constant coefficients on the whole real line and to apply the previous results of Banica [1]. However, the proof at this stage is restricted to a very particular structure of the tree. The main goal of our project is to extend the class of trees/graphs where these dispersive properties of Strichartz type hold and to apply these estimates to solve some nonlinear Schrödinger equations on the considered network.

The problems we address here enter in the framework of quantum graphs. The name quantum graph is used for a graph considered as a one-dimensional singular variety and equipped with a differential operator. These quantum graphs arise as simplified models in mathematics, physics, chemistry, and engineering (e.g., nanotechnology and microelectronics), when one considers propagation of waves through a quasi-one-dimensional system that looks like a thin neighborhood of a graph. We can mention in particular the quantum wires and thin waveguides. We refer to the survey paper of Kuchment, *Quantum graphs: an introduction and a brief survey*, 2008, and the references therein for more information on this topic.

2 Results

Between 16/10/2000 and 20/11/2010 Liviu Ignat visited Henri Poincare Institute in Paris with partial support from LEA Math-Mode project. V. Banica also visited IMAR between 20/11/2010 and 27/11/2010. During those visits we analyzed the proposed subject and we hope that it will be submitted early to publication.

There is one preprint containing the partial results we obtained until now.

1. V. Banica, L. Ignat, DISPERSION FOR QUANTUM WIRES

There is another paper accepted that entry in proposed research theme:

2. L. Ignat, D. Stan, DISPERSIVE PROPERTIES FOR DISCRETE SCHRÖDINGER EQUATIONS, accepted in J. Fourier Analysis and Application

References

[1] V. Banica. Dispersion and Strichartz inequalities for Schrödinger equations with singular coefficients. *SIAM J. Math. Anal.*, 35(4):868–883 (electronic), 2003.

[2] J. Ginibre and G. Velo. The global Cauchy problem for the nonlinear Schrödinger equation revisited. *Ann. Inst. H. Poincaré Anal. Non Linéaire*, 2(4):309–327, 1985.

[3] R.S. Strichartz. Restrictions of Fourier transforms to quadratic surfaces and decay of solutions of wave equations. *Duke Math. J.*, 44:705–714, 1977.