ECCM 2010
IV European Conference on Computational Mechanics
Palais des Congrès, Paris, France, May 16-21, 2010

Approximation Results for Quasistatic Contact Problems

M. Cocou[1], L. Badea[2]

[1] *Laboratoire de Mécanique et d'Acoustique C.N.R.S. and Aix-Marseille Université, U.F.R.-M.I.M., Marseille, France, cocou@lma.cnrs-mrs.fr*
[2] *Institute of Mathematics of the Romanian Academy, Bucharest, Romania, Lori.Badea@imar.ro*

This work deals with the mathematical and numerical analysis of a system of evolution variational inequalities representing a unified approach to quasistatic contact problems in linear elasticity, which constitutes a generalization of the cases studied in [4] and [5].

The general results presented here are applied to a quasistatic unilateral contact problem with nonlocal friction but various quasistatic contact problems can be analyzed, including unilateral contact, normal compliance conditions with friction or more complex interaction laws, as, for example, interface laws coupling unilateral contact, adhesion and nonlocal friction between two elastic bodies [6].

Using an implicit time discretization scheme and some estimates of the incremental solutions, approximation and existence results are proved for a class of abstract implicit evolution variational inequalities involving a nonlinear operator.

To solve the general elliptic quasi-variational inequalities of the second kind that are obtained by the previous incremental procedure, some subspace correction algorithms are introduced, for which global convergence is proved and error estimates are established.

If the subspaces are the finite element spaces of the fine grid associated with a decomposition of the domain, or with the space corresponding to the coarse mesh, these algorithms are in fact one- and two-level Schwarz methods which are shown to have an optimal convergence rate, depending on the overlapping and mesh parameters. Schwarz methods are widely applied for solving linear problems, because they provide robust and efficient solution methods but their generalization to non-linear problems as, for example, quasi-variational inequalities, is not straightforward.

Finally, we remark that applications of these methods to other types of convex sets in general abstract spaces and to monotone minimizing functionals have been presented, e.g., in [1], [2]. Also, the case where the inequality contains extra terms which do not stem from the minimization of a functional has been investigated in [3].

References

[1] L. Badea, Convergence rate of a multiplicative Schwarz method for strongly nonlinear inequalities, in: *Analysis and optimization of differential systems*, V. Barbu, I. Lasiecka, D. Tiba, C. Varsan Editors, Kluwer Academic Publishers, 2003, pp. 31–42.

[2] L. Badea, Convergence rate of a Schwarz multilevel method for the constrained minimization of non-quadratic functionals, *SIAM J. Numer. Anal.*, 44, 2006, pp. 449–477.

[3] L. Badea, Schwarz methods for inequalities with contraction operators, *J. Comp. Appl. Math.*, 215, 2008, pp. 196–219.

[4] A. Capatina, M. Cocou, M. Raous, A class of implicit variational inequalities and applications to frictional contact, *Math. Meth. Appl. Sci.*, 32, 2009, pp. 1804–1827.

[5] M. Cocou, E. Pratt, M. Raous, Formulation and approximation of quasistatic frictional contact, *Int. J. Engrg. Sci.*, 34, 1996, pp. 783–798.

[6] M. Raous, L. Cangémi, M. Cocou, A consistent model coupling adhesion, friction, and unilateral contact, *Comput. Meth. Appl. Mech. Engrg.*, 177, 1999, pp. 383–399.