

Report - 2010

Existence of travelling waves for nonlocal reaction - diffusion equations

LEA Math Mode project

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Results. We have studied integro-differential operators from Mathematical Biology, which correspond to nonlocal reaction-diffusion equations of the type

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + F(u, J(u)),$$

where

$$J(u) = \int_{-\infty}^{\infty} \phi(x-y) u(y, t) dy,$$

$\phi : \mathbb{R} \rightarrow \mathbb{R}$ is a bounded nonnegative function, with compact support. A travelling wave solution for the above nonlocal reaction-diffusion equations has the form $u(x, t) = w(x - ct)$. It satisfies the equation

$$w'' + cw' + F(w, J(w)) = 0,$$

with the limits at infinity $w(\pm\infty) = w_{\pm}$, where the constant $c \in \mathbb{R}$ is the wave speed and should be found together with the function w . The Fredholm property and the properness in weighted Holder spaces have been proved for the integro-differential operator from the left-hand side of the above equality [1]. This allows one to define the topological degree. We have worked with the linearized operators and have employed the associated limiting operators. These methods and results will be applied to the study of travelling wave solutions of the nonlocal reaction-diffusion equations in the case when $J(w, J(w)) = f(w)J(w) - g(w)$ [2]. To do this, we combine the Leray-Schauder method with a priori estimates.

Elaborated papers:

1. N. Apreutesei, V. Volpert, Properness and topological degree for non-local reaction-diffusion operators, submitted.
2. N. Apreutesei, V. Volpert, Existence of travelling waves for a class of integro-differential equations from population dynamics, in preparation.

Travels:

During 2010, the Romanian participant in the project has visited Institut Camille Jordan (August 25 - September 6). A part of the paper [1] was discussed and written at that moment.

Proposal for 2011:

We will continue our collaboration in 2011. We intend to finish paper [2] and to go on with systems of integro-differential equations. Fredholm property, properness and existence of travelling waves will be studied for the nonlinear operators which govern this system.

We require mutual visits to continue our work: one visit of the Romanian participant in France (2 weeks in October) and one visit of the French participant in Romania (one week in May). One estimates the total cost of the two travels for 2011: 3000 euros.